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EQUIVALENT VIBRATION SYSTEM FOR RECTANGULAR TLD WITH VIBRATION AROUND VERTICAL AXIS

Teruhiko TAKANISHI[1], Toshiya SONODA[2] and Hiroshi TADA[3]

[1]Member of JSCE, Dr. of Engineering, Professor, Dept. of Civil Engineering, Kyushu Institute of Technology (Sensui-cho 1-1, Tobata-ku, Kitakyushu-shi 804 , Japan)
[2]Member of JSCE, Associate Professor, Dept. of Civil Engineering, Oita National College of Technology (Oaza-maki 1666, Oita-shi 870-01 , Japan)
[3]Member of JSCE, Technician, Dept. of Civil Engineering, Kyushu Institute of Technology (Sensui-cho 1-1, Tobata-ku, Kitakyushu-shi 804 , Japan)

Knowledge of an equivalent vibration system for water mechanism in TLD is useful in calculating earthquake response of structures having TLD. Equivalent vibration systems have already been well researched concerning horizontal vibration of rectangular and cylindrical TLD. When a structure, the axes of which rigidity and gravity do not align, is struck by an earthquake, TLD vibrates around a vertical axis with the motion of the structure. In this paper, an equivalent vibration system is obtained theoretically for when a rectangular TLD vibrates around a vertical axis. Then, an equivalent vibration system for water in rectangular TLD is obtained by experiment. Finally, by comparing them with the theoretical value and confirming the agreement of both values, the validity of the theoretical equations is shown.

Key Words *: equivalent vibration system, rectangular TLD, rotation*

1. INTRODUCTION

Recently, by installing a vibration controller named TLD (Tuned Liquid Damper) onto the main towers of bridges, skyscrapers or high towers, attempts to damp off vibrations when these structures suffer from earthquakes or strong winds have been made. Up to now, however, the study on vibration control by TLD has mainly focused on horizontal vibration both theoretically and experimentally [1),2),3),4),5)].

The authors [6),7),8),9)] have theoretically studied and reported on the case of stationary and non-stationary vibrations in one horizontal direction or two horizontal directions by using a three-story frame model, and have found that the theoretical values are in agreement with those obtained by experiments within a weak non-linear region. When the two centers of gravity and rigidity of high buildings or towers do not coincide with each other, the structures, subjected to earthquakes or strong winds, suffer from rotational vibration around the vertical axis as well as horizontal vibration.

Therefore, when installed on a structure in which the gravitational center is not coincident with the center of rigidity, the TLD not only rotates but also vibrates horizontally at the same time. In calculating the response of a structure with TLD which rotates around a vertical axis, it would be beneficial if an equivalent vibration system of rotating TLD is known.

In this paper, first, an equivalent vibration system of rectangular TLD which rotates around a vertical axis is obtained theoretically. Next, by performing the experiment using TLD for the damped-free vibration which rotates around a vertical axis, an equivalent vibration system is obtained. Finally, by confirming that the two results correspond quite well, the validity of the theory is shown.

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2. THEORY OF AN EQUIVALENT VIBRATION SYSTEM OF TLD

At first, the free vibration of a rectangular tank as shown in **Fig.1** is considered. L is the length of

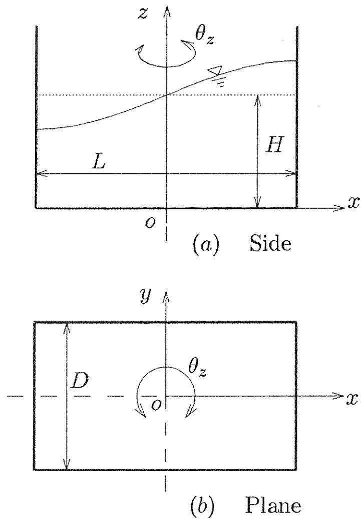

Fig. 1 Coordinates of Rectangular TLD

the tank, D is the width and the water level is H . The tank is regarded as rigid, the contained liquid is assumed to not have compression and viscocity, and there is no rotating motion. Furthermore, in this paper, the case in which a contained liquid vibrates linearly is considered. The case of non-linearity has yet to be studied.

Generally, the vibration of the liquid in the tank can be divided into two types of vibration modes, namely, one corresponding to the natural vibration mode (henceforth called sloshing mode) and the other corresponding to the vibration mode of a rigid body (rigid vibration mode). In this paper, when a rectangular tank rotates around a vertical axis,as shown in **Fig.1**, the equivalent moment of inertia and rotational spring constant for these two types of modes are calculated and an equivalent vibration system for the contained liquid is obtained.

(1) The Case of Sloshing Mode

When dynamic water pressure in the liquid is represented by σ , the basic equation for the free vibration is shown as

$$\frac{\partial^2\sigma}{\partial x^2}+\frac{\partial^2\sigma}{\partial y^2}+\frac{\partial^2\sigma}{\partial z^2}=0 \qquad (1)$$

The boundary conditions at both side-walls are shown as follows:

$$\left.\frac{\partial\sigma}{\partial x}\right|_{x=\frac{L}{2}}=\left.\frac{\partial\sigma}{\partial x}\right|_{x=-\frac{L}{2}}=0 \qquad (2)$$

$$\left.\frac{\partial\sigma}{\partial y}\right|_{y=\frac{D}{2}}=\left.\frac{\partial\sigma}{\partial y}\right|_{y=-\frac{D}{2}}=0 \qquad (3)$$

The boundary conditions at the bottom and free surface are represented by the following equations:

$$\left.\frac{\partial\sigma}{\partial z}\right|_{z=0}=0 \qquad (4)$$

$$\left(\frac{\partial^2\sigma}{\partial t^2}+g\frac{\partial\sigma}{\partial z}\right)_{z=H}=0 \qquad (5)$$

where g is gravitational acceleration.

When the (i,j)th natural circular frequency is n_{ij} ; the amplitude of the normal coordinates is Ψ_{ij} ; the vibration mode of dynamic water pressure is $\Gamma_{ij}(x,y,z)$ and the density of the contained liquid is ρ , the solution of Eq.(1) which satisfies the conditions of Eqs.(2)～(5) can be shown as

$$\sigma=\sum_i\sum_j\sigma_{ij}(x,y,z)$$
$$=\sum_i\sum_j n_{ij}^2\Psi_{ij}\Gamma_{ij}(x,y,z)\sin n_{ij}t \qquad (6)$$

where,

$$\Gamma_{2i-1,2j-1}(x,y,z)=\frac{\rho H}{\kappa_{2i-1,2j-1}H}\times\frac{\sin\lambda_{2i-1}x\sin\mu_{2j-1}y\cosh\kappa_{2i-1,2j-1}z}{\sinh\kappa_{2i-1,2j-1}H} \qquad (7)$$

$$\Gamma_{2i-1,2j}(x,y,z)=\frac{\rho H}{\kappa_{2i-1,2j}H}\times\frac{\sin\lambda_{2i-1}x\cos\mu_{2j}y\cosh\kappa_{2i-1,2j}z}{\sinh\kappa_{2i-1,2j}H} \qquad (8)$$

$$\Gamma_{2i,2j-1}(x,y,z)=\frac{\rho H}{\kappa_{2i,2j-1}H}\times\frac{\cos\lambda_{2i}x\sin\mu_{2j-1}y\cosh\kappa_{2i,2j-1}z}{\sinh\kappa_{2i,2j-1}H} \qquad (9)$$

$$\Gamma_{2i,2j}(x,y,z)=\frac{\rho H}{\kappa_{2i,2j}H}\times\frac{\cos\lambda_{2i}x\cos\mu_{2i}y\cosh\kappa_{2i,2j}z}{\sinh\kappa_{2i,2j}H} \qquad (10)$$

$$\lambda_i=\frac{i}{L}\pi\quad,\quad\mu_j=\frac{j}{D}\pi \qquad (11)$$

$$\kappa_{i,j}=\sqrt{\lambda_i^2+\mu_j^2} \qquad (12)$$

In addition, when x,y and z components of the (i,j)th vibration mode are assumed as $U_{ij}(x,y,z),V_{ij}(x,y,z)$ and $W_{ij}(x,y,z)$ respectively, the displacements of water particle, u,v,w, in the direction of x,y,z can be expressed as follows: That is to say, by using σ in Eq.(6), the equation of motion of the liquid is integrated twice with respect to the time .

$$u=\sum_i\sum_j u_{ij}(x,y,z,t)$$
$$=\sum_i\sum_j\Psi_{ij}U_{ij}(x,y,z)\sin n_{ij}t \qquad (13)$$

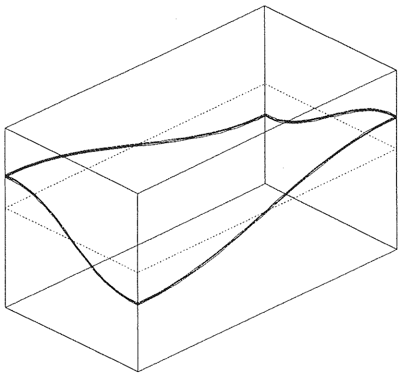

Fig. 2 (1,1)th Vibration Mode

$$v = \sum_i \sum_j v_{ij}(x, y, z, t) = \sum_i \sum_j \Psi_{ij} V_{ij}(x, y, z) \sin n_{ij} t \quad (14)$$

$$w = \sum_i \sum_j w_{ij}(x, y, z, t) = \sum_i \sum_j \Psi_{ij} W_{ij}(x, y, z) \sin n_{ij} t \quad (15)$$

When the equivalent moment of inertia of the (i, j)th mode is shown as J_{ij}^z; the equivalent spring constant is k_{ij}^z and the equivalent rotational angular amplitude is a_{ij}^z, the relationship among them can be expressed by the following equations:

$$M_{ij}^{z,max} = k_{ij}^z a_{ij}^z \quad (16)$$

$$T_{ij}^{z,max} = \frac{1}{2} M_{ij}^{z,max} a_{ij}^z \quad (17)$$

$$n_{ij}^2 = \frac{k_{ij}^z}{J_{ij}^z} \quad (18)$$

where, $T_{ij}^{z,max}$ is maximum kinetic energy T_{ij}^z of the liquid of the (i, j)th mode and expressed as

$$T_{ij}^z = \frac{1}{2}\rho \int_0^H \int_{-\frac{D}{2}}^{\frac{D}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left\{ \dot{u}_{ij}^2 + \dot{v}_{ij}^2 + \dot{w}_{ij}^2 \right\} dx\, dy\, dz \quad (19)$$

Also, $M_{ij}^{z,max}$ is the maximum moment of hydrodynamic water pressure against the walls around the vertical axis in **Fig.1** and M_{ij}^z can be calculated from the following equation using Eq.(6):

$$M_{ij}^z = \int_0^H \int_{-\frac{L}{2}}^{\frac{L}{2}} \left\{ \sigma_{ij}\left(x, \frac{D}{2}, z, t\right) x - \sigma_{ij}\left(x, -\frac{D}{2}, z, t\right) x \right\} \times dx\, dz$$
$$+ \int_0^H \int_{-\frac{D}{2}}^{\frac{D}{2}} \left\{ -\sigma_{ij}\left(\frac{L}{2}, y, z, t\right) y + \sigma_{ij}\left(-\frac{L}{2}, y, z, t\right) y \right\} \times dy\, dz \quad (20)$$

The maximum moment of dynamic water pressure against the walls becomes 0 for the $(2i-1, 2j)$th, $(2i, 2j-1)$th and $(2i, 2j)$th modes. Therefore, the case of only $(2i-1, 2j-1)$th moment of dynamic water pressure is left as non-zero value, which is expressed as

$$M_{2i-1,2j-1}^{z,max} = \frac{4(-1)^{i+j} \rho\, n_{2i-1,2j-1}^2 \Psi_{2i-1,2j-1}}{\lambda_{2i-1}^2 + \mu_{2j-1}^2} \times \left(\frac{1}{\lambda_{2i-1}^2} - \frac{1}{\mu_{2j-1}^2} \right) \quad (21)$$

The $(1, 1)$th vibration mode of the TLD liquid is shown in **Fig.2**, as an example.

If $T_{ij}^{z,max}$ and $M_{ij}^{z,max}$ are given, $n_{i,j}$ can be obtained by using Eq.(22).

$$n_{2i-1,2j-1}^2 = g\kappa_{2i-1,2j-1} \tanh \kappa_{2i-1,2j-1} H \quad (22)$$

Hence, from Eqs.(16)~(18), the equivalent moment of inertia $J_{2i-1,2j-1}^z$ and the equivalent rotational spring constant $k_{2i-1,2j-1}^z$ can be obtained. They can be expressed as follows:

$$J_{2i-1,2j-1}^z = 64\rho LDH \left(\frac{LD}{H}\right)^2 \times \frac{(\kappa_{2i-1,2j-1}H) \tanh(\kappa_{2i-1,2j-1}H)}{\{(2i-1)(2j-1)\pi^2\}^4} \times \left\{ \frac{\left(\frac{2j-1}{2i-1}\right)^2 - \left(\frac{D}{L}\right)^2}{\left(\frac{2j-1}{2i-1}\right)^2 + \left(\frac{D}{L}\right)^2} \right\}^2 \quad (23)$$

$$k_{2i-1,2j-1}^z = J_{2i-1,2j-1}^z n_{2i-1,2j-1}^2 \quad (24)$$

Concerning a square tank ($D = L$), as to the equivalent moment of inertia, $J_{ij}^z = 0$ when $i = j$.

Now, as shown in **Fig.6**, when a rectangular tank is rotated around a vertical axis with the angular acceleration of $\ddot{\theta}_z(t)$, the response is obtained by the following equations, assuming that the normal coordinate for the $(2i-1, 2j-1)$th mode are set as $\xi_{2i-1,2j-1}(t)$.

$$J_{2i-1,2j-1}^z \ddot{\xi}_{2i-1,2j-1}(t) + k_{2i-1,2j-1}^z \xi_{2i-1,2j-1}(t) = -J_{2i-1,2j-1}^z \ddot{\theta}_z(t) \quad (25)$$

The displacements of water particle in the direction x, y and z are

$$u_{2i-1,2j-1}(x, y, z, t) = \frac{\Psi_{2i-1,2j-1}}{a_{2i-1,2j-1}^z} \times U_{2i-1,2j-1}(x, y, z) \xi_{2i-1,2j-1}(t) \quad (26)$$

$$v_{2i-1,2j-1}(x, y, z, t) = \frac{\Psi_{2i-1,2j-1}}{a_{2i-1,2j-1}^z} \times V_{2i-1,2j-1}(x, y, z) \xi_{2i-1,2j-1}(t) \quad (27)$$

$$w_{2i-1,2j-1}(x,y,z,t)=\frac{\Psi_{2i-1,2j-1}}{a^z_{2i-1,2j-1}}\times W_{2i-1,2j-1}(x,y,z)\xi_{2i-1,2j-1}(t) \quad (28)$$

And the dynamic water pressure is

$$\begin{aligned}\sigma_{2i-1,2j-1}(x,y,z,t) &= -\rho\int\frac{\partial^2}{\partial t^2}\{w_{2i-1,2j-1}(x,y,z,t)\}\,dz \\ &= -\frac{\Psi_{2i-1,2j-1}}{a^z_{2i-1,2j-1}}\Gamma_{2i-1,2j-1}(x,y,z)\ddot{\xi}_{2i-1,2j-1}(t)\end{aligned} \quad (29)$$

The moment of dynamic water pressure against the walls caused by the rotation around the vertical axis can be expressed as follows, integrating Eq.(29) by using σ in the Eq.(20):

$$\begin{aligned}M^z_{2i-1,2j-1}(t) &= -\frac{\Psi_{2i-1,2j-1}}{a^z_{2i-1,2j-1}}\ddot{\xi}_{2i-1,2j-1}(t) \\ &\times\Bigg[\int_0^H\int_{-\frac{L}{2}}^{\frac{L}{2}}\Big\{\Gamma_{2i-1,2j-1}(x,\frac{D}{2},z)x - \Gamma_{2i-1,2j-1}(x,-\frac{D}{2},z)x\Big\}dxdz \\ &-\int_0^H\int_{-\frac{D}{2}}^{\frac{D}{2}}\Big\{\Gamma_{2i-1,2j-1}(\frac{L}{2},y,z)y - \Gamma_{2i-1,2j-1}(-\frac{L}{2},y,z)y\Big\}dydz\Bigg] \\ &= -J^z_{2i-1,2j-1}\ddot{\xi}_{2i-1,2j-1}(t)\end{aligned} \quad (30)$$

If Eq.(25) is taken into consideration, the above equation can be written as

$$M^z_{2i-1,2j-1}(t)= k^z_{2i-1,2j-1}\xi_{2i-1,2j-1}(t)+J^z_{2i-1,2j-1}\ddot{\theta}_z(t) \quad (31)$$

From this, it can be found that $M^z_{2i-1,2j-1}(t)$ consists of the total sum of the moment proportional to the normal coordinates $\xi_{2i-1,2j-1}(t)$ and the moment proportional to the rotational angular acceleration $\ddot{\theta}_z(t)$.

(2) The Case of Rigid Vibration Mode

As is shown in **Fig.1**, when a rectangular TLD rotates around a vertical axis with the angular displacement $\theta_z(t)=\Theta_z\sin\omega_z t$, dynamic water pressure σ can be obtained by solving the basic eq.(1) under the following boundary conditions:

On both walls,

$$\left.\frac{\partial\sigma}{\partial x}\right|_{x=\frac{L}{2}}=\left.\frac{\partial\sigma}{\partial x}\right|_{x=-\frac{L}{2}}=-\rho\omega_z^2\Theta_z y\sin\omega_z t \quad (32)$$

$$\left.\frac{\partial\sigma}{\partial y}\right|_{y=\frac{D}{2}}=\left.\frac{\partial\sigma}{\partial y}\right|_{y=-\frac{D}{2}}=\rho\omega_z^2\Theta_z x\sin\omega_z t \quad (33)$$

The boundary conditions of the bottom and the free surface are the same as Eqs.(4) and(5).

The solution σ which satisfies the basic equation and the boundary conditions can be obtained by this equation:

$$\begin{aligned}\sigma = \omega_z^2\Theta_z\sin\omega_z t\Bigg[&\sum_{j=0}^{N_{0x}}A_{0,2j+1}\sin\nu_{x0,2j+1}x \times\sin\mu_{2j+1}y\cosh\eta_0 z \\ &+\sum_{j=N_{0x}+1}^{\infty}A_{0,2j+1}\sinh\nu_{x0,2j+1}x\sin\mu_{2j+1}y\cosh\eta_0 z \\ &+\sum_{s=1}^{\infty}\sum_{j=0}^{\infty}A_{s,2j+1}\sinh\nu_{xs,2j+1}x\sin\mu_{2j+1}y\cos\eta_s z \\ &+\sum_{j=0}^{N_{0y}}B_{0,2j+1}\sin\nu_{y0,2j+1}y\sin\lambda_{2j+1}x\cosh\eta_0 z \\ &+\sum_{j=N_{0y}+1}^{\infty}B_{0,2j+1}\sinh\nu_{y0,2j+1}y\sin\lambda_{2j+1}x\cosh\eta_0 z \\ &+\sum_{s=1}^{\infty}\sum_{j=0}^{\infty}B_{s,2j+1}\sinh\nu_{ys,2j+1}y \times\sin\lambda_{2j+1}x\cos\eta_s z\Bigg]\end{aligned} \quad (34)$$

where,

$$\nu^2_{xs,2j+1}=\eta_s^2+\mu^2_{2j+1} \quad (s=1,2,\cdots\ ,\ j=1,2,\cdots) \quad (35)$$

$$\nu^2_{ys,2j+1}=\eta_s^2+\lambda^2_{2j+1} \quad (s=1,2,\cdots\ ,\ j=1,2,\cdots) \quad (36)$$

$$\eta_0 H\tanh\eta_0 H=\frac{H}{g}\omega_z^2 \quad (37)$$

$$\eta_s H\tan\eta_s H=-\frac{H}{g}\omega_z^2\ ,\ (s=1,2,\cdots) \quad (38)$$

$$\lambda_{2j+1}=\frac{2j+1}{L}\pi\ ,\ \mu_{2j+1}=\frac{2j+1}{D}\pi \quad (j=1,2,\cdots) \quad (39)$$

When $\eta_0>\mu_{2j+1}$, $\nu^2_{x0,2j+1}=\eta_0^2-\mu^2_{2j+1}$ (40)

Then,

$$A_{0,2j+1}=-16(-1)^j\rho\frac{L}{D}\frac{1}{\mu^2_{2j+1}} \times\frac{\sinh\eta_0 H}{(\nu_{x0,2j+1}L\cos\nu_{x0,2j+1}\frac{L}{2})(\sinh 2\eta_0 H+2\eta_0 H)} \quad (j=1,2,\cdots,N_{0x}) \quad (41)$$

N_{0x} is maximum value of j satisfying $\eta_0>\mu_{2j+1}$.

When $\eta_0<\mu_{2j+1}$, $\nu^2_{x0,2j+1}=-\eta_0^2+\mu^2_{2j+1}$ (42)

Then,

$$A_{0,2j+1}=-16(-1)^j\rho\frac{L}{D}\frac{1}{\mu^2_{2j+1}}$$

$$\times \frac{\sinh \eta_0 H}{(\nu_{x0,2j+1} L \cosh \nu_{x0,2j+1} \frac{L}{2})(\sinh 2\eta_0 H + 2\eta_0 H)}$$
$$(j = N_{0x}+1, N_{0x}+2, \cdots) \quad (43)$$

And,

$$A_{s,2j+1} = -16(-1)^j \rho \frac{L}{D} \frac{1}{\mu^2_{2j+1}}$$
$$\times \frac{\sin \eta_s H}{(\nu_{xs,2j+1} L \cosh \nu_{xs,2j+1} \frac{L}{2})(\sin 2\eta_s H + 2\eta_s H)}$$
$$(s = 1, 2, \cdots \ , \ j = 1, 2, \cdots) \quad (44)$$

When $\eta_0 > \lambda_{2j+1}$, $\nu^2_{y0,2j+1} = \eta_0^2 - \lambda^2_{2j+1}$ (45)

Then,

$$B_{0,2j+1} = 16(-1)^j \rho \frac{D}{L} \frac{1}{\lambda^2_{2j+1}}$$
$$\times \frac{\sinh \eta_0 H}{(\nu_{y0,2j+1} D \cos \nu_{y0,2j+1} \frac{D}{2})(\sinh 2\eta_0 H + 2\eta_0 H)}$$
$$(j = 1, 2, \cdots, N_{0y}) \quad (46)$$

N_{0y} is the maximum value j satisfying $\eta_0 > \lambda_{2j+1}$.

When $\eta_0 < \lambda_{2j+1}$, $\nu^2_{y0,2j+1} = -\eta_0^2 + \lambda^2_{2j+1}$ (47)

Then,

$$B_{0,2j+1} = 16(-1)^j \rho \frac{D}{L} \frac{1}{\lambda^2_{2j+1}}$$
$$\times \frac{\sinh \eta_0 H}{(\nu_{y0,2j+1} D \cosh \nu_{y0,2j+1} \frac{D}{2})(\sinh 2\eta_0 H + 2\eta_0 H)}$$
$$(j = N_{0y}+1, N_{0y}+2, \cdots) \quad (48)$$

And,

$$B_{s,2j+1} = 16(-1)^j \rho \frac{D}{L} \frac{1}{\lambda^2_{2j+1}}$$
$$\times \frac{\sin \eta_s H}{(\nu_{ys,2j+1} D \cosh \nu_{ys,2j+1} \frac{D}{2})(\sin 2\eta_s H + 2\eta_s H)}$$
$$(s = 1, 2, \cdots \ , \ j = 1, 2, \cdots) \quad (49)$$

Assuming that the moment of dynamic water pressure against the walls by rotational motion around the vertical axis is $\bar{M}^z_{0\omega}$, the moment can be calculated by referring to Eq.(20) and using Eq.(34).

Thus, the obtained $\bar{M}^z_{0\omega}$ is expressed as

$$\bar{M}^z_{0\omega} = -\bar{J}^z_{0\omega}\ddot{\theta}_z = \omega_z^2 \Theta_z \bar{J}^z_{0\omega} \sin \omega_z t \quad (50)$$

and in the above equation, if $\omega_z \to 0$, it can be described as

$$\bar{M}^z_0 = -\bar{J}^z_0 \ddot{\theta}_z \quad (51)$$

where, $\bar{J}^z_0$ means the equivalent moment of inertia around the vertical axis for the rigid vibration mode, which is expressed as

$$\bar{J}^z_0 = \frac{32\rho H}{\pi^5} \sum_{j=0}^{\infty} \frac{1}{(2j+1)^5}$$
$$\times \left\{ L^4 \tanh\left(\frac{\lambda D}{2}\right) + D^4 \tanh\left(\frac{\mu L}{2}\right) \right\}$$

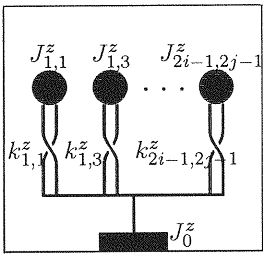

Fig. 3 Equivalent Vibration System of Rotational Vibration

$$-\frac{1}{12}\rho DHL(L^2 + D^2) \quad (52)$$

(3) Equivalent Moment of Inertia for Fixed Water

The equivalent moment inertia proportional to the rotational angular acceleration of the liquid, or the equivalent moment of inertia for fixed water, J^z_0, can be obtained as follows:

The moment of total dynamic water pressure against the walls around the vertical axis as concerns sloshing vibration is described as follows, by Eq.(31):

$$M^z_\infty = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} M^z_{2i-1,2j-1} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}$$
$$\left\{ k^z_{2i-1,2j-1}\xi_{2i-1,2j-1}(t) + J_{2i-1,2j-1}\ddot{\theta}_z(t) \right\} \quad (53)$$

Then, the moment of dynamic water pressure as to the fixed water around the vertical axis can be expressed as Eq.(51).

Therefore, the moment of dynamic water pressure, M^z_0, against walls proportional to the rotational angular acceleration $\ddot{\theta}_z(t)$ can be obtained by using the second term of the right side of Eq.(53) and Eq.(51), as follows:

$$M^z_0 = -\left\{ \bar{J}^z_0 - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} J_{2i-1,2j-1} \right\} \ddot{\theta}_z(t) \quad (54)$$

Therefore, now assuming that

$$M^z_0 = -J^z_0 \ddot{\theta}_z(t), \quad (55)$$

the equivalent moment of inertia for fixed water, J^z_0, can be obtained by the following equation:

$$J^z_0 = \bar{J}^z_0 - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} J^z_{2i-1,2j-1} \quad (56)$$

From the above, the equivalent vibration system for the liquid when the rectangular TLD rotates around the vertical axis can be modeled, as shown in **Fig.3**, as a vibration system which consists of three elements: the equivalent moment of inertia $J^z_{2i-1,2j-1}$ for the sloshing vibration, the

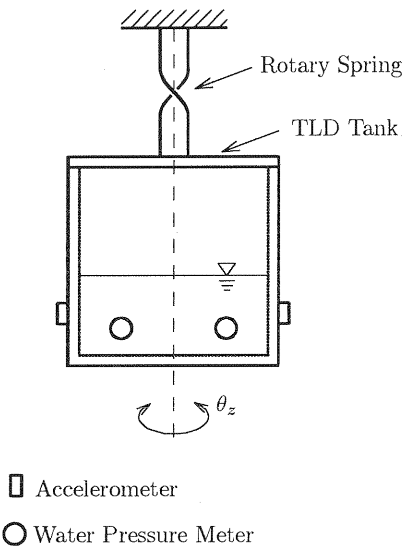

Fig. 4 Diagram of Device

equivalent rotational spring constant $k^z_{2i-1,2j-1}$ and the equivalent moment of inertia for fixed water J^z_0 .

3. EXPERIMENTS ON FREE VIBRATION

(1) Outline of Experiment

As is shown in **Fig.4**, the rectangular TLD, which was filled with water to an appropriate depth, was connected to a plate spring fixed at the top, and TLD was turned slightly around the vertical axis. After that, TLD was released quietly, so that the TLD itself could generate rotational damped free vibration. The natural frequencies of this vibration system were measured, while varying the shape of TLD, the twisting rigidity of plate spring, and the depth of water. Measurement of the natural frequencies was made by using two accelerometers (Strain type AS-2C, manufactured by Kyowa Dengyo) which were set at the points of symmetrical position as shown in **Fig.4**. Dynamic water pressure gauges (PGM-02KG by Kyowa Dengyo) were used to measure the dynamic water pressure on the walls. Two kinds of TLD with a thickness of 0.5 cm were made of acrylic plates: rectangular and square tanks. Inside dimensions were 30×12 ×30 cm ($L \times D \times$ height) and 30×30×30 cm, respectively. The depth of the liquid was varied at 0, 4.5, 6, 7.5, 9, and 12 cm; six levels in all. The plate spring of phosphor bronze plate was 1.2 mm thick, 20 mm

Table 1 Springs for the rectangular TLD

Spring Length (cm)	Natural Frequency of Tank (Hz)	Spring Constant K^z_s (N·m/rad)
6.0	2.03	6.409
7.5	1.78	4.932
10.0	1.61	4.048
11.0	1.60	3.987
12.0	1.43	3.166
14.0	1.41	3.091
20.0	1.18	2.147
24.0	1.06	1.740

Table 2 Springs for the Square TLD

Spring Length (cm)	Natural Frequency of Tank (Hz)	Spring Constant K^z_s (N·m/rad)
4.0	1.64	9.536
5.0	1.45	7.449
6.0	1.34	6.352
8.0	1.14	4.566
9.0	1.11	4.351
12.0	0.94	3.155
24.0	0.70	1.721

wide and the length was changed as is shown in **Tables 1 and 2**.

In our experiment, the total pressure against the walls of the TLD was at most 1.5 kPa, which is so small that deformation of the tank is negligible and the tank was regarded as rigid. Indeed, judging from the TLD tanks actually fixed onto the existing structures, the rigidity of the tank for the total pressure is adequate. Therefore, the tank can be regarded as rigid.

(2) Calculating Method of $J^z_{1,1}, J^z_0, k^z_{1,1}$

By using the natural periods of rotational vibration of TLD tank–spring system obtained by our experiment, the constants of the vibration system were calculated as shown below.

An equivalent vibration system to the vibration system shown in **Fig.4** can be expressed as **Fig.6** ($i = j = 1$), taking the (1,1)th vibration into consideration, where K^z_s is the rotational spring constant of the plate spring (twisting rigidity), and J^z_s is the moment of inertia of TLD, accelerometers etc., excluding the contained water. Equation of motion of the equivalent vibration system for the free vibration can be described as below, assuming that the rotation angle of the tank is θ_z and that the normal coordinates are ξ, when the damping is neglected.

$$J^z_{1,1}(\ddot{\theta}_z + \ddot{\xi}) + k^z_{1,1}\xi = 0 \quad (57)$$

Equation of motion for the rotational free vibration of TLD–spring system is

$$(J^z_s + J^z_0)\ddot{\theta}_z + K^z_s\theta_z = k^z_{1,1}\xi \quad (58)$$

In the above equation, the damping term was neglected for the following reason. Though the damping constants were deduced by the free vibration experiment, (**Fig.5** shows an example of the damped free vibration curves towards the circumferential acceleration of the tank), each of them was found to be within the range of 0.003 ~0.008. Therefore, judging from the necessary accuracy for the calculation of an equivalent vibration system, the influence of the damping constants on the natural frequencies of the damped free vibration can be considered almost zero. For this reason the influence by the damping was neglected.

By substituting the following equations into Eqs.(57) and (58),

$$\theta_z = \Theta_z e^{int} \quad , \quad \xi = Z e^{int} \quad (59)$$

and, taking into consideration Eq.(24), the n^z_1 of the first natural circular frequency of TLD–spring system can be expressed as

$$(n^z_1)^2 = \frac{2(n_{1,1})^2 K^z_s}{\left\{(n_{1,1})^2(J^z_s + J^z_0 + J^z_{1,1}) + K^z_s\right\} + \sqrt{\left\{(n_{1,1})^2(J^z_s + J^z_0 + J^z_{1,1}) + K^z_s\right\}^2 - 4(n_{1,1})^2 K^z_s(J^z_s + J^z_0)}} \quad (60)$$

Changing the liquid depth H and spring constant K^z_s in various values, the experiments were performed on a rotational damped free vibration of TLD–spring system to obtain the n^z_1 of the natural circular frequency of the vibration systems concerned. When n^z_1 is obtained for the values of K^z_s, and when the depth H and the moment of inertia of the tank J^z_s etc. are known, the most probable values of $J^z_{1,1}$, J^z_0 of the equivalent vibration system can be calculated from Eq.(60) by using, for example, the least-square method. However, as Eq.(60) is a non-linear equation with respect to $J^z_{1,1}, J^z_0$, the equations are converted into inearized equations employing approximate values of $J^{z(0)}_{1,1}$, $J^{z(0)}_0$, as shown below, and the most probable values were obtained by iteration. For a starting value of the iteration, the theoretical values from Eqs.(23) and (56) were adopted. Now Eq.(60) is described as

$$(n^z_1)^2 = f(J^z_0, J^z_{1,1}) \quad (61)$$

This equation is approximately expressed as,

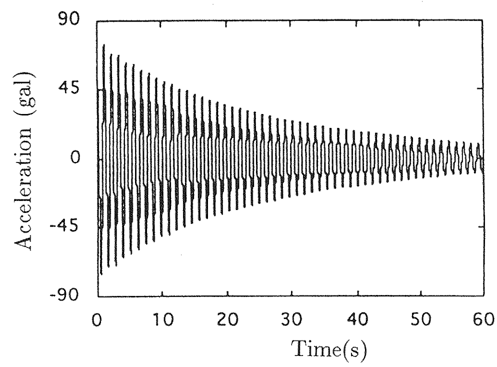

Fig. 5 An example of the damped free vibration curve of the acceleration towards the circumference of TLD (Square TLD;spring length=12 cm ; water depth=9 cm)

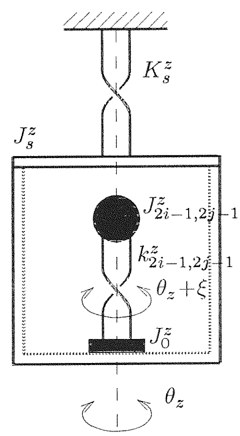

Fig. 6 Equivalent Vibration System for Rotational Vibration

$$(n^z_1)^2 = f(J^{z(0)}_0, J^{z(0)}_{1,1}) + \frac{\partial}{\partial J^z_0} f(J^{z(0)}_0, J^{z(0)}_{1,1})\Delta J^z_0 + \frac{\partial}{\partial J^z_{1,1}} f(J^{z(0)}_0, J^{z(0)}_{1,1})\Delta J^z_{1,1} \quad (62)$$

where,

$$\Delta J^z_0 = J^z_0 - J^{z(0)}_0 \quad , \quad \Delta J^z_{1,1} = J^z_{1,1} - J^{z(0)}_{1,1} \quad (63)$$

The constants required for the equivalent moment of inertia are decided as follows: J^z_s is calculated from the mass of the tank, etc. and the geometrical shape. $n_{1,1}$ is calculated by Eq.(22). K^z_s is calculated as $K^z_s = 4\pi^2 J^z_s/(T^z_s)^2$, by using J^z_s and T^z_s, the natural frequencies obtained from

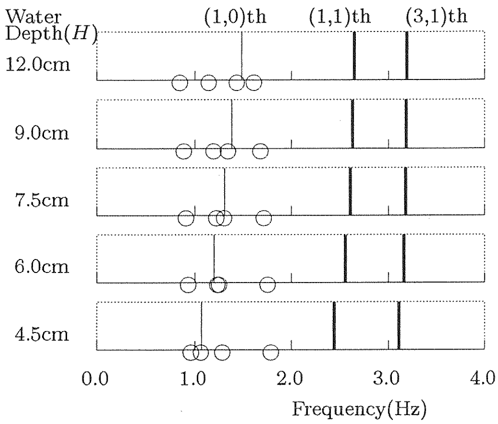

Fig. 7 TLD – Spring System and Natural Frequency for Rectangular TLD

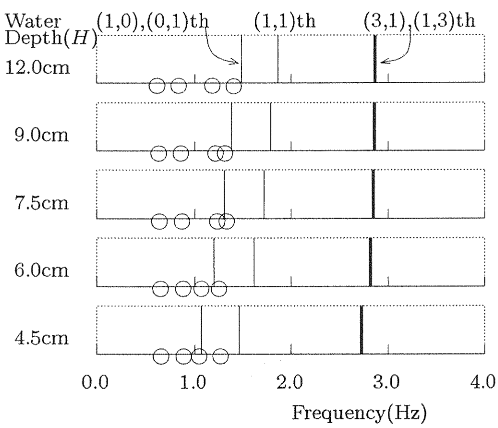

Fig. 8 TLD – Spring System and Natural Frequency for Square TLD

the experiment of the rotational damped free vibration for the empty vessel (depth $H = 0$).

Using these constants and the natural periods of TLD-spring system obtained from the experiment under the given water depth, the equivalent moment of inertia $J^z_{1,1}, J^z_0$ was calculated by the procedures as mentioned above. The spring constant K^z_s used in the experiment is shown in **Tables 1 and 2**.

(3) The Results of the Experiments

When the depth is changed, theoretical values of natural frequency of the (i, j) th mode of TLD liquid are shown by the vertical lines in **Fig.7** for the rectangular tank and in **Fig.8** for the square tank. Thick line represents the theoretical value of the natural frequency because the moment of

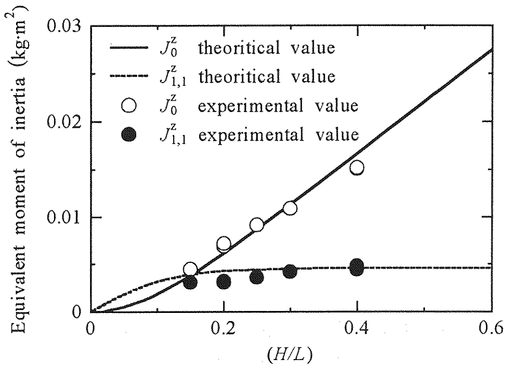

Fig. 9 Equivalent Moment of Inertia $(D/L = 0.4)$

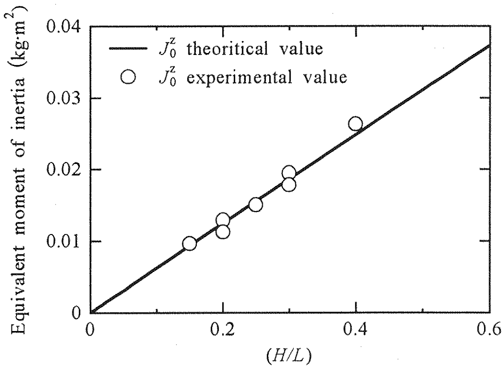

Fig. 10 Equivalent Moment of Inertia $(D/L = 1.0)$

inertia around the vertical axis does not become zero, while the thin line represents the theoretical value when it becomes zero. Also, ○ mark in the figure is the first natural frequency of the rotational vibration of TLD-spring system when various kinds of plate springs are employed. Both figures show that each value of the first natural frequency of TLD-spring system is smaller than the theoretical value of the natural frequency of the liquid in TLD; as for **Fig.7**, the (1,1)th, for **Fig.8**, the(1,3), (3,1)th. This is because special consideration has been made so that the equivalent moment of inertia of TLD for the higher vibration mode may not affect the experimental results. This was achieved by choosing the plate springs so that the first natural frequency of TLD-spring system would become smaller than the minimum natural frequency of liquid in TLD whose moment of inertia is not zero (the [1,1] th mode in case of the rectangular, and the [1,3] [3,1] th mode in case of the square tank).

In **Figs.9 and 10**, values of the equivalent moment of inertia obtained from the experiment,

$J^z_{1,1}$ and J^z_0, are shown together with the theoretical values. The theoretical values are shown by solid lines (J^z_0) and by dotted lines ($J^z_{1,1}$), while the experimental values are shown by ○ (J^z_0) and ● ($J^z_{1,1}$) . **Fig.9** shows the case of the aspect ratio of D/L=0.4 and **Fig.10** shows the case of $D/L = 1$. In the case of **Fig.10**, $J^z_{1,1}$ becomes zero theoretically, so that only the value of J^z_0 is shown. **Fig.9** shows that the values $J^z_{1,1}$ obtained from our experiment are generally smaller than the theoretical ones, and, on the contrary, that the values J^z_0 tend to be a little larger. This is considered to have been caused by the neglect of the influences of the moment of inertia response to the natural frequency of the liquid, which is higher than the (3.1)th, when J^z_0 and $J^z_{1,1}$ are calculated after the natural frequency of TLD-spring system is experimentally obtained, as shown in **3.(2)**. Specifically, the smaller H/L is, the more the influence of the moment of inertia on $J^z_{1,1}$ becomes and when H/L=0.15, it becomes more than 10 percent.

Generally, when the ratio of H/L is small, the ratio of the moment of inertia J^z_0 to the fixed water becomes small. As the ratio of H/L becomes larger, the ratio of the equivalent moment of inertia ($J^z_{1,1}$) for the sloshing vibration also becomes smaller, so that the experimental definition cannot be accurate. From the viewpoint of the accuracy of the experiment, it is desirable that the moment of inertia of TLD tank itself be as small as possible. When H/L is small, this condition becomes inevitable, and the minimum value concerning how far the experimental value can follow the theoretical value is difficult to describe accurately, as it depends on the improvement of the experimental devices and of the exactness of the measuring method.

The above being considered, our experimental results can be safely said to be in good agreement with the theoretical values as to all the depths as well as when H/L is different.

4. SUMMARY

A theoretical equivalent vibration system of a contained liquid was obtained when a rectangular TLD was turned with vibration around a vertical axis. Next, the equivalent moment of inertia for the TLD liquid was obtained by the experiment model. Then the results were compared. It was confirmed that the two are in good agreement with each other, which has shown the validity of the theoretical equations.

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