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NONSTATIONARY RANDOM ANALYSIS WITH COUPLING VIBRATION OF BENDING AND TORSION OF SIMPLE GIRDER BRIDGES UNDER MOVING VEHICLES

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According to experimental results of dynamic field tests, a response of external girder is considerably different from internal one's. This difference is come from influence of torsional vibration coupling with bending vibration of a bridge. In this study, simultaneous nonstationary random vibrations of a bridge and moving vehicles are theoretically analyzed by means of random vibration theory, taking account of roadway roughness and coupling vibration of bending and torsion. RMS values of random response of girder bridges are calculated and compared with those from simulation analysis.

Key Words : *traffic-induced vibration, coupling vibration of torsion and bending, nonstationary random vibration, roadway roughness, RMS values*

1. INTRODUCTION

Dynamic response of highway bridges under moving vehicles has been investigated in connection with impact factor used in strength design of bridges[1)-8)]. Traffic-induced vibration of highway bridges is affected considerably by not only dynamic characteristics of bridge and vehicle but also roadway roughness. Even assuming that a sequence of roadway roughness is a stationary random process, traffic-induced vibration of bridges shows properties of nonstationary ones, because vibrating vehicles are moving on bridges[4),5),7),9),10)]. For theoretical analyses of dynamic response of highway bridges under moving vehicles taking account of roadway roughness, random vibration theory is utilized as well as simulation method[1)-8)]. One of authors presented an analytical method of simultaneous nonstationary random vibrations of both bridge and moving vehicles by means of random vibration theory[10)], and proposed formulae of impact factor[11)].

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Meanwhile, according to experimental results of dynamic field tests, dynamic response of external girder is considerably different from internal one's[12),13)]. This difference comes from influence of torsional vibration coupled with bending vibration of a girder bridge. When a moving vehicle is running on the lane eccentric from a shear center of cross section of a bridge with more than two traffic lanes, tortional vibration coupled with bending vibration of a bridge must be considered for estimating dynamic response of a bridge. Torsional vibration also has a large effect on estimating vibration serviceability of pedestrians because sidewalk is located at the edge of deck.

In this study, simultaneous nonstationary random vibrations of both bridge and moving vehicles are theoretically analyzed by means of random vibration theory, taking account of roadway roughness and coupling vibration of bending and torsion. The validity of this method is investigated by comparing root mean square (RMS) values of dynamic response of a bridge based on the present procedure with those due to simulation analytical method. RMS values of the internal girder are compared with those of the external one. The influence of characteristics of moving vehicles on dynamic response of a highway bridge is investigated.

2. ANALYTICAL PROCEDURE

(1) Fundamental equations of motion

Fundamental equations of motion for a simple girder bridge subjected to loadings of a series of moving vehicles are firstly derived under the following assumptions.

① A simple girder bridge is a plane system, having a certain cross section over the whole length.

② Each moving vehicle is a two-degree-of-freedom damped sprung-mass system which has front and rear axles.

③ A sequence of the roughness of roadway surface is a stationary random process.

As shown in **Fig.1**, the center of gravity G of a bridge cross section is defined as the origin, the y coordinate axis is taken in the direction of vertical downward. Disregarding vibration of horizontal direction, simultaneous equations for bending vibration and torsional vibration can be written as follows;

$$EI_z\frac{\partial^4 v}{\partial x^4}+mA\frac{\partial^2 v}{\partial t^2}+mAz_s\frac{\partial^2 \varphi}{\partial t^2}=q_y(t,x)$$

$$EC_w\frac{\partial^4 \varphi}{\partial x^4}-GK\frac{\partial^2 \varphi}{\partial x^2}+mAz_s\frac{\partial^2 v}{\partial t^2}+mI_s\frac{\partial^2 \varphi}{\partial t^2}=m_x(t,x) \tag{1}$$

where,

m :mass per unit volume of a bridge,
A :area of cross section of a bridge,
z_s : z coordinate of the shear center,
EI_z :bending rigidity around z axis,
GK :torsional rigidity, EC_w :warping rigidity,
$q_y(t,x)$:external force in the direction of vertical downward,
$m_x(t,x)$:external moment around x axis (longitudinal axis of a bridge),
I_s :inertia moment around the shear center ($=I_G+A(z_s^2+y_s^2)$),
I_G :inertia moment around the gravity center.

The dynamic vertical deflection v and torsional deflection φ at an arbitrary point x of longitudinal axis of a bridge and at any time t are expressed by the following equations, employing generalized coordinates $a_n(t)$ and $c_n(t)$, and normal mode functions $v_n(x)$ and $\varphi_n(x)$ satisfied with the boundary conditions of a bridge, respectively.

$$\left.\begin{aligned}v(t,x)&=\sum_n a_n(t)v_n(x)\\ \varphi(t,x)&=\sum_n c_n(t)\varphi_n(x)\end{aligned}\right\} \tag{2}$$

The simultaneous differential equations of bending and torsional vibration of a bridge under moving vehicles on the lane eccentric from the shear center by taking account of viscous damping of a bridge can be written as follows;

$$\left.\begin{aligned}&\ddot{a}_n(t)+2h_{bn}p_{bn}\dot{a}_n(t)+p_{bn}^2a_n(t)+z_s\ddot{c}_n(t)\\ &\qquad=\frac{2}{mAl_b}\sum_{j=1}^{h}\sum_{s=1}^{2}\sum_{k=1}^{ax(s)}v_n(x_{jsk})P_{jsk}(t)\\ &\ddot{c}_n(t)+2h_{tn}p_{tn}\dot{c}_n(t)+p_{tn}^2c_n(t)+\frac{z_s}{\gamma_s^2}\ddot{a}_n(t)\\ &\qquad=\frac{2}{mI_sl_b}\sum_{j=1}^{h}\sum_{s=1}^{2}\sum_{k=1}^{ax(s)}\varphi_n(x_{jsk})P_{jsk}(t)e\end{aligned}\right\} \tag{3}$$

where,

$p_{bn}=\left(\frac{n\pi}{l_b}\right)^2\left[\frac{EI_z}{mA}\right]^{1/2}$:the n-th natural circular frequency of bending vibration of a bridge,

$p_{tn}=\left(\frac{n\pi}{l_b}\right)\left[\frac{GK}{mI_s}\left\{\left(\frac{n\pi}{k}\right)^2+1\right\}\right]^{1/2}$, $k=l_b\left[\frac{GK}{EC_w}\right]^{1/2}$:the n-th natural circular frequency of torsional vibration,

h_{bn},h_{tn} :damping constant of bending and torsional vibration,

$\gamma_s^2=I_s/A$,

e :eccentric distance between the shear center and the loading point of vehicles,

h :number of vehicles,

$x_{jsk}=v(t-t_j)-\lambda_{jsk}$,

$P_{jsk}(t)=\frac{1}{ax(s)}P_{js}(t)$, $P_{js}(t)$ are the loading forces of front and rear wheels,

$ax(s)$:number of axles of front and rear wheels, where, $ax(1)=1, ax(2)=2$.

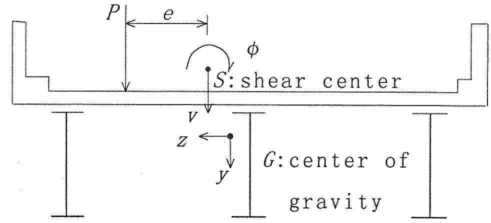


Fig. 1 Cross section of model bridge

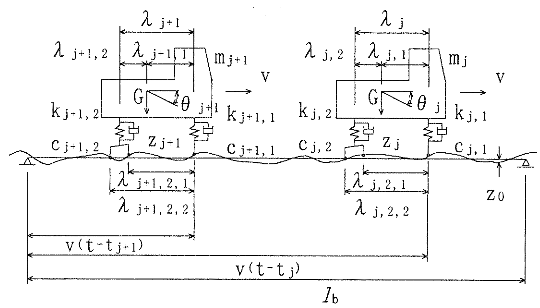


Fig.2 Analytical models of a bridge and moving vehicles

As shown in **Fig.2**, if each moving vehicle is idealized in a two-degree-of-freedom sprung-mass system, the motion of the vehicle can be described in terms of the vertical displacement $z_j(t)$ of the center of gravity G and the rotational angle $\theta_j(t)$ about it of the j-th moving vehicle as follows;

$$\left.\begin{aligned} & m_j\ddot{z}_j(t)+\sum_{s=1}^{2}v_{js}(t)=0 \\ & m_jr_j^2\ddot{\theta}_j(t)-\sum_{s=1}^{2}(-1)^s\lambda_{js}v_{js}(t)=0 \\ & \qquad (j=1,2,\ \ldots\ldots,\ h) \end{aligned}\right\} \quad (4)$$

where,

$$\begin{aligned} v_{js}(t) &= k_{vjs}\{z_j-(-1)^s\lambda_{js}\theta_j-\frac{1}{ax(s)}\sum_{m=1}^{ax(s)}y_{vjsm}\} \\ &+c_{vjs}\{\dot{z}_j-(-1)^s\lambda_{js}\dot{\theta}_j-\frac{1}{ax(s)}\sum_{m=1}^{ax(s)}\dot{y}_{vjsm}\} \end{aligned} \quad (5)$$

where,

$m_jr_j^2$: the mass moment of inertia of vehicle.

The other symbols are referred in **Fig.2** and the displacement y_{vjsm} of the loading point of the moving vehicle is expressed as follows;

$$\begin{aligned} y_{vjsm} &= v(t,x_{jsm})+\varphi(t,x_{jsm})e-z_0(x_{jsm}) \\ &= \sum_g a_g(t)v_g(x_{jsm})+\sum_g c_g(t)\varphi_g(x_{jsm})e-z_0(x_{jsm}) \\ & \qquad (j=1,2\cdots,h,\quad s=1,2,\quad m=1,ax(s)) \end{aligned} \quad (6)$$

The loading forces of front and rear wheels are expressed as follows;

$$P_{js}(t)=\left(1-\frac{\lambda_{js}}{\lambda_j}\right)m_jg+v_{js}(t) \qquad (j=1,2,\cdots,h,\quad s=1,2) \quad (7)$$

where, g :the acceleration of gravity.

By using Eqs.(5),(6) and (7), simultaneous differential equations of a bridge and moving vehicles on roadway roughness can be derived from Eqs.(3) and (4).

When the mean values of nonstationary random process $a_n(t), c_n(t), z_j(t), \theta_j(t)$ at an arbitrary time t are expressed as $\bar{a}_n(t), \bar{c}_n(t), \bar{z}_j(t), \bar{\theta}_j(t)$, they are dynamic response taking no account of roadway roughness. Expressing centered random values from the mean values as $\tilde{a}_n(t), \tilde{c}_n(t), \tilde{z}_j(t), \tilde{\theta}_j(t)$, nonstationary random process can be expressed as follows;

$$\left.\begin{aligned} a_n(t)=\bar{a}_n(t)+\tilde{a}_n(t),\quad c_n(t)=\bar{c}_n(t)+\tilde{c}_n(t) \\ z_j(t)=\bar{z}_j(t)+\tilde{z}_j(t),\quad \theta_j(t)=\bar{\theta}_j(t)+\tilde{\theta}_j(t) \end{aligned}\right\} \quad (8)$$

The dynamic response of a bridge at an arbitrary point x is expressed as follows;

$$y(t,x)=\bar{y}(t,x)+\tilde{y}(t,x) \quad (9)$$

Substituting Eqs.(8) and (9) into Eqs. (3) and (4), the differential equations for centered random response are derived as follows;

$$\left.\begin{aligned} & \ddot{\tilde{a}}_n(t)+2h_{bn}p_{bn}\dot{\tilde{a}}_n(t)+p_{bn}^2\tilde{a}_n(t)+z_s\ddot{\tilde{c}}_n(t)= \\ & \qquad \frac{2}{mAl_b}\sum_{j=1}^{h}\sum_{s=1}^{2}\sum_{k=1}^{ax(s)}v_n(x_{jsk})\tilde{P}_{jsk}(t) \\ & \ddot{\tilde{c}}_n(t)+2h_{tn}p_{tn}\dot{\tilde{c}}_n(t)+p_{tn}^2\tilde{c}_n(t)+\frac{z_s}{\gamma_s^2}\ddot{\tilde{a}}_n(t)= \\ & \qquad \frac{2}{mI_sl_b}\sum_{j=1}^{h}\sum_{s=1}^{2}\sum_{k=1}^{ax(s)}\varphi_n(x_{jsk})\tilde{P}_{jsk}(t)e \\ & m_j\ddot{\tilde{z}}_j(t)+\sum_{s=1}^{2}\tilde{v}_{js}(t)=0, \\ & m_jr_j^2\ddot{\tilde{\theta}}_j(t)-\sum_{s=1}^{2}(-1)^s\lambda_{js}\tilde{v}_{js}(t)=0 \\ & \qquad (j=1,2,\ \ldots\ldots,\ h) \end{aligned}\right\} \quad (10)$$

where,

$$\left.\begin{aligned} & \tilde{P}_{jsk}(t)=\frac{1}{ax(s)}\tilde{v}_{js}(t) \\ & \tilde{v}_{js}(t)=k_{vjs}\{\tilde{z}_j-(-1)^s\lambda_{js}\tilde{\theta}_j-\frac{1}{ax(s)}\sum_{m=1}^{ax(s)}\tilde{y}_{vjsm}\} \\ & \qquad +c_{vjs}\{\dot{\tilde{z}}_j-(-1)^s\lambda_{js}\dot{\tilde{\theta}}_j-\frac{1}{ax(s)}\sum_{m=1}^{ax(s)}\dot{\tilde{y}}_{vjsm}\} \\ & \tilde{y}_{vjsm}=\sum_g\tilde{a}_g(t)v_g(x_{jsm}) \\ & \qquad +\sum_g\tilde{c}_g(t)\varphi_g(x_{jsm})e-z_0(x_{jsm}) \end{aligned}\right\} \quad (11)$$

(2) System of linear differential equations

The state vector $w(t)$ associated with both bridge and moving vehicles as well as the external force vector $z(t)$ are defined as Eq.(12).

$$\left.\begin{aligned} w(t) &= \{\tilde{a}_1;\tilde{a}_2;\cdots;\tilde{a}_n;\dot{\tilde{a}}_1;\dot{\tilde{a}}_2;\cdots;\dot{\tilde{a}}_n; \\ & \quad \tilde{c}_1;\tilde{c}_2;\cdots;\tilde{c}_n;\dot{\tilde{c}}_1;\dot{\tilde{c}}_2;\cdots;\dot{\tilde{c}}_n; \\ & \quad \tilde{z}_1;\dot{\tilde{z}}_1;\tilde{\theta}_1;\dot{\tilde{\theta}}_1;\cdots;\tilde{z}_h;\dot{\tilde{z}}_h;\tilde{\theta}_h;\dot{\tilde{\theta}}_h\} \\ &= \{w_1;w_2;\cdots;w_{2n};w_{2n+1};w_{2n+2};\cdots; \\ & \quad w_{4n};w_{4n+1};w_{4n+2};\cdots;w_{4n+4h-1};w_{4n+4h}\} \\ z(t) &= \{z_0[v(t-t_{v1})];z_0[v(t-t_{v1})-\lambda_{121}]; \\ & \quad z_0[v(t-t_{v1})-\lambda_{122}];\dot{z}_0[v(t-t_{v1})]; \\ & \quad \dot{z}_0[v(t-t_{v1})-\lambda_{121}];\dot{z}_0[v(t-t_{v1})-\lambda_{122}]; \\ & \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots; \\ & \quad z_0[v(t-t_{vh})];z_0[v(t-t_{vh})-\lambda_{h21}]; \\ & \quad z_0[v(t-t_{vh})-\lambda_{h22}];\dot{z}_0[v(t-t_{vh})]; \\ & \quad \dot{z}_0[v(t-t_{vh})-\lambda_{h21}];\dot{z}_0[v(t-t_{vh})-\lambda_{h22}]\} \end{aligned}\right\} \quad (12)$$

Using Eq.(12), Eq.(10) can be expressed in a matrix form as Eq.(13).

$$\dot{w}(t)=A(t)w(t)+B(t)z(t) \quad (13)$$

Equation(13) is regarded as a linear differential equation, and has the following initial conditions for moving vehicles at time t_{v1}, t_{v2}, … and t_{vh}.

$$\left.\begin{aligned} w(t_{v1}) &= w_{01} = \{0;\ldots;0;w_{4n+1};\ldots;w_{4n+4};0;\ldots\ldots;0\} \\ w(t_{v2}) &= w_{02} = \{0;\ldots;0;w_{4n+1};\ldots\ldots;w_{4n+8};0;\ldots;0\} \\ & \cdots\cdots\cdots\cdots\cdots\cdots, \\ w(t_{vh}) &= w_{0h} = \{0;\ldots;0;w_{4n+1};\ldots\ldots\ldots\ldots;w_{4n+4h}\} \end{aligned}\right\} \quad (14)$$

According to the theory of a linear differential equation[14], the solution of Eq.(13) can be given under the initial conditions of Eq.(14) as follows;

$$\boldsymbol{w}(t) = \boldsymbol{\Phi}(t,t_0)\boldsymbol{w}_{0k} + \int_{t_0}^{t} \boldsymbol{\Phi}(t,\tau)\boldsymbol{B}(\tau)z(\tau)d\tau \qquad (15)$$

where, $\boldsymbol{\Phi}(t,\tau)$ is a transition matrix, and k is the number of moving vehicles entering the bridge by time t.

(3) Covariance matrix of response

The covariance matrix $\boldsymbol{R}_w(t_1,t_2)$ of the state vector $\boldsymbol{w}(t)$ can be written as Eq.(16).

$$\left.\begin{aligned} \boldsymbol{R}_w(t_1,t_2) &= \mathrm{E}\left[\boldsymbol{w}(t_1)\boldsymbol{w}^T(t_2)\right] \\ &= \boldsymbol{\Phi}(t_1,t_0)\mathrm{E}\left[\boldsymbol{w}_{0,k_1}\boldsymbol{w}_{0,k_2}^T\right]\boldsymbol{\Phi}^T(t_2,t_0) \\ &+ \int_{t_0}^{t_1}\boldsymbol{\Phi}(t_1,\tau)\boldsymbol{B}(\tau)\mathrm{E}\left[z(\tau)\boldsymbol{w}_{0,k_2}^T\right]\boldsymbol{\Phi}^T(t_2,t_0)\,d\tau \\ &+ \int_{t_0}^{t_2}\boldsymbol{\Phi}(t_1,t_0)\mathrm{E}\left[\boldsymbol{w}_{0,k_1}z^T(s)\right]\boldsymbol{B}^T(s)\boldsymbol{\Phi}^T(t_2,s)ds \\ &+ \int_{t_0}^{t_1}\int_{t_0}^{t_2}\boldsymbol{\Phi}(t_1,\tau)\boldsymbol{B}(\tau)\mathrm{E}\left[z(\tau)z^T(s)\right]\boldsymbol{B}^T(s)\boldsymbol{\Phi}^T(t_2,s)\,d\tau ds \end{aligned}\right\} \qquad (16)$$

Where, E[ ] is the linear operator of the mean value and the superscript T denotes the transpose of a vector or a matrix.

The initial conditions for the bridge and each moving vehicle are assumed as follows ;

Ⅰ) The bridge remains at rest till the first vehicle enters its span.

Ⅱ) Each vehicle has been moving on the roadway having the statistically same surface roughness as those of the bridge roadway. Consequently, it can be expected that a stationary random vibration has been already produced in each vehicle before entering the span of the bridge.

Under the initial conditions mentioned above, the covariance matrix of the state vector $\boldsymbol{w}(t)$ can be given by the following equation based on the Wiener-Khintchine relations between the spectral density and the covariance of a stationary random process.

$$\begin{aligned} \boldsymbol{R}_w(t_1,t_2) &= \boldsymbol{\Phi}(t_1,t_0)\left[\begin{array}{c|cccc} \boldsymbol{0} & & & \boldsymbol{0} & \\ \hline & \mathrm{E}\left[\boldsymbol{w}_{10}\boldsymbol{w}_{10}^T\right] & & & \\ & & \ddots & & \\ \boldsymbol{0} & & & \mathrm{E}\left[\boldsymbol{w}_{k0}\boldsymbol{w}_{k0}^T\right] & \\ & & & & \ddots \\ & & & & \mathrm{E}\left[\boldsymbol{w}_{h0}\boldsymbol{w}_{h0}^T\right] \end{array}\right]\boldsymbol{\Phi}^T(t_2,t_0) \\ &+ \int_{-\infty}^{\infty}\left\{\boldsymbol{H}^*(t_1,\omega)_{\tau 1}\boldsymbol{x}_{\tau 1,k}^{(T)} + \cdots\cdots + \boldsymbol{H}^*(t_1,\omega)_{\tau k}\boldsymbol{x}_{\tau k,k}^{(T)}\right\}\hat{S}_{z_0}\left[\begin{array}{c|ccc} \boldsymbol{0} & \boldsymbol{H}_1^T(t_{v1},\omega) & & \boldsymbol{0} \\ & & \ddots & \\ & \boldsymbol{0} & & \boldsymbol{H}_k^T(t_{vk},\omega) \end{array}\right]d\omega\,\boldsymbol{\Phi}^T(t_2,t_0) \\ &+ \boldsymbol{\Phi}(t_1,t_0)\int_{-\infty}^{\infty}\left[\begin{array}{ccc} & \boldsymbol{0} & \\ \hline \boldsymbol{H}_1(t_{v1},\omega) & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{H}_k(t_{vk},\omega) \end{array}\right]\hat{S}_{z_0}\left\{\boldsymbol{x}_{k,s1}\boldsymbol{H}^{*T}(t_2,\omega)_{s1} + \cdots + \boldsymbol{x}_{k,sk}\boldsymbol{H}^{*T}(t_2,\omega)_{sk}\right\}d\omega \\ &+ \int_{-\infty}^{\infty}\left\{\boldsymbol{H}(t_1,\omega)_{\tau 1}\left[\boldsymbol{x}_{\tau 1,s1}\boldsymbol{H}^{*T}(t_2,\omega)_{s1} + \cdots + \boldsymbol{x}_{\tau 1,sk}\boldsymbol{H}^{*T}(t_2,\omega)_{sk}\right] + \cdots\cdots\right. \\ &\qquad \left. + \boldsymbol{H}(t_1,\omega)_{\tau k}\left[\boldsymbol{x}_{\tau k,s1}\boldsymbol{H}^{*T}(t_2,\omega)_{s1} + \cdots + \boldsymbol{x}_{\tau k,sk}\boldsymbol{H}^{*T}(t_2,\omega)_{sk}\right]\right\}\hat{S}_{z_0}d\omega \end{aligned} \qquad (17)$$

where, $\boldsymbol{H}(t_1,\omega)_{\tau m} = \int_{t_{vm}}^{t_{v,m+1}} \boldsymbol{\Phi}_k(t_1,\tau)\boldsymbol{B}(\tau)e^{-j\omega\tau}d\tau$, $\quad \mathrm{E}\left[\boldsymbol{w}_{k0}\boldsymbol{w}_{k0}^T\right] = \int_{-\infty}^{\infty}\boldsymbol{H}_k(t_{vk},\omega)\boldsymbol{x}_{ekk}\hat{S}_{z0}\boldsymbol{H}_k^{*T}(t_{vk},\omega)d\omega$

$\boldsymbol{H}_k(t_{vk},\omega) = \int_{-\infty+t_{vk}}^{t_{vk}} \boldsymbol{\Phi}_k(t_{vk},\xi)\boldsymbol{B}_k e^{-j\omega\xi}d\xi$, $\quad \hat{S}_{z_0} = \dfrac{1}{2\pi v}S_{z_0}\left(\dfrac{\omega}{2\pi v}\right)$, $\quad \boldsymbol{x}_{epq} = \begin{bmatrix} \boldsymbol{x}_{pq} & j\omega\boldsymbol{x}_{pq} \\ -j\omega\boldsymbol{x}_{pq} & \omega^2\boldsymbol{x}_{pq} \end{bmatrix}\exp\left\{j\omega(t_{vp}-t_{vq})\right\}$

$$\boldsymbol{x}_{pq} = \begin{bmatrix} 1 & \exp\left\{-j\dfrac{\omega}{v}\lambda_{q21}\right\} & \exp\left\{-j\dfrac{\omega}{v}\lambda_{q22}\right\} \\ \exp\left\{j\dfrac{\omega}{v}\lambda_{p21}\right\} & \exp\left\{j\dfrac{\omega}{v}\left(\lambda_{p21}-\lambda_{q21}\right)\right\} & \exp\left\{j\dfrac{\omega}{v}\left(\lambda_{p21}-\lambda_{q22}\right)\right\} \\ \exp\left\{j\dfrac{\omega}{v}\lambda_{p22}\right\} & \exp\left\{j\dfrac{\omega}{v}\left(\lambda_{p22}-\lambda_{q21}\right)\right\} & \exp\left\{j\dfrac{\omega}{v}\left(\lambda_{p22}-\lambda_{q22}\right)\right\} \end{bmatrix} \qquad (18)$$

From Eq.(17), RMS values of the deflections of the bridge at an arbitrary point x can be expressed as follows;

$$R_y(t,t) = \sum_i\sum_k v_i(x)v_k(x)\,\mathrm{E}\left[\tilde{a}_i(t)\,\tilde{a}_k(t)\right] + s^2\sum_i\sum_k \varphi_i(x)\varphi_k(x)\,\mathrm{E}\left[\tilde{c}_i(t)\,\tilde{c}_k(t)\right] + 2s\sum_i\sum_k v_i(x)\varphi_k(x)\mathrm{E}\left[\tilde{a}_i(t)\,\tilde{c}_k(t)\right] \qquad (19)$$

where, s : the distance between the shear center and noted point.

3. NUMERICAL ANALYSIS

(1) Analytical models

a) A simple girder bridge

Structural quantities of a model bridge with span length of 40.4m are listed in **Table 1**. These quantities are estimated for a full cross section of the bridge with two lanes. The 1st. mode to the 3rd. one of bending and torsional vibration, respectively, are considered in analysis. These degrees of vibration modes are determined by confirming that the dynamic responses of a bridge by means of simulation analysis agree with the experimental results of field test[13].

b) A moving vehicle

The vehicle model of two-degree-of-freedom sprung-mass system with one front axle and two rear axles is used as shown in **Fig.2**. The geometrical dimensions of the vehicle model is also shown in **Fig.3** and its mechanical quantities are listed in **Table 2**. The mass moment of inertia is calculated based on an assumption of dividing the weight in the ratio 1:4 to the front axle and rear one. The spring constant k_{js} and the damping coefficient c_{js} are calculated on assumptions of the natural frequency f_v =3.0Hz and the damping constant h_v =0.03, respectively, and are divided in the ratio 1:4 to front axle and rear one. Mechanical quantities of 20tf vehicle are modeled appropriately from an actual dump truck. Those of 25tf vehicle are chosen from literature as vehicle length of 11.0m, the farthest distance between front and rear axles of 7.0～8.8m and adjacent distance of the tandem wheels of 1.8m. **Table 2** shows the case that the farthest distance between front and rear axles is 7.0m. The vehicles have a constant speed of 10m/s. This bridge model has two traffic lanes and the running position of the vehicle is eccentric at 1.442m from the shear center.

c) Power spectra of roadway roughness

The power spectral density function of road surface roughness may be usually expressed by a formula as follows;

$$S_{z0}(\Omega) = \frac{\alpha}{\Omega^n + \beta^n} \tag{20}$$

where α is the parameter of smoothness, n is the parameter of distribution of the power in frequency domain, β is the shape parameter as for $S_{z0}(\Omega)$ must not become infinity at $\Omega = 0$.

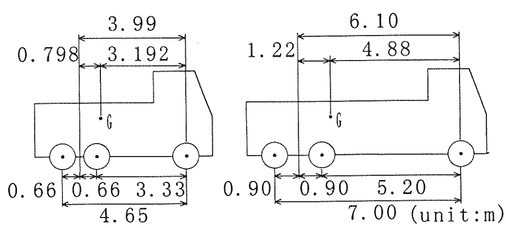
Three kinds of power spectra are used in this analysis as shown in **Fig.4**, in which straight broken lines indicate the boundaries of ISO estimate of roadway roughness. Roadway roughness No.① is modeled based on the measurement roughness at Yasugawa bridge on Meishin Expressway just after completion[7]. No.② is defined insafety side from some measurement roughnesses on Hanshin Expressway[11]. No.③ is defined as the mean

Table 1 Structural properties of model bridge

Span length	(m)	40.4
Weight per unit length	(kN/m)	74.01
Area of section	(m^2)	2.553
Moment of inertia of area	(m^4)	0.2197
Polar moment of inertia x Weight per unit length	(kN·m)	579.1
Torsion constant	(m^4)	5.479×10^{-2}
Warping constant	(m^6)	1.126
Damping constant for 1st. and 2nd. modes		0.026
Natural frequency		
Bending vibration 1st	(Hz)	2.35
2nd	(Hz)	9.42
3rd	(Hz)	21.19
Torsional vibration 1st	(Hz)	3.86
2nd	(Hz)	10.16
3rd	(Hz)	19.89

Table 2 Dynamic properties of model vehicle

Total weight	(kN)(tf)	196(20)	245(25)
Mass moment of inertia	$(kN\cdot m^2)$	499.2	1458.6
Spring constant	k_{j1} (kN/m)	1421.2	1776.5
	k_{j2} (kN/m)	5684.9	7106.1
Damping coefficient	c_{j1} (kN·s/m)	4.528	5.655
	c_{j2} (kN·s/m)	18.101	22.618
Natural frequency	(Hz)	3.0	3.0
Damping constant		0.03	0.03



(1) 20tf (196kN)   (2) 25tf (245kN)

Fig.3 Dimension of moving vehicles

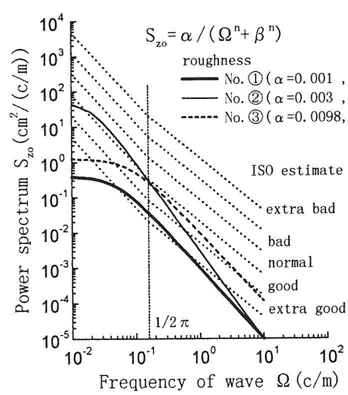


Fig.4 Power spectra of roadway roughness

value of many measurement roughnesses on the various short or medium-span bridges[15].

(2) Analytical results

In the above formulation a series of moving vehicles is considered, however in the following numerical examples, only a single moving vehicle is considered. RMS values of random response are calculated in terms of physical quantities of stress.

a) Comparison with simulation method

A RMS value of stress response is estimated as non-dimensional that dynamic RMS value is divided by the maximum static response, because stress response is related to impact factor used in strength design. Non-dimensional RMS values at the span center of the bridge by the present theory are compared with those by simulation analysis of Monte Carlo method as shown in **Fig.5**. The result of simulation method is estimated from frequent calculations using 100 samples of roadway roughness. Both results are in good agreement, and the present theoretical procedure is validated.

In **Fig.5** the RMS values of the internal and external girder are shown, respectively, and non-dimensional RMS value of the external girder is smaller than that of the internal one. A response of the external girder is larger than that of the internal one due to torsional component added to bending component under an eccentric loading of vehicle. In this analytical result, dynamic RMS response of torsional component does not increase as the static response compared with that of bending component. Also this phenomenon will be discussed in the following section d).

b) Effect of roadway roughness

Non-dimensional RMS values under the conditions of three kinds of roadway roughness are shown in **Fig.6** . There is a remarkable difference among RMS values due to the condition of roadway roughness. It is obvious that characteristics of roadway roughness have a great influence on traffic-induced vibration of a bridge including torsional vibration as well as bending one.

c) Comparison of new and old live load

Non-dimensional RMS values due to two types of vehicles of 20tf and 25tf are shown in **Fig.7**. In the case of 20tf vehicle, the non-dimensional RMS value of the external girder is smaller than that of the internal one as described in section a) and shown in **Fig.5**. On the other hand, in the case of 25tf vehicle with a different distance between front and rear axles, the non-dimensional RMS value of the external one becomes larger than that of the internal one over the loading position of about 3/10 of span length. Also the non-dimensional RMS values due to 25tf vehicle are smaller than those due to 20tf vehicle, and this shows that the increase of dynamic response is smaller than that of static response with increasing of the vehicle's weight. This is also affected that the distance between axles of 25tf vehicle is larger than that of 20tf vehicle.

In **Fig.8**, the analytical results are shown under the conditions that the farthest distance of axles of 25tf vehicle is changed to 7.0m, 7.8m and 8.8m, respectively. There is no remarkable difference among non-dimensional RMS values, and this shows that effects of the farthest distance defined for new live load on traffic-induced vibration of bridge are small.

d) Effect of natural frequency of vehicle

According to the past studies, it is well known that the relationship of the natural frequency between the vertical bending vibration of a bridge and the vibration of a moving vehicle affects the dynamic response of the bridge. In the analytical bridge model of this study, it is expected that not only the vertical bending vibration but also the torsional vibration of the bridge is resonant with the vibration of a moving vehicle because of low natural frequency of the torsional vibration. As described in section a), however, the bending vibration of the bridge is predominant over the torsional vibration, because the natural frequency of the moving vehicle is closer to that of the bending vibration than to that of the torsional vibration. In this section, effects of the relationship between the natural frequencies are examined by changing the natural frequency of the vehicle due to the changing the spring constant, still keeping the mass constant.

Analytical results are shown in **Fig. 9** . When the natural frequency of the vehicle f_v is close to that of the bending vibration of the bridge f_b=2.35Hz, non-dimensional RMS value of the bridge is largest, because the bending vibration of the bridge is resonant with the vibration of the vehicle. On the other hand, when the natural frequency of the vehicle f_v is close to that of the torsional vibration of the bridge f_t=3.86Hz, the response of the external girder becomes larger a little bit than that of the internal one due to the torsional response, but the response of the bridge does not attain resonant with the tortional vibration. This reason is that the moving vehicle eccentric from the shear center raises not only the torsional vibration but also the bending vibration. However, these analytical results are just for one model bridge, and it is expected that there are some cases of other bridges in which effects of the torsional vibration become large.

e) Effect of weight of vehicle

So far it has been believed that effects of the traffic-induced vibration on bridges are relatively smaller in heavy bridges such as concrete bridges than in light bridges such as steel bridges. So, an impact factor that is provided as multiplying ratio of static response of bridges

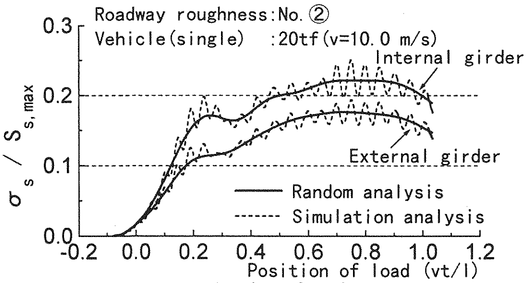


Fig.5 R.M.S value of random response

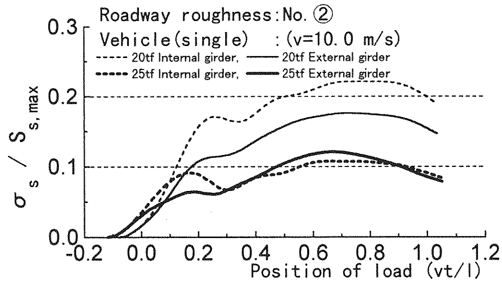


Fig.7 R.M.S value of random response

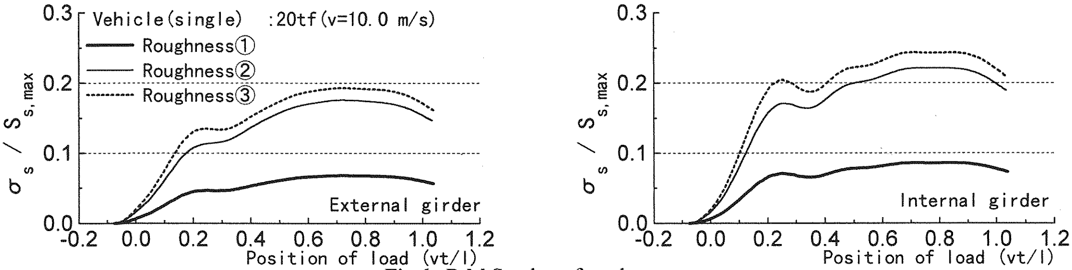


Fig.6 R.M.S value of random response

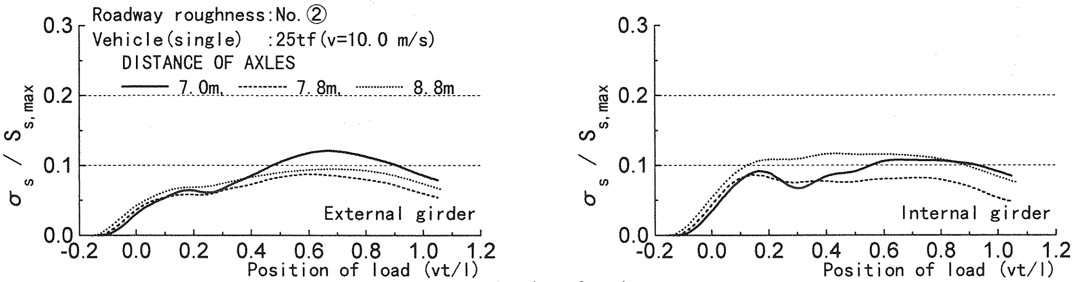


Fig.8 R.M.S value of random response

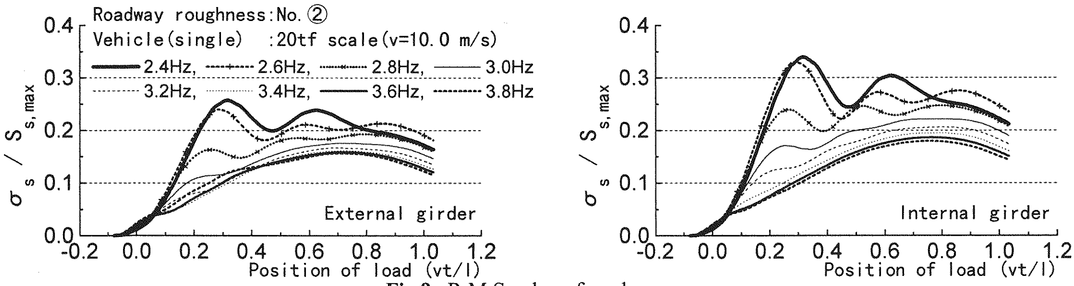


Fig.9 R.M.S value of random response

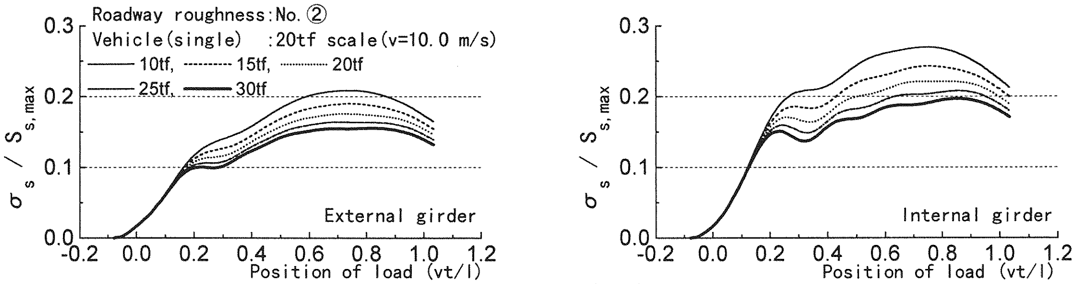


Fig.10 R.M.S value of random response

to dynamic response due to live load, is smaller in concrete bridges than in steel bridges[16]. Meanwhile, authors have to dynamic response due to live load, is smaller in concrete bridges than in steel bridges[16]. Meanwhile, authors have reported that the impact factor of steel bridges is not so different from that of concrete bridges based on the simultaneous nonstationary random vibration analysis considering only bending vibration of bridges[17].

In this section, using the present analytical procedure, dynamic response of the model bridge is analyzed under the condition of moving vehicles with different weights keeping the natural frequency constant, such as different weight ratios of vehicle to bridge. Analytical results are shown in **Fig.10**. The ordinate represents non-dimensional response that dynamic RMS value is divided by the maximum static response. The heavier weight of the moving vehicle, the smaller non-dimensional RMS value. When the simultaneous vibration of a bridge and a moving vehicle is considered, the analytical results of non-dimensional dynamic response of a bridge related to impact factor change due to weight ratio, and differ from a general understanding[1),4),17)]. These analyses are carried out in one model bridge, and it is necessary to do more detail investigations.

4. CONCLUSIONS

The simultaneous nonstationary random vibration with coupling of bending and torsion of bridge is formulated and the analytical results are summarized as follows.

(1)Analytical method of simultaneous nonstationary random vibration of a bridge under moving vehicles is modified being able to take account of coupling vibration of bending and torsion.

(2)RMS value of dynamic response due to the present procedure is in good agreement with that due to simulation analysis method and the validity of the present method is verified.

(3)When the traffic-induced vibrations are considered, parameter of roadway roughness is significant even in the case including the torsional vibration.

(4)In the case of 20tf vehicle the non-dimensional RMS value of stress response of the external girder is smaller than that of the internal one. On the other hand, in the case of 25tf vehicle the former one becomes larger than the latter one over the loading position of about 3/10 of span length.

(5)In this model girder bridge, the resonant behavior of a torsional vibration of a bridge with a vibration of a moving vehicle is not obvious than that of a bending vibration.

(6)Non-dimensional RMS value of the bridge that dynamic response is divided by the maximum static response tends to become smaller, when the vehicle's weight becomes heavier.

REFERENCES

1) Yamada, Y. and Kobori, T.: Study on highway bridge impact due to random moving vehicles, *Proc. of JSCE*, No.119, pp.1-9, 1965. (in Japanese).
2) Yamada, Y. and Kobori, T.: Dynamic response of highway bridges due to live load by spectral analysis, *Proc. of JSCE*, No.148, pp.40-50, 1967. (in Japanese).
3) Narita, N., Katsuragi, M. and Emoto, M.: Impact factor of bridges taking account of roadway roughness, *Civil Engineering Journal*, Vol.20, No.3, pp.27-32, 1978. (in Japanese).
4) Yoshimura, T., Hikosaka, H. and Uchitani, T.: Non-stationary random response of highway bridges under a single moving load, *Proc. of JSCE*, No.258, pp.35-44, 1977. (in Japanese).
5) Hikosaka, H., Yoshimura, T. and Uchitani, T.: Non-stationary random response of bridges to a series of moving vehicle loads. *Theo. and Appl. Mech.* Vol.28, pp.379-386, 1980.
6) Honda, H., Kobori, T. and Yamada, Y.: Dynamic factor of highway steel girder bridges, *IABSE Proceedings*, P-98, pp.57-75, 1986.
7) Okabayashi, T. and Yamaguchi, Z.: Mean square response analysis of highway bridges under a series of moving vehicles. *Proc. of JSCE*, No.334, pp.1-11, 1983. (in Japanese).
8) Yokoyama, K. and Inoue, J.: Traffic mode and impact coefficients of highway bridges, *Proc. of JSCE*, No.422/I-14(note), pp.391-394, 1990. (in Japanese).
9) Frýba, L.: Non-stationary response of a beam to a moving random force, *J. Sound and Vibration*, Vol.46, No.3, pp.323-338, 1976.
10) Kawatani, M. and Komatsu, S.: Nonstationary random response of highway bridges under a series of moving vehicles, *Structural Eng./Earthquake Eng.*, Vol.5, No.2, pp.285-292, (*Proc. of JSCE*, No.398/I-10), 1988.
11) Kawatani, M. and Shimada, R.: Nonstationary random response and impact of girder bridges under moving vehicles, *Proc. of JSCE*, No.398/I-10, pp.303-309, 1988. (in Japanese).
12) Kawatani, M., Komatsu, S. and Sasaki, T.: Dynamic response characteristics of plate girder bridge subject to moving motor vehicle, *Proc. of JSCE*, No.392/I-9, pp.351-358, 1988. (in Japanese).
13) Kawatani, M., Nishiyama, S. and Yamada, Y.: Dynamic analysis of highway girder bridges under moving vehicles, *Proc. 4th East Asia-Pacific Conference on Structural Engineering & Construction*, pp.1857-1862, 1993.
14) Soong, T.T.: Random Differential Equations in Science and Engineering, *Academic Press. Inc.*, New York, pp.152-179, 1973.
15) Honda, H., Kajikawa, Y. and Kobori, T.: Spectra of road surface roughness on bridges, *J. of Struct. Div., Proc. of ASCE*, Vol.108, No.ST9, pp.1956-1966, 1982.
16) Japan Road Association: Japanese Specifications for Highway Bridges , December, 1996.
17) Kawatani, M. Kobayashi, Y. and Takamori, K.: Nonstationary random analysis and impact coefficients of highway bridges under moving vehicles, *Proc. 5th East Asia-Pacific Conference on Structural Engineering & Construction*, pp.345-350, 1995.