Structural Eng./Earthquake Eng., JSCE, Vol. 15, No. 1, 73s—84s, 1998 April
(J. Struct. Mech. Earthquake Eng., JSCE, No. 591/ I —43)

STRUCTURAL SYSTEM IDENTIFICATION USING THE H_∞ FILTER

Tadanobu SATO [1] and Kai QI [2]

[1] Member of JSCE, Dr. of Eng., Professor, Disaster Prevention Research Institute, Kyoto University (Gokasho, Uji, Kyoto 611-0011, Japan)

[2] Member of JSCE, Dr. of Eng., Researcher, Disaster Prevention Research Institute, Kyoto University (Gokasho, Uji, Kyoto 611-0011, Japan)

Identification algorithms were proposed that use the H_∞ filter to identify linear structural systems. Characteristics of the H_∞ filter for structural system identification were studied in detail by checking digital simulation results obtained by using the H_∞ and Kalman filters. Application of the proposed identification algorithms to SDOF and MDOF structural systems shows that the identified parameters obtained with the H_∞ filter converge faster and closer to the exact values than do those obtained with the Kalman filter. The H_∞ filter is more robust than the Kalman filter for identifying linear structural systems.

Key Words: *identification, H_∞ filter, Kalman filter, structural system, convergence, robust*

1. INTRODUCTION

The Kalman filter, based on the least-squares error, has been widely used as the time marching algorithm for structural system identification. The exact stochastic properties of disturbances must be known a priori in order to obtain optimal estimates with the Kalman filter; but, it is very difficult to know the exact stochastic properties of the disturbance in advance. The state estimate may be degraded if uncertainty of the disturbance statistics exists. The identification algorithm using the Kalman filter is started with an initial guess at the parameters and with the initial error covariance matrix. Convergence of the algorithm as well as the final values are known to depend, to a great extent, on this initial guess [1)].

Over the past several years, H_∞ control has received extensive attention and has been applied successfully in aerospace and mechanical engineering as a robust control approach [2)]. The H_∞ filtering problem, which is based on the H_∞ criterion, also has been solved from many standpoints [3), 4), 5)]. The H_∞ filtering problem is a state estimation problem of minimizing the maximum energy in the estimation error over all the disturbance trajectories. The state estimation based on this criterion is valid when a significant uncertainty in the disturbance statistics exists.

A robust filter, the H_∞ filter has been used to identify the parameters of the linear structural systems in our research. Identification algorithms are proposed for linear structural systems for which the acceleration, velocity and displacement of every floor is available; for linear structural systems for which only the velocity and displacement of each floor is available; and for linear structural systems for which the velocity and displacement responses of some floors are available for identification. The performance of the H_∞ filter is checked in detail by comparing the identified results obtained with the identification algorithms using the H_∞ and Kalman filters. Digital simulation results show that the characteristics of the H_∞ filter are better than those of the Kalman filter for structural system identification.

2. BACKGROUND OF THE H_∞ FILTER

Consider a system described by

$$x_{t+1} = A_t x_t + B_t \omega_t \quad (1)$$

$$y_t = C_t x_t + D_t v_t \quad (2)$$

$$u_t = L_t x_t \quad (3)$$

where $x_t \in \mathbf{R}^n$ is the state vector, $y_t \in \mathbf{R}^q$ the measurement and $u_t \in \mathbf{R}^p$ the vector to be estimated. The exogenous signals $\omega_t \in \mathbf{R}^m$ and $v_t \in \mathbf{R}^\ell$ respectively are the process and measurement noises. Moreover, we assume that $R_t := D_t D_t^T > 0$ holds for any t.

Let $\hat{u}_t$ be the estimate of u_t, and we assume that the estimate of the initial state x_0 is a priori given by $\bar{x}_0$. For the minimax filtering problem, the estimate of $\hat{u}_t$ tries to minimize the squared estimation error $\sum_{t=0}^{N}\|u_t - \hat{u}_t\|^2$, while the triple ($\bar{x}_0$, ω_t, v_t) tries to maximize the squared estimation error. Because arbitrary large values of $\|\omega_t\|$, $\|v_t\|$ and $\|x_0\|$ cause arbitrary large value of the estimation error, we define the cost function J as follows:

$$J(\hat{u}; x_0, \omega, v) = \sum_{t=0}^{N}\left\|u_t - \hat{u}_t\right\|^2 - \gamma^2\left(\sum_{t=0}^{N}\left\|\omega_t\right\|^2 + \sum_{t=0}^{N}\left\|v_t\right\|^2 + \left\|x_0 - \hat{x}_0\right\|_{\Pi^{-1}}^2\right) \quad (4)$$

The second term in the right-hand side is the penalty term on ω_t, v_t and x_0; γ is a positive constant which represents the magnitude of the penalty. Π is a positive definite weighting matrix which represents the uncertainty of the initial state x_0. From the game theoretic viewpoint, we can say that the filtered estimate $\hat{u}_t$ and the triple (ω_t, v_t, x_0) are the minimizing and maximizing polices of J, respectively [6].

The finite-horizon H_∞ filtering problem is to find estimates of u_t and x_t based on the measurement set $\{y_0, \cdots, y_t\}$ such that

$$\sup_{\omega, \upsilon, x_0} \frac{\sum_{k=0}^{N}\left\|u_t - \hat{u}_t\right\|^2}{\sum_{t=0}^{N}\left(\left\|\omega_t\right\|^2 + \left\|\upsilon_t\right\|^2\right) + (x_0 - \bar{x}_0)^T \Pi^{-1}(x_0 - \bar{x}_0)} < \gamma^2 \quad (5)$$

This condition is equivalent to

$$J(\hat{u}; x_0, \omega, v) < 0,$$

$$\sum_{t=0}^{N}\left\|\omega_t\right\|^2 + \sum_{t=0}^{N}\left\|v_t\right\|^2 + \left\|x_0 - \hat{x}_0\right\|_{\Pi^{-1}}^2 \neq 0 \quad (6)$$

The central H_∞ filter which satisfies the above H_∞ bound is given by [6]

$$\hat{x}_t = \bar{x}_t + K_t(y_t - C_t \bar{x}_t) \quad (7)$$

$$\bar{x}_{t+1} = A_t \hat{x}_t, \quad \hat{x}_0 = \bar{x}_0 \quad (8)$$

$$\hat{u}_t = L_t \hat{x}_t \quad (9)$$

$$K_t = \bar{P}_t C_t^T R_t^{-1} \quad (10)$$

$$\bar{P}_t = (P_t^{-1} + C_t^T R_t^{-1} C_t)^{-1} \quad (11)$$

in which $\hat{x}_t$ is the estimate of the system state vector x_t, and K_t the gain of the H_∞ filter at time t. $\bar{x}_{t+1}$ is the predicted value of the system state vector at time t+1. The covariance matrix P_t satisfies the Riccati difference equation

$$P_{t+1} = A_t P_t \left\{I_n + (C_t^T R_t^{-1} C_t - \gamma^{-2} L_t^T L_t) P_t\right\}^{-1} A_t^T + B_t B_t^T, \quad P_0 = \Pi \quad (12)$$

and a condition matrix V_t defined as

$$V_t := \gamma^2 I_p - L_t P_t (I_n + C_t^T R_t^{-1} C_t P_t)^{-1} L_t^T > 0 \quad (13)$$

in which I_n and I_p are identical matrices with dimensions of n×n and p×p. γ is selected as the minimum value which satisfy Eq. (13).

If γ in Eq. (12) tends to the infinite, the covariance matrix P_t becomes

$$P_{t+1} = A_t \left\{P_t^{-1} + C_t^T R_t^{-1} C_t\right\}^{-1} A_t^T + B_t B_t^T \quad (14)$$

Equation (14) is exactly the Riccati difference equation of the Kalman filter. Therefore, the H_∞ filter is a modified version of the Kalman filter.

3. ALGORITHM FOR STRUCTURAL SYSTEM IDENTIFICATION

(1) Algorithm for the case of a linear structural system for which the acceleration, velocity and displacement of each floor is available

In the identification of a structural system with n degrees of freedom, the measurement equation of the identification algorithm can be derived from the motion equation of the structural system. The motion equation is given by

$$\ddot{z}_t = \Theta_t H_t - \ddot{Z}_t \quad (15)$$

where $\Theta_t = \left[-M^{-1}C \ \ -M^{-1}K\right]$ and $H_t = \left[\dot{z}_t \ \ z_t\right]^T$ respectively are the n×$2n$ matrix of the structural parameters and the vector of the structural responses with $2n$ elements. M is the $n \times n$ mass matrix, C the damping matrix and K the stiffness matrix of the multiple degree of freedom (MDOF) system; $\ddot{z}_t$, $\dot{z}_t$ and z_t respectively are the n element vectors of the acceleration, velocity and displacement responses relative to the ground, and $\ddot{Z}_t$ is the ground motion acceleration. The measurement equation in the identification algorithm is given as[7]

$$y_t = C_t x_t + D_t v_t \quad (16)$$

in which $y_t = \ddot{z}_t + \ddot{Z}_t$. x_t, the system state vector with $2n^2$ elements to be identified, is

$$x_t = \{\theta(1, 1), \cdots, \theta(1, 2n), \theta(2, 1), \cdots, \theta(2, 2n), \cdots, \cdots, \theta(n, 1), \cdots, \theta(n, 2n)\}^T \quad (17)$$

and C_t is the measurement matrix with the dimensions of $n \times 2n^2$, given as

$$C_t = \begin{bmatrix} H_t^T & 0^T & \cdots & 0^T \\ 0^T & H_t^T & \cdots & 0^T \\ \vdots & \vdots & \ddots & \vdots \\ 0^T & 0^T & \cdots & H_t^T \end{bmatrix} \quad (18)$$

In Eq. (17), $\theta(i, j)$ represents the ij th element of matrix Θ in Eq. (15).

The system transfer equation is given by

$$x_{t+1} = I x_t + B_t \omega_t \quad (19)$$

in which I is the identical matrix.

When the responses of acceleration, velocity and displacement, as well as the ground motion acceleration can be obtained, the structural system defined by Eqs. (19) and (15) can be identified using the H_∞ filter given by Eqs. (7)~(12).

(2) Algorithm for the case of a linear structural system for which only the velocity and displacement of each floor is available

Assume that only the structural responses of velocity and displacement are available for the structural system identification. The structural parameters to be identified are the damping coefficient and stiffness matrices. As the mass matrix is assumed to be given, we therefore identify the natural frequency and damping constant of each story of the n DOF structural system defined by[8]

$$h_i = \frac{c_i}{2\sqrt{m_i k_i}}, \quad \omega_i = \sqrt{\frac{k_i}{m_i}}, \quad i = 1, \cdots, n \quad (20)$$

The state vector to be identified is defined by

$$x_t = \{\cdots\ z_i\ \dot{z}_i\ h_i\ \omega_i\ \cdots\}^T, \quad i = 1, \cdots, n \quad (21)$$

The state transfer equation is expressed as a non-linear equation of x_t;

$$\dot{x}_t = g(x_t) + B_t \omega_t \quad (22)$$

To apply the H_∞ filter to the system transfer equation defined by Eq. (22), the equation must be linearized by a proper linearization scheme as follows (see Appendix for details):

$$x_t = A_{t-1} x_{t-1} + d_{t-1} + B_t \omega_t \quad (23)$$

The system transfer matrix in Eq. (23) is given by

$$A_{t-1} \approx I + F_{t-1} dt, \quad F_{t-1} = \left. \frac{\partial g_i}{\partial x_j} \right|_{x_{t-1} = \hat{x}_{t-1}} \quad (24)$$

in which dt is the integration time interval, and d_{t-1} a constant term developed by the linearization process given by

$$d_{t-1} = \left(e^{F_{t-1} dt} - I\right) F_{t-1}^{-1} \{g(\hat{x}_{t-1}) - F_{t-1} \hat{x}_{t-1}\} \quad (25)$$

The pre-estimator of the state variable vector is given by the equation;

$$\bar{x}_t = A_{t-1} \hat{x}_{t-1} + d_{t-1} \quad (26)$$

The measurement equation is given by

$$y_t = C_t x_t + D_t \upsilon_t \quad (27)$$

in which y_t is the $2n$ observation vector defined by

$$y_t = \{\cdots\ z_i\ \dot{z}_i\ \cdots\}^T, \quad i = 1, \cdots, n \quad (28)$$

and C_t is the $2n \times 4n$ measurement matrix given as

$$C_t = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & [0] & \cdots & [0] \\ [0] & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \cdots & [0] \\ \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & \cdots & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{bmatrix} \quad (29)$$

By applying the H_∞ filter to the structural system defined by Eqs. (23) and (27), the damping coefficient and frequency of the structural system can be identified when only the velocity and displacement responses of the structural system are used for identification.

(3) Algorithm for the case of a linear structural system for which only the velocity and displacement of some floors are available

In the practical application of the structural system identification scheme, usually only the responses of some floors are available as observation data. To identify this kind of system, we just redefine the measurement matrix C_t in Eq. (27) and the measurement vector y_t.

The measurement matrix in Eq. (27) is a $2n \times 4n$

matrix consisting of n^2 sub-matrices with dimensions of 2 × 4. If the velocity and displacement responses of the i th floor are available for the identification, the ii th sub-matrix of the measurement matrix C_t is given by

$$C_{t;ii} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{30}$$

If the responses of j floors (j<n, j is the total number of floors for which responses are available for identification) are obtained for structural system identification, the measurement matrix C_t is given as a 2j×4n matrix with the sub-matrix $C_{t;ii}$ defined by Eq. (30).

The observation vector y_t is defined by

$$y_t = \{\cdots \ z_i \ \ \dot{z}_i \ \cdots\}^T \tag{31}$$

in which i refers to the floor whose responses is available for structural system identification.

By applying the H_∞ filter to the system defined by Eqs. (23), (27), (30) and (31), the damping coefficients and natural frequencies of each floor can be identified for a partly-observed structural system.

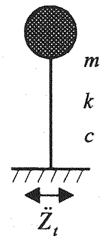

Fig. 1 SDOF system model

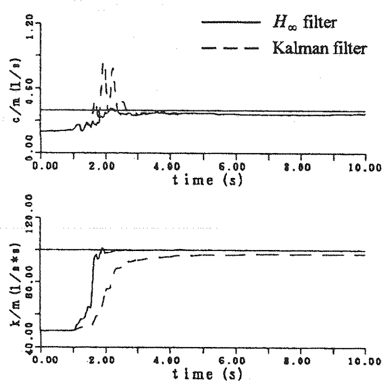

Fig. 2 Identified para. of the SDOF system obtained with the H_∞ and Kalman filters (R=1.0, $P_0 = diag\{0.001 \quad 0.1\}$, γ =3.9)

4. STRUCTURAL SYSTEM IDENTIFICATION

The identification algorithms developed were applied to different structural systems. The seismic responses of these systems were simulated as the observed data in the identification. The El Centro NS (1940) earthquake record with a scaled peak value of 50.0 gal was used as the input excitation. Identification results obtained using the H_∞ filter are compared with those obtained using the Kalman filter to show the performance of the H_∞ filter in the identification of structural systems.

In practical application of identification, we have to assume the initial values $\bar{x}_0$ and P_0, and noise covariance R_t, as design values of the filter. But we do not know the real parameters of the system to be identified. Therefore, the sensitivity of the filter to the initial value is a very important characteristics for the filter. Therefore, the effect of design values of the filter on the identification results are checked in detail in our digital simulation.

(1) Identification for the case of a single degree of freedom (SDOF) structural system

Assume that the responses of acceleration, velocity and displacement of the SDOF linear structural system are available for identification. **Fig. 1** shows the model of the SDOF system used to generate the observation time history. The parameters are m=1.0, c=0.4 and k=100.0. The initial value of P_t was set as $P_0 = diag\{0.001 \quad 0.1\}$. The initial value of the state vector, $\bar{x}_0$, is assumed to be given by 50% of the real value. Pink noise (frequency band: 0-25 Hz) with a standard deviation set at 5% of the standard deviation of the structural response is used as the measurement noise. **Fig. 2** shows the results when the noise covariance, defined by $R_t = D_t D_t^T$, is set as 1.0. The parameters identified using the H_∞ filter converge faster and closer to the real values than those identified using the Kalman filter. The H_∞ filter gain is more sensitive to the identification error than the Kalman gain.

In the digital simulation, the initial value of the state vector, $\bar{x}_0$, is set at different values to check the effect of the initial value of the state vector on the identified results when the covariance value R_t changes from 0.01 to 1.6. In this case, we used the

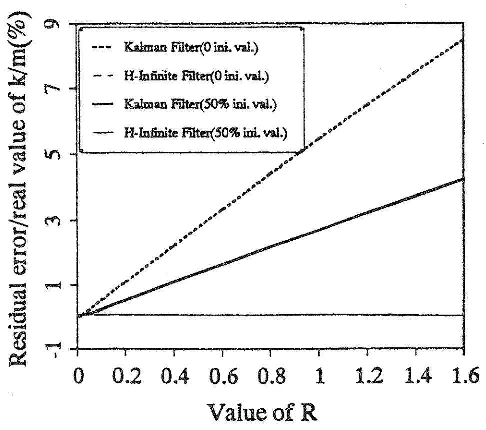

Fig. 3 Residual error of the identified para. *k/m* when $\bar{x}_0=0$ and is 50% of the real value (R=1.0, $P_0 = diag\{0.001 \quad 0.1\}$)

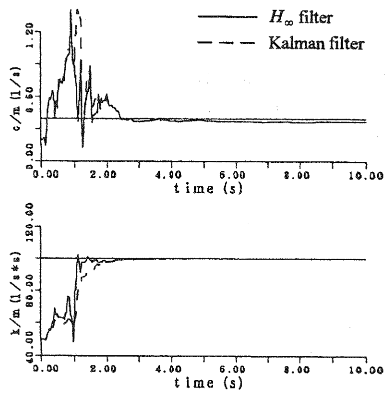

Fig. 4 Identified para. of the SDOF system obtained with the H_∞ and Kalman filters (R=0.01, $P_0 = diag\{0.001 \quad 0.1\}$, γ =2.7)

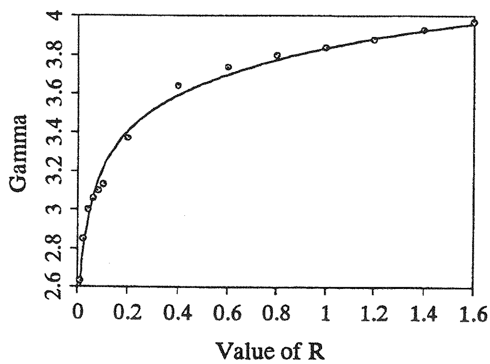

Fig. 5 Minimum gamma values versus value R_t for the SDOF system identification ($\bar{x}_0$=is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$)

same set of observation data. In **Fig. 3**, the solid lines show the ratio of the residual error of the identified values of *k/m* to the real values obtained with the algorithms using the H_∞ (thin solid line) and Kalman (bold solid line) filters when the initial state vector $\bar{x}_0$ is given by 50% of the real value. The dashed lines in **Fig. 3** shows the residual error ratio of the identified values of *k/m* obtained with the algorithms using the H_∞ (thin dashed line consistent with the thin solid line) and Kalman (bold dashed line) filters when the vector $\bar{x}_0$ is set at zero. If the initial value of the system state vector, $\bar{x}_0$, is set far from the real value, the performance of the Kalman filter would deteriorates, whereas the residual error of the identified parameter *k/m* for the H_∞ filter is very small. Compared with the Kalman filter, the H_∞ filter performs better for the identification of the parameter *k/m* when the initial value of the state vector can not be set near the real value.

The effect of the noise covariance, R_t, on the identified results obtained using the H_∞ and Kalman filters also can be checked using the results shown in **Fig. 3**. In the case of the Kalman filter, when the value of R_t becomes large, a large residual error in the identified value is expected. The bold lines (solid and dashed) in **Fig. 3** show that the residual errors of the identified parameter *k/m* increase rapidly when R_t changes from 0.01 to 1.6.

For the H_∞ filter, the residual error of the identified parameter *k/m* is very small and is not affected by the value of R_t. When R_t is set to be small, the identified parameters converge quickly, but oscillation is large before the identified parameters to be converged, as shown in **Fig. 4** (R=0.01). **Fig. 5** shows the minimum value of γ which satisfies Eq. (13) when R_t changes from 0.01 to 1.6. For the H_∞ filter, γ works as an adapter to limit the identified error of *k/m* under a certain level. For both the H_∞ and Kalman filters, however, the residual error of *c/m* increases as the value of R_t increases. There is no obvious difference between the residual errors obtained using either filter, as shown in **Fig. 6** ($P_0 = diag\{0.001 \quad 0.1\}$, $\bar{x}_0$=50% of the real value).

The effect of the level of the measurement noise on structural system identification also is checked in the digital simulation. The initial value of P_0 is set at $P_0 = diag\{0.001 \quad 0.1\}$. Pink noise with a standard deviation of up to 50% of the standard deviation of the structural response is added to the structural

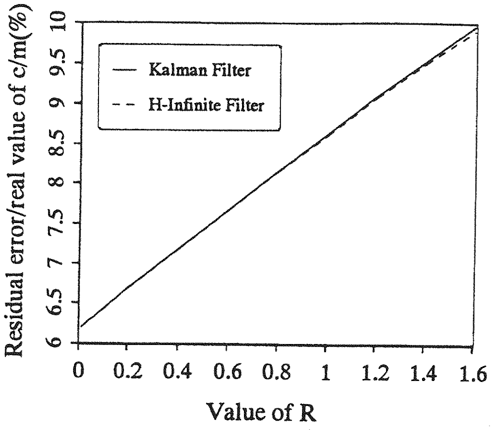

Fig. 6 Residual error of the identified para. of *c/m* of the SDOF system versus value R_t ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$)

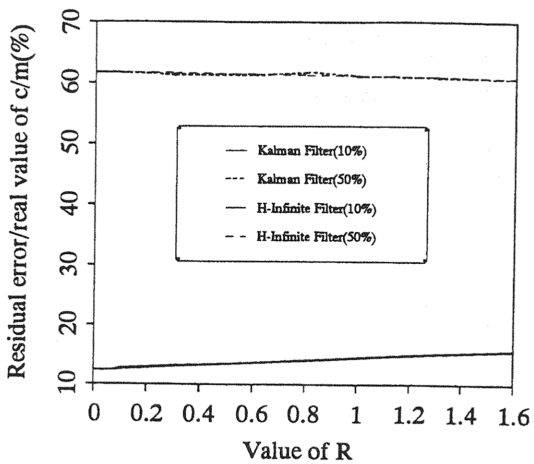

Fig. 8 Residual error of the identified para. *c/m* for different measurement noise levels ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$)

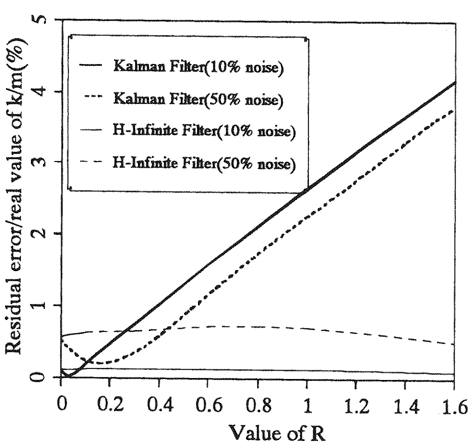

Fig. 7 Residual error of the identified para. *k/m* for different measurement noise levels ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$)

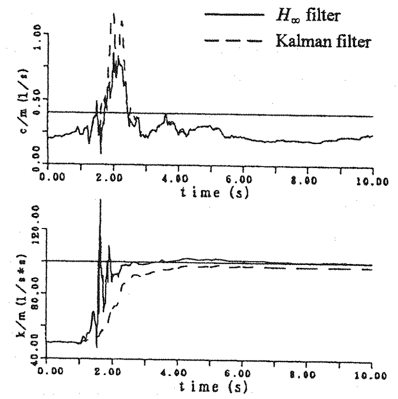

Fig. 9 Identified para. of the SDOF system when observation data are contaminated heavily by measurement noise ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$, γ =3.84)

responses as measurement noise. **Fig. 7** shows the ratios of the residual error of *k/m* to the real value when R_t changes from 0.001 to 1.6 for cases in which the standard deviation of the measurement noise is set at 10% and 50% that of the structural responses. The bold curves (solid and dashed) obtained with the Kalman filter show that the residual errors have minimum values at specific values of R_t. When R_t is beyond a certain range, the residual errors of *k/m* increase rapidly. The performance of the H_∞ filter is better than that of the Kalman filter. The residual error of *k/m* obtained with the H_∞ filter is not affected by the value of R_t even when the observation data are contaminated by different levels of measurement noise. The H_∞ filter is more robust than the Kalman filter. For the identified parameter *c/m*, residual error increases as the level of measurement noise increases. There is no obvious difference between the identification results obtained with either filter, as shown in **Fig. 8**. But, for the residual error of the identified parameter *c/m,* if the observation data is contaminated heavily by measurement noise (e.g., the standard deviation of the measurement noise is set at 30% that of the

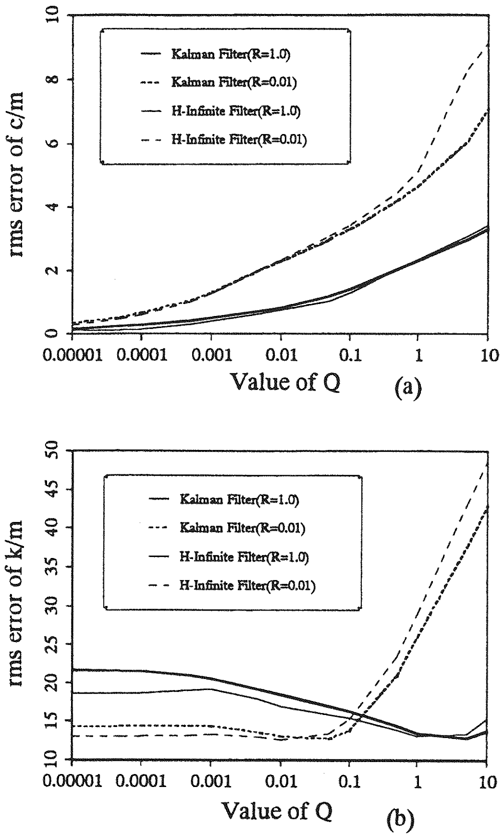

Fig. 10 Effect of process noise on rms value of the identified para. of the SDOF system ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$)

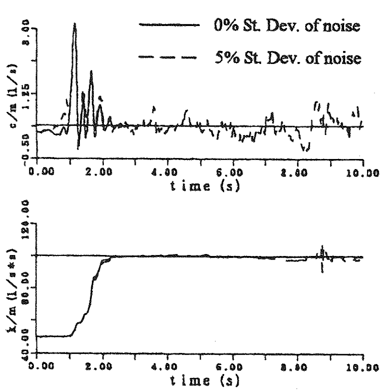

Fig. 11 Identified para. of the SDOF system under different measurement noise levels when process noise exists ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$, R=1.0, Q=0.001)

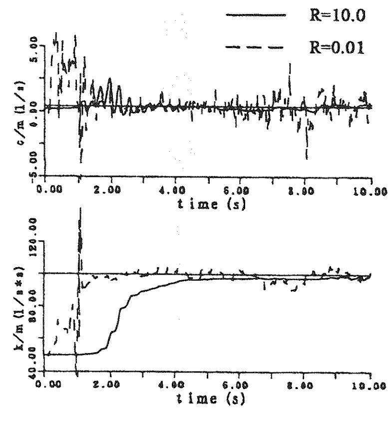

Fig. 12 Identified para. of the SDOF system with different value R_t when process noise exists ($\bar{x}_0$ =is 50% of the real value, $P_0 = diag\{0.001 \quad 0.1\}$, Q=0.001)

structural responses), it is very difficult for either filter to obtain the correct identified value of *c/m*, as shown in **Fig. 9**.

The effect of process noise on identification was investigated by setting different values of B_t. **Fig. 10** shows the rms values of the identified *k/m* and *c/m* values obtained with the H_∞ and Kalman filters for R_t=1.0 and R_t=0.01 when value $Q_t = B_t^T B_t$ changes from 0.00001 to 10.0. The standard deviation of the measurement noises is set at 5% of the standard deviation of the structural response. The rms values of the identified parameters become very large when process noise exists. The rms values of the identified parameters obtained with the H_∞ filter, in particular, increase rapidly as the value of Q_t increases when Q_t is larger than 0.1. **Fig. 11** shows the time history of the identified parameter *k/m* obtained with the H_∞ filter when the standard deviation of the measurement noises is set at zero and 5% of the standard deviation of the structural response, and Q_t is 0.001. The H_∞ filter can not remove contaminating noises from the observation data for the identification of structural parameters. The reason is that Q_t acts as a forgetting factor which fades away the effect of pre-information on the post-estimator of the state variable vector calculated from the observation data. Increasing R_t decreases the rms value of the identified parameter; but, it leads to slow convergence of the identified value *k/m*. The identified value c/m can not converge even if the value of R_t is set very high, as in **Fig. 12** (Q_t=0.001).

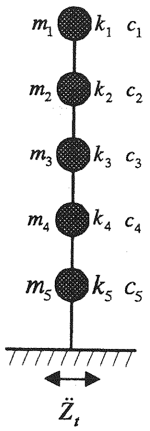

Fig. 13 5 DOF system model

(2) Identification for the case of a 5 DOF linear structural system for which the acceleration, velocity and displacement responses of each floor are available

Assume that all the responses of acceleration, velocity, and displacement of the MDOF linear system are available for the identification. In this case, the respective dimensions of state vector x_t, given by Eq. (17), and measurement matrix C_t, given by Eq. (18), are $2n^2 \times 1$ and $n \times 2n^2$. If the number of degrees of freedom increases, the dimensions of the variables in the identification program increase rapidly. A large amount of computer memory is needed, leading to difficulties in calculation. To cope with this, we divided the system defined by Eqs. (19) and (16) into n sub-systems

$$x_{t+1}^i = x_t^i + B_t^i \omega_t^i \quad (i=1,\ n) \tag{32}$$

$$y_t^i = C_t^i x_t^i + D_t^i \upsilon_t^i \quad (i=1,n) \tag{33}$$

where $x_t^i = \{\theta(i,\ 1),\ \cdots,\ \theta(i,\ 2n)\}^T$ and C_t^i H_t^T are vectors with $2n$ elements. The dimensions of all the variables in the program therefore can be reduced effectively; e.g., the dimension of P is reduced from $2n^2 \times 2n^2$ to $2n \times 2n$. To identify the parameters defined by the respective n sub-systems, we can identify the parameters of the structural system without defining the large dimensions of the variables.

As shown in **Fig. 13**, the 5 DOF linear structural system with parameters $m_i = 0.12553$, $c_i = 0.07$, $k_i = 24.5$ $(i = 1,\ \cdots,\ 5)$ was used to generate the observation time history. Pink noise with a standard deviation of 5% of the standard deviation of the structural response is added to the simulated structural responses as measurement noise. The initial value of the state vector, $\bar{x}_0$, is assumed to be given by 50% of the real value. The initial covariance matrix is given by

$$P_0 = diag\{\cdots \quad px_{0;i}^2 \quad \cdots\} \quad (i = 1, \cdots, 10) \tag{34}$$

in which $x_{0;i}$ is the ith component of the initial value of $\bar{x}_0$, and p is set at 1.0. The covariance of noise, R_t, is set at diag$\{\cdots\ 0.01\ \cdots\}$.

The state vector, x, is identified with the identification algorithms for both the H_∞ and Kalman filters. The simulation results show that the H_∞ filter performs better for an MDOF structural system identification. **Fig. 14** shows the time history

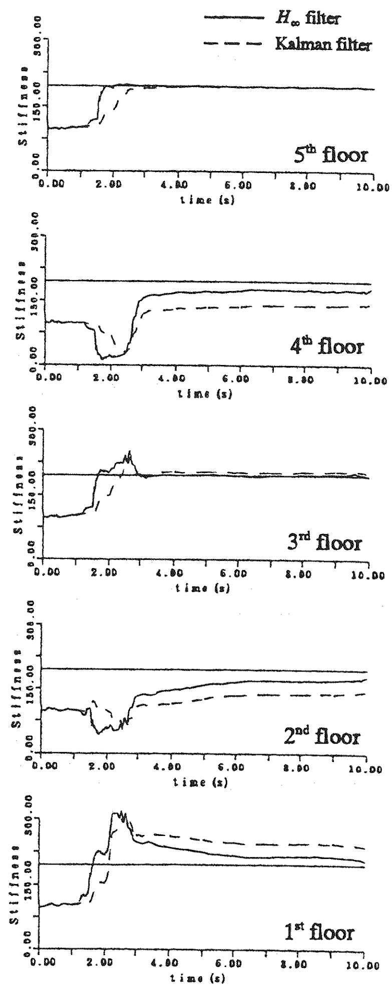

Fig. 14 Identified para. *k/m* of the 5 DOF system obtained with the H_∞ and Kalman filters ($R=diag\{\cdots\ 0.01\ \cdots\}$, p=1.0, $\bar{x}_0$=is 50% of the real value, γ=15.6)

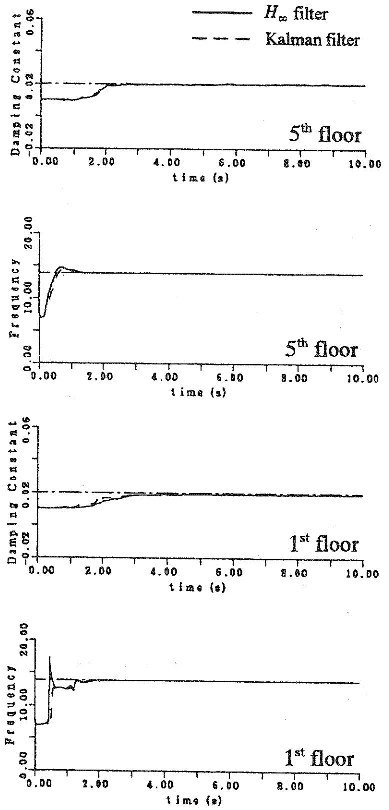

Fig. 15 Identified para. of the 5 DOF system obtained when initial values are set properly (R=*diag*{$\cdots$ 0.1 $\cdots$}, p=1.0, $\bar{x}_0$=is 50% of the real value, γ=46.9)

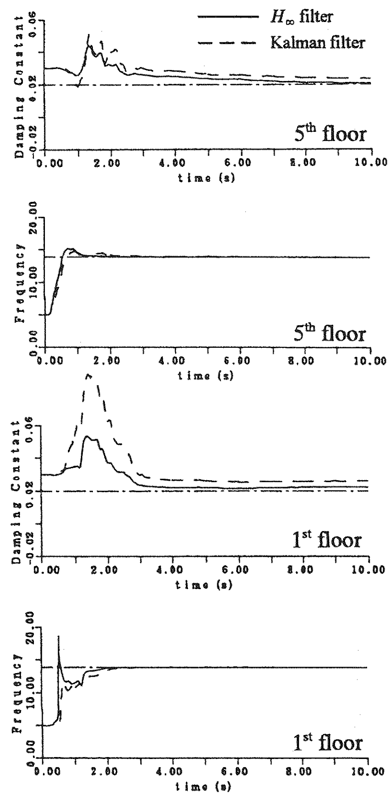

Fig. 16 Identified para. of the 5 DOF system when initial values are not set properly (R=*diag*{$\cdots$ 0.1 $\cdots$}, p=1.0, $\bar{x}_0 = diag\{\cdots\ 0.03\ \ 5.0\ \cdots\}$, γ=34.7)

of the identified parameter *k*/*m* for each floor of the 5 DOF structural system. The identified parameters obtained with the H_∞ filter converge faster, and residual error is smaller than the values obtained with the Kalman filter.

The design values of the filter, such as noise covariance R_t and the initial value of state vector $\bar{x}_0$, heavily affect the performance of the filter. The identified parameters of some floor in this case could not converge because the design values of the filter were not set properly. Good identified results can be obtained if we set suitable design values.

(3) Identification for a 5 DOF linear structural system for which all the floor responses of velocity and displacement are available

Assume that only the responses of velocity and displacement are available for each floor in the identification of a 5 DOF linear system. The parameters of the structural model used to generate the observed data are the same as in the model in **Fig. 13**. Pink noise with a standard deviation of 5% of the standard deviation of the structural response is used as measurement noise. The initial covariance matrix also is given by Eq. (34).

Fig. 15 shows the identified parameters of the damping coefficients and frequencies of the 1st and 5th floors of the structural system when the initial covariance matrix, defined by Eq. (34), can be set properly (in this case, the initial value of the state vector is 50% of the real value and p=1.0). Simulation results show that the performance of the response is used as measurement noise. The initial H_∞ filter is better than that of the Kalman filter. If the initial value, P_0, is set properly, good identified results can be obtained with algorithms using the H_∞ and Kalman filters. The identified parameter *k*/*m* obtained with the H_∞ filter converges a little faster. When the initial value, P_0, can not be set properly, the residual error of the identified values obtained with the Kalman filter is very large. **Fig.**

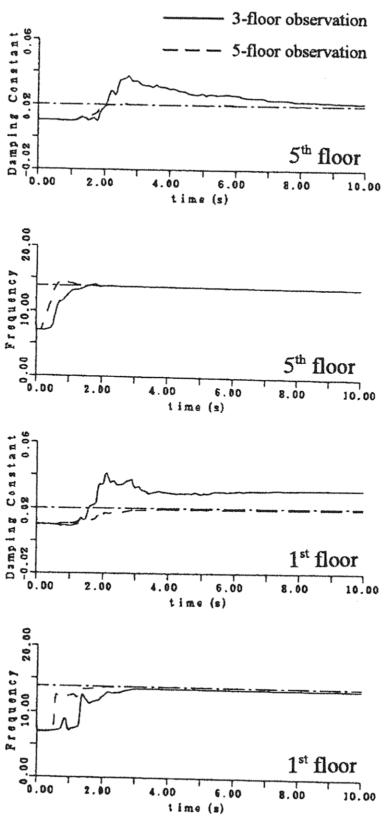

Fig. 17 Identified para. of the 5 DOF system for completely observed and partly observed systems using the Kalman filter ($R=diag\{\cdots\ 0.1\ \cdots\}$, p=1.0, $\bar{x}_0$=is 50% of the real value)

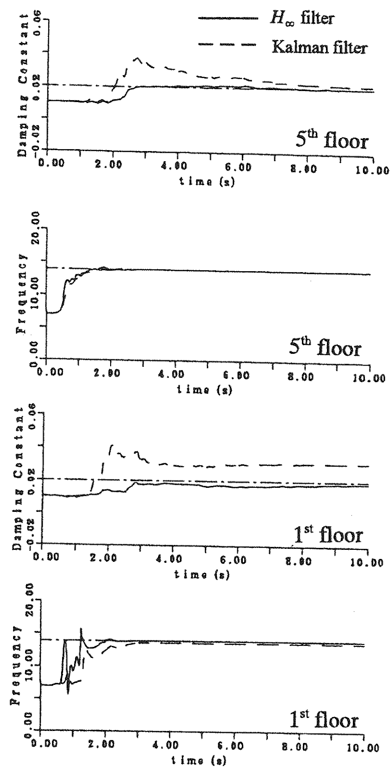

Fig. 18 Comparison of the identified para. of 5 DOF system with 3-floor observation data obtained with the H_∞ and Kalman filters ($R=diag\{\cdots\ 0.1\ \cdots\}$, p=1.0, $\bar{x}_0$=is 50% of the real value, γ=27.5)

16 shows the identified parameters of the damping coefficients and frequencies of the 1st and 5th floors of the structure when the initial covariance matrix can not be set properly (in this case, the initial values of damping and frequency are set at 0.03 and 5.0, and p=1.0). The figure also shows that the H_∞ filter gives very good identification results even if the initial value, P_0, does not guarantee the accuracy of identification results when we use the algorithm with the Kalman filter. The identification algorithm with the H_∞ filter is more robust than that with the Kalman filter for structural system identification.

(4) Identification for a 5 DOF linear structural system for which the responses of velocity and displacement of some floors are available

In the identification of multiple-floor structural system, the dynamic responses of the structure of all-floor or partly-floor should be obtained as observation data. It is not practical to measurement the all-floor responses of the structure. If we can identify the parameters of the structural system with measurement from limited floors of the structure is what we concern in this subsection.

Assume that only the responses of velocity and displacement of the 1st, 3rd, and 5th floors are available for the identification of a 5 DOF linear system. The parameters of the structural model used to generate the observed data are the same as those in **Fig. 13**. Pink noise with a standard deviation set at 5% of the standard deviation of the structural value of the state vector, $\bar{x}_0$, is assumed to be given by 50% of the real value. The initial covariance matrix is given by

$$P_0 = diag\left\{\cdots\ px_{0i}^2\ \cdots\right\} \quad (i=1,\cdots,6) \qquad (35)$$

Fig. 17 shows the identification results of the 1st

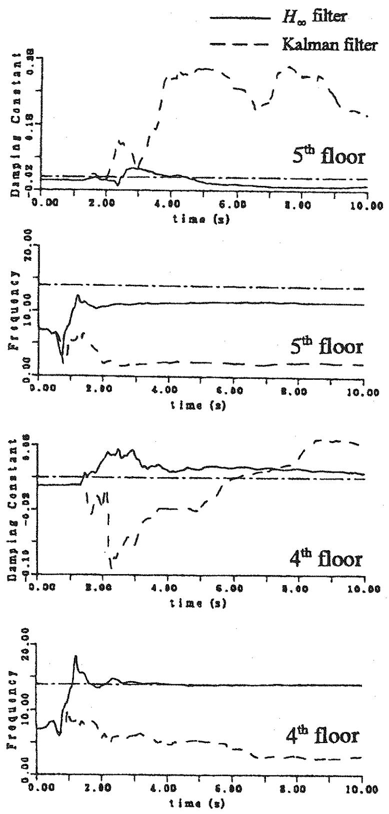

Fig. 19 Identified para. of a 5 DOF system with 2-floor observation data obtained with the H_∞ and Kalman filters (R=$diag\{\cdots\ 0.1\ \cdots\}$, p=1.0, $\bar{x}_0$=is 50% of the real value, γ =37.2)

and 5th floors of the structure obtained with the Kalman filter for cases in which all the floor responses or only responses of 3 floors are obtained for the identification. The solid curves show that the identified parameters for the 3-floor-only observation data converge more slowly and that the residual errors are larger than when the responses for all the floors are available for the identification. Identification efficiency for a partly observed system decreases because the system has less information than one that has all the observed data.

Fig. 18 shows the time histories of identified parameters of the 1st and 5th floors of an structural system when only the responses of the 1st, 3rd, and 5th floors are available for the identification using both the H_∞ and Kalman filters. The simulation results indicate that the H_∞ filter avoids a decrease in identification efficiency. The algorithm guarantees that the identified parameters converge faster and closer to the real values than those obtained with the algorithm using the Kalman filter.

The case of only responses of the 1st and 5th floors being available for the identification also was checked by digital simulation. **Fig. 19** shows the identified parameters of the 4th and 5th floors. Because there is not enough information for system parameter identification, i.e., the number of floors, whose responses are available for the identification of multiple-floor structural system, is under a certain level, neither of the identification algorithms using the H_∞ or Kalman filters produce good results. The parameter of ω_5, however, converges very near to the real value when the H_∞ filter is used.

5. CONCLUSION

Structural identification algorithms are proposed using the H_∞ filter to identify the parameters of linear structural systems. These algorithms were applied to structural systems which had completely or partly observed structural seismic responses. Identification results of the digital simulations show that the performance of the H_∞ filter in structural system identification is better than that of the Kalman filter. The conclusions of this study are as follows:

(1) The identified parameters of the structural system obtained with the H_∞ filter converge faster and closer to the real values of the structural systems than do those obtained with the Kalman filter.

(2) A large noise covariance R_t leads to a larger residual error in the identified parameters when the Kalman filter is used. The residual error in the identified parameter k/m when the H_∞ filter is used is very small and is not affected by the value of R_t.

(3) The initial value of the system state vector has no obvious effect on the identified parameter k/m obtained with the H_∞ filter, whereas the performance of the Kalman filter deteriorates when there is an unsuitable initial system state vector.

(4) For the Kalman filter, the initial covariance matrix, P_0, must be set very carefully to get good identification results. In the algorithm using the H_∞ filter, the identified parameters converge faster and the residual error is very small even when the initial covariance matrix,

P_0, does not guarantee the accuracy of identification results if P_0 is used for the algorithm with the Kalman filter.

(5) For identification of a system with partly observed str, ctural responses, the algorithm using the H_∞ filter performs better than the one using the Kalman filter. The H_∞ filter prevents a decrease in identification efficiency, but when sufficient information is not available for the identification, even the H_∞ filter can not guarantee identification results.

(6) The H_∞ filter is more robust than the Kalman filter for the identification of linear structural systems.

APPENDIX:

Linearization and discretization of equation $\dot{x} = g(x)$:

The first order expansion of the above equation with respect to $\hat{x}_{t-1}$ gives

$$\dot{x} \approx g(\hat{x}_{t-1}) + F_{t-1}(x - \hat{x}_{t-1}) \quad \text{(A-1)}$$

The general solution of Eq. (A-1) is expressed by

$$x_\tau = e^{F_{t-1}\tau}\left\{\int_{t-1}^{\tau} c_{t-1} e^{-F_{t-1}\tau} d\tau + e^{F_{t-1}(t-1)} x_{t-1}\right\} \quad \text{(A-2)}$$

in which

$$c_{t-1} = g(\hat{x}_{t-1}) - F_{t-1}\hat{x}_{t-1} \quad \text{(A-3)}$$

Placing $\tau = t$ in Eq. (A-2) gives the formula of Eq. (23);

$$x_t = A_{t-1}x_{t-1} + d_{t-1} \quad \text{(A-4)}$$

$$A_{t-1} = e^{F_{t-1}dt} \quad \text{(A-5)}$$

$$d_{t-1} = \int_{t-1}^{t} e^{F_{t-1}(t-\tau)} c_{t-1} d\tau \quad \text{(A-6)}$$

in which dt is the time interval.

REFERENCES

1) GHANEM, R. and SHINOZUKA, M.: Structural-System Identification. I: Theory, *J. of Engr. Mech, ASCE*, Vol. 121, No. 2, pp 255-264, 1995.

2) STOORVOGEL, A.: *The H_∞ Control Problem, A State Space Approach*, Prentice Hall, 1992.

3) TAKABA, K. and KATAYAMA, T.: Discrete-time H_∞ Algebraic Riccati Equation and Parametrization of all H_∞ Filters, *International J. of Control,* Vol. 64, No. 6, pp. 1129-1149, 1996.

4) PARTINGTON, J. R.: Robust Identification and Interpolation in H_∞, *International J. of Control,* Vol. 54, No. 5, pp. 1281-1290, 1991.

5) NAGPAL, K. M. and KHARGONEKAR, P. P.: Filtering and Smoothing in an H_∞ Setting, *IEEE Transactions on Automatic Control,* Vol. 36, No. 2, pp. 152-166, 1991.

6) TAKABA, K.: *Studies on H_∞ Filtering Problems for Discrete-Time System*, Doctoral Dissertation, Department of Applied Mathematics and Physic, Kyoto, University, January 1996.

7) SATO, T. and KIKUKAWA, M.: A Linear Algorithm to Identify the Nonlinear Structural System Equations, *Journal of JSCE* (in Japanese, in press).

8) SATO, T. and TAKEI, K.: Real Time Robust Algorithm for Structural System with Time-Varying Dynamic Characteristics, *Proceedings of SPIE's Symposium on Smart Structures and Materials*, San Diego, California, USA, March 2-6, pp. 393-404, 1997.

(Received May 15, 1997)