

SHAPE OPTIMIZATION BY BIOLOGICAL GROWTH STRAIN METHOD - MINIMIZATION OF STRESS CONCENTRATION

Kai-Lin HSU¹ and Taketo UOMOTO²

¹Member of JSCE, M. Eng., Graduate Student, Institute of Industrial Science., University of Tokyo (7-22-1 Roppongi, Minato-ku, Tokyo 106, Japan)

²Member of JSCE, Dr. Eng., Professor, Institute of Industrial Science., University of Tokyo (7-22-1 Roppongi, Minato-ku, Tokyo 106, Japan)

With the advent of structural optimization, the researches for shape optimization have also been under development with growing interest. In this research, in order to minimize the stress concentration, one optimal shape design method originated from the concept of biological adaptation process was proposed. The characteristic of this new method is in no need of structural sensitivity analysis. The algorithm and availability of this method was explained and verified in this paper. In addition, the excellence of computation efficiency of this method was also illustrated.

Key Words : *optimal shape design, stress concentration, simulated biological growth*

1. INTRODUCTION

(1) Literature review

By the notch stress theory developed by Baud¹⁾, Neuber²⁾ and Schack³⁾, etc., the shape optimization problem of minimizing stress concentration in the continuous structures (also termed as stress optimal design) can be focused on the adjacent area where stress concentration occurs instead of dealing with the whole design structure. With this understanding on the optimal shape design methods of minimizing stress concentration, various attempts⁴⁾ have been made either from proposing various numerical optimization techniques or finding the practical application cases. However, for most of those approaches, in order to effectively find the optimum, structural sensitivity analysis is often requisite. Yet, by observing the essence of this calculation process, the computation efficiency of those optimization techniques using structural sensitivity, especially on the computation time, is significantly detrimental. Nevertheless, up to date, there is very little reliable report related to such numerical deficiency caused by bulky computation for optimal shape design methods.

On the other hand, from the comparative viewpoint on the mechanism of minimizing stress concentration, it is found that the adaptation

process of biological tissues by shape variation to their environment is very effective in reducing stress concentration in the biological structures. Such kind of high similarity between the biological adaptation process and the stress optimal design in the continuous structures has motivated the researchers to propose the methods based on their simulated biological growth rules. However, their methods (e.g. Umentai and Hirai⁵⁾) were generally proposed to move the design points by some choice on the direction and magnitude of the nodal updating vectors. Such kind of choice on nodal updating vectors generally needs the subjective judgment of the developers, which may make these methods unfriendly to ordinary users. As a result, our interest of this research is to propose one new optimal shape design method by avoiding the choice on the direction and magnitude of the nodal updating vectors, the essence of which is more close to the nature of the biological adaptation process.

(2) Scope of this research

Based on the aforementioned questions, the work of this research is to describe one gradientless optimal shape design method called "biological growth strain method (BGSM)" by simulating the biological adaptation process. By this proposed method, the stress concentration of design

structures can be effectively minimized through the illustrative examples. In addition, in order to verify the excellence on computation efficiency, especially for saving computation time, numerical comparison between BGSM and one sensitivity-based optimality approach method was also given as the pioneering work for the efficiency comparison of optimal shape design methods. It is worth noting that the structures under discussion in this paper be limited to two-dimensional structures. But, the algorithm of BGSM can be easily extended to three-dimensional structures by partly modification on the structural freedom of design structures and the number of design variables without extensive difficulty.

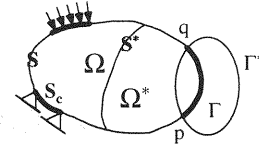
(3) Composition of this paper

Hence, the paper consists of three parts: first of all, the mathematical formulation and numerical process of the design problem by finite element method as well as the characteristics of BGSM will be given in Chapter 2 and 3 respectively; secondly, the numerical verification of the proposed method and application on various fields of structural design are illustrated in Chapter 4 and finally the numerical comparison between this proposed method and one sensitivity-based optimality criteria approach is given in Chapter 5 and the concluding remarks is included in Chapter 6.

2. FORMULATION OF STRESS OPTIMAL DESIGN

For a linear, elastic, homogeneous and continuous body, the stress optimal design problem is schematized in **Fig.1**. Ω and S is the domain of design structure and its boundary. Based on the notch stress theory, the optimization process is only considered on the subdomain of the design structure Ω^* and its boundary S^* . As shown in **Fig.1**, the selected design profile between p and q is defined as Γ and its allowed variation domain Γ^* due to the constructional or technical consideration. Based on these notations, the mathematical interpretation of stress optimal design problem in the continuous body can be expressed as Eq.(1a) up to (1c) by minimizing the quadratic difference between the reference stress σ_{ref} (e.g. the average of the stresses along Γ) and the equivalent stress $\bar{\sigma}$ along Γ within Γ^* . Here, Eq.(1a) is defined as objective function.

$$\text{Min} \left\{ \int_{\Gamma} (\bar{\sigma} - \sigma_{ref})^2 \delta \Gamma \right\} \quad (1a)$$



Ω : Given Domain Ω^* : Design Domain $\subset \Omega$
 S : Boundary of Ω S^* : Boundary of Ω^*
 Γ : Design Profile S_c : Fixed Boundary
between p and q Γ^* : Variation Domain

Fig.1 Sketch of stress optimal design problem for its analytical formulation

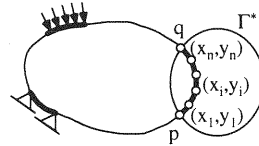


Fig.2 Sketch of stress optimal design problem by selecting the finite design points along Γ within Γ^*

On the condition that

$$\Gamma \subset \Gamma^* \quad (1b)$$

$$\bar{\sigma} \leq \bar{\sigma}'' \quad (1c)$$

Where $\bar{\sigma}''$: the allowable stress. However, as pointed out in the recent structural optimization researches, the classical optimal shape design approach on the continuous bodies led to complicated differential and integral equations which could be only solved in special cases. As a result, the approaches using numerical methods like finite element method (FEM), boundary element method (BEM) or finite difference method (FDM) with the combination of mathematical programming or optimality criteria techniques are most favored. Here, with the use of FEM, the analytical formulation in Eq.(1a) up to (1c) needs to be expressed in the numerical form of the following formulation from Eq.(2a) up to (2c) by assigning finite number of design points along Γ , the sketch of which is shown in **Fig.2**.

$$\text{Min} \left\{ \sum_{(x_i, y_i), i=1..n} (\bar{\sigma}_i - \sigma_{ref})^2 \right\} \quad (2a)$$

$$(x_i, y_i) \in \Gamma^* \quad (2b)$$

$$\bar{\sigma}_i = \bar{\sigma}(x_i, y_i) \leq \bar{\sigma}'' \quad (2c)$$

where (x_i, y_i) : the coordinates of each design point i along the design profile Γ , $i = 1..n$ and $\bar{\sigma}_i$ is the equivalent stress at each design point i . During the process of optimization, if the objective function Eq.(2a) can be satisfied (i.e. the value of

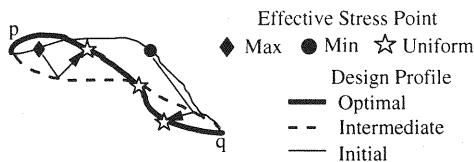


Fig.3 Illustration for curvature variation on the design profile based on Baud's postulation

the objective function converges) and the allowable variation domain Γ^* as well as the allowable stress $\bar{\sigma}^u$ is not violated as defined in Eq.(2b) up to (2c). Then, the result of this design can be considered as an optimum.

3. CHARACTERISTICS OF BIOLOGICAL GROWTH STRAIN METHOD

(1) Similarity between stress optimal design and biological adaptation process

Through years of researches, "uniform stress distribution design (or equi-strength design)" - the hypothesis postulated by Baud showed its practical validation by analytical demonstration⁶⁾. Based on this validated Baud's postulation, the stress optimal design can be achieved by varying the outer-surface shape (i.e. curvature of the design points along the design profile) of the design structure. By iteratively repeating the above process, the outer-surface shape can be considered to be optimized once if the conditions (Eq.(2a) up to Eq.(2c)) are satisfied. The iterative optimization process can be illustrated by Fig.3. On the other hand, by looking at the load carriers in animals or plants like bones or brunches, it is undoubtedly convinced that the load carriers develop their optimum by well adapting themselves to their environment at a certain loading conditions⁷⁾. With the further inspect on those biological adaptation process, several interesting aspects can be summarized in the following points: (1) the state of constant stress can be found on the surface of those well-adapted biological structures and (2) the adaptation process of these biological tissues is carried out by growth or atrophy of their live tissues near the stress-concentrated surface. By comparing these observed facts on the biological adaptation process with stress optimal design process, it is not difficult to find out the high similarity between them on the optimality condition and the optimization process. Here, it is worth mentioning that the gradients for objective function and the constraint conditions is usually necessary for the conventional stress optimal design^{8),9),10)} while the biological optimization process needs no gradient information. Thus, it was motivated to propose one gradientless

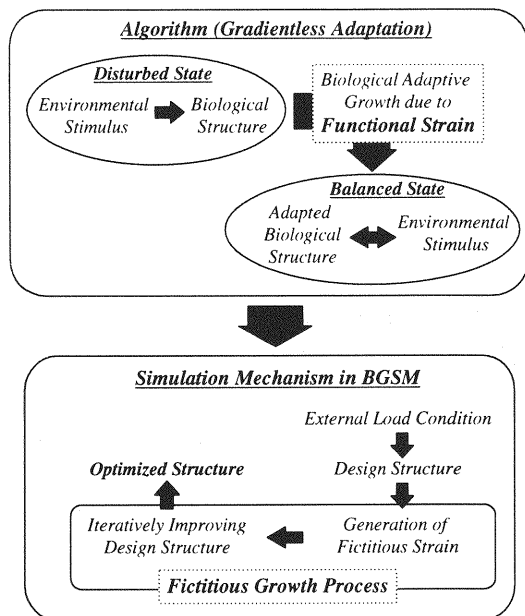


Fig.4 Simulation on Biological Self-Adaptation Process

stress optimal design method by simulating the biological adaptation process to update the design variables instead of the gradient optimization techniques, the simulation mechanism of which is illustrated in Fig.4. According to the research on the biological self-adaptation process, when the external stimulus disturbs biological structures, the biological structures will adapt themselves in an iterative way by generating functional strain until the balanced state between the adapted biological structures and the external stimulus can be found. Therefore, the simulation mechanism of BGSM is proposed as follow; when the external load condition is applied on the design structure, one parameter called fictitious strain is generated in the updating process (fictitious growth process). In this research, this parameter is called "biological growth strain" while the fictitious growth process is called "biological growth strain analysis". With this parameter, the design structure can be updated iteratively until the optimized structure is found.

(2) Biological growth strain^{11),12)}

The concept of biological growth strain is from the observation: the biological structures vary their shape to adjust the stress distribution along the stress-concentrated surface by growth or atrophy of the tissues near the surface. Therefore, the principle for simulation is stated as that, with the use of FEM, if the element equivalent stress is greater than the reference stress, this element should swell; otherwise, if smaller than the reference stress, the element should shrink. Therefore, after each

iteration, the difference between reference stress and element equivalent stress can be reduced based on the definition of biological growth strain. In this research, the maximum principal stress or von Mises stress is chosen as the equivalent stress according to the type of material. For ductile material, von Mises stress is used while maximum principal stress is used for brittle material. Besides, with the consideration on the effect of strength ratio (i.e. the ratio of compressive strength F_c to tensile strength F_t in two dimensional case) in brittle material, the ratio of principal stress to its uniaxial strength with the correspondent sign is also introduced in the definition of biological growth strain. The mathematical definition of the above descriptions is given in Eq.(3) for principal biological growth strain matrix $\{\epsilon_{mn}^{BP}\}_j$

$$\{\epsilon_{mn}^{BP}\}_j = \begin{Bmatrix} \epsilon_{11}^{BP} & 0 \\ 0 & \epsilon_{22}^{BP} \end{Bmatrix}_j \quad (3)$$

$$= \begin{cases} \text{for ductile material: } \frac{\bar{\sigma}_j - \sigma_{ref}}{\sigma_{ref}} \nabla h \delta_{mn}, & \text{if element } j \in \Gamma^* \\ \text{for brittle material: } \frac{\bar{\sigma}_j - \sigma_{ref}}{\sigma_{ref}} \left| \frac{\sigma_k}{f_s} \right| \nabla h \delta_{mn}, & \text{if element } j \in \Gamma^* \\ \{0\} & \text{if element } j \notin \Gamma^* \end{cases}$$

where $j : j^{th}$ design element within Γ^* ; $\bar{\sigma}_j$: equivalent stress within j^{th} design element; σ_{ref} : mean of equivalent stress of all the design elements; $\sigma_k : k^{th}$ principal stress, $k=1..2$; f_s : uniaxial compressive or tensile strength; ∇h : constant for search step; δ_{mn} : Kronecker delta and m, n : dummy index complying with summation convention¹³. According to our experience, the value of ∇h is suggested to be used from 0.05 to 0.2. In order to express the biological growth strain in a general way in stead of the form of principal strain as expressed in Eq.(3), the transformation of coordinates from the principal coordinate system (1-2) to global coordinate system (x-y) is carried out as shown in Eq.(4),

$$\{\epsilon_{mn}^B\}_j = \begin{Bmatrix} \epsilon_x^B & \epsilon_{xy}^B \\ \epsilon_{yx}^B & \epsilon_y^B \end{Bmatrix}_j \quad (4)$$

$$= [A]^T \{\epsilon_{mn}^{BP}\}_j [A]$$

where $\{\epsilon_{mn}^B\}_j$: biological growth strain matrix within j^{th} design element and T: transpose of matrix and coordinate transformation matrix $[A]$ is defined

as

$$[A] = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \quad (5)$$

where ϕ : the rotation angle between global coordinate system (x-y) and principal coordinate system (1-2).

(3) Algorithm of biological growth strain analysis

In the process of optimization, the technique to update design variables in the iterative design is one of the major research topics. In this newly proposed method, the process of updating design variables is provided in the calculation algorithm called "biological growth strain analysis", the derivation of which is explained in the following : first, the strain occurring in the structure is assumed to be composed of elastic strain ϵ_{ij}^E and biological growth strain ϵ_{ij}^B as below

$$\epsilon_{ij} = \epsilon_{ij}^E + \epsilon_{ij}^B \quad (6)$$

The elastic strain-stress relationship is known as follow

$$\sigma_{ij} = D_{ijkl} \epsilon_{kl}^E \quad (7)$$

where D_{ijkl} : fourth-order material modulus tensor; by introducing Eq.(6) into Eq.(7), Eq.(7) can be rewritten as

$$\sigma_{ij} = D_{ijkl} \epsilon_{kl}^E - D_{ijkl} \epsilon_{kl}^B \quad (8)$$

By expressing Eq.(8) in the form of matrix, Eq.(8) becomes

$$\{\sigma\} = [D]\{\epsilon\} - [D]\{\epsilon^B\} \quad (9)$$

In the process of shape optimization with the introduction of biological growth strain, one assumption is introduced here; that is, there is no work generated by traction and body force. Based on this assumption, the general formulation for the principle of virtual work can be rewritten as

$$\int_{\Omega_e} \{\bar{\epsilon}\}^T \{\sigma\} \partial \Omega_e = 0 \quad (10)$$

where $\{\bar{\epsilon}\}$: the virtual strain. In addition, the strain-displacement relationship in FEM model is also introduced. For the two-dimensional case

$$\begin{aligned} \epsilon_{ij} = \{\epsilon\} &= \{\epsilon_x \ \epsilon_y \ \epsilon_{xy}\}^T \\ &= \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial x} & \dots & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial y} & \dots & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & \dots & \dots & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \\ \{u_{1x} \ u_{1y} \ \dots \ u_{nx} \ u_{ny}\}^T &= [B]\{u\} \end{aligned} \quad (11)$$

where $[B]$: the relationship matrix between element strain vector $\{\epsilon\}$ and element nodal displacement vector $\{u\}$. Furthermore, by substituting Eq.(9) and (11) into Eq.(10) and rearranging it, the result can be expressed as

$$\begin{aligned} \{\bar{u}\}^T \left[\int_{\Omega_e} [B]^T [D] [B] d\Omega_e \right] \{u\} \\ = \{\bar{u}\}^T \left[\int_{\Omega_e} [B]^T [D] \{\epsilon^B\} d\Omega_e \right] \end{aligned} \quad (12)$$

where $\{\bar{u}\}^T$: transpose of virtual nodal displacement vector. By eliminating $\{u\}$, the element governing equation can be established as below

$$[K]\{u\} = \{\Delta g\} \quad (13)$$

where $[K]$: element stiffness matrix and the equivalent nodal force vector $\{\Delta g\}$ is formulated as

$$\{\Delta g\} = \int_{\Omega_e} [B]^T [D] \{\epsilon^B\} d\Omega_e \quad (14)$$

As introduced by the assumption of no work exerted by traction or body force, the element nodal displacement vector $\{u\}$ is generated by the equivalent nodal force vector $\{\Delta g\}$, which is the function of $\{\epsilon^B\}$. Because the generation of $\{\epsilon^B\}$ is to update the design structure, $\{u\}$ should be regarded as the nodal coordinate updating vector. Then, the global governing equation is given by superimposing every element governing equation. As a result, the nodal coordinate updating vector $\{u\}$ can be obtained by solving the global governing equation superimposed by Eq.(13). Therefore, the shape of the structure is updated by adding the nodal coordinate updating vectors to the old coordinates of nodes.

The computational flow of BGSM is illustrated in Fig.5. The initial configuration of a design structure can be input by means of a preprocessor (i.e. mesh generator, initial input of configuration of structures). Next, a structural analysis within elastic

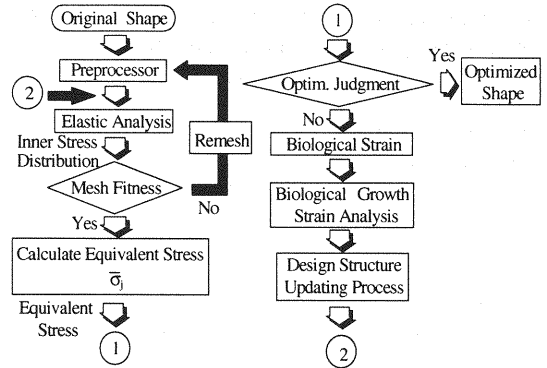


Fig.5 Computational flow of BGSM

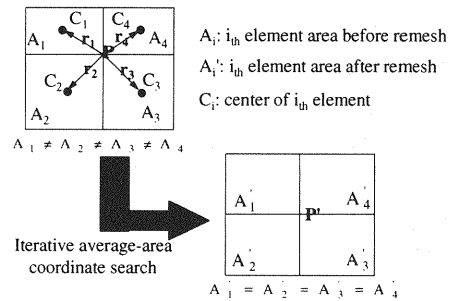


Fig.6 Sketch of the iterative average-area coordinate search

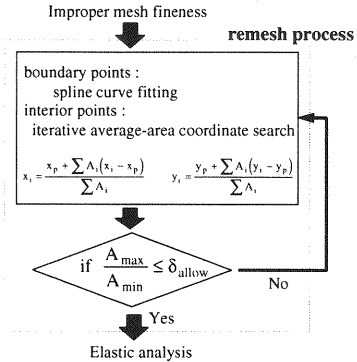


Fig.7 Flow of remesh process

limit is carried out by FEM. As a result, the scatter of the inner stress within the structure can be calculated. Then, the process of the optimization will be iteratively executed in accordance with the execution order shown in Fig.5. The termination of this iterative computation can be recognized when the conditions in Eq.(2a) up to (2c) are satisfied.

(4) Evaluation on mesh distortion and remesh

During the shape optimization by FEM, one common problem is the distortion of meshes which leads to the enlargement of the loss of precision of analysis. In order to maintain the reliability of the analysis, it is essential to adjust the mesh

discretization acceptably at each iteration. Based on this consideration, 2 steps (also shown in Fig.5) are necessary to be adopted. The first one is to evaluate the fitness of present FE element discretization. For two-dimensional cases, the evaluation criteria is the uniformity of element area. The way of evaluation is to judge if the ratio of the maximum element area to minimum element area within the variation domain Γ^* is greater than the given limit. If the ratio is under the given limit, the present process of optimization can be continued; however, if not, the second step to adjust the improper mesh discretization (i.e. remesh) is adopted. In fact, there are several approaches for adaptive remesh - h method, p method, r method or the combination h-p method^[4]. The characteristic of p method is to increase the number of the nodes and h method is to increase the number of elements while r method is to relocate the coordinates of nodes without adding total number of nodes and elements. Generally speaking, the accuracy of h method, p method or their combination h-p method is higher but the time and cost involved in the calculation also increase substantially. Conversely, with a reasonable number of nodes and elements at the initial mesh, r method can be thought as a practical way for remesh. The algorithm of r method for remesh in BGSM is explained here; for the points on the boundaries, spline curve fitting is adopted to adjust the interval length between the boundary points with an approximate curve on the initial curve of boundary while, for the nodes within the design domain, one method called "iterative average-area coordinate search" is used to relocate the position of interior nodes, the concept and the formulation of which are schematized in Fig.6 as well as Fig.7 separately. By our experience using the proposed remesh method, the accuracy of mesh discretization during the shape optimization is really good enough to be accepted. One illustrative example by using this remesh method is given in Fig.8.

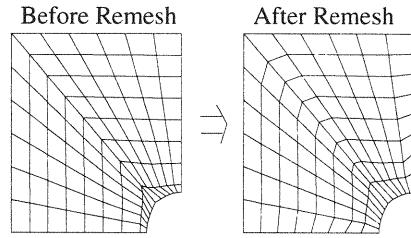


Fig.8 Illustration for the effect of the present remesh method

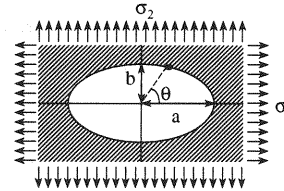
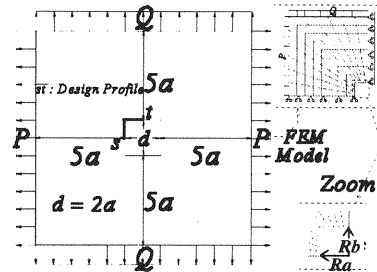
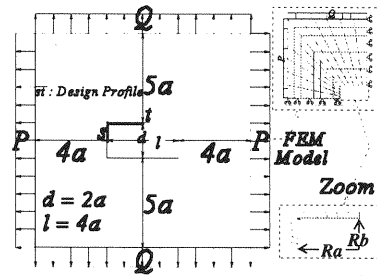


Fig.9 Hole in a plate under biaxial stress field



(a) square hole



(b) rectangle hole

Fig.10 Sketch of FEM model for the hole in an infinite plate under biaxial stress with different initial shapes

4. NUMERICAL VERIFICATION

(1) Verification on correctness of BGSM

After explaining the details of BGSM, the availability of this method is necessary to be verified before applying this method to general structures. To verify the availability of this method, the design problem to find out the optimal shape of the hole in a plate under biaxial stress is considered, which is illustrated in Fig.9. According to the elasticity theory^[5], the tangential stress for the elliptical hole in an infinite plate under biaxial stress is given as

$$\sigma_{\theta} = \sigma_1 \frac{(1+k)^2 \sin^2 \theta - k^2}{\sin^2 \theta + k^2 \cos^2 \theta} + \sigma_2 \frac{(1+k)^2 \cos^2 \theta - 1}{\sin^2 \theta + k^2 \cos^2 \theta} \quad (15)$$

where k : axis ratio ($=b/a$; b : vertical axis and a : transversal axis) and θ : directional angle. With the introduction of the assumption $\sigma_2/\sigma_1 = k$, Eq.(15) can be rewritten as follow

$$\begin{aligned} \sigma_{\theta} &= \sigma_1 \frac{(1+k)^2 \sin^2 \theta + k(1+k)^2 \cos^2 \theta - k^2 - k}{\sin^2 \theta + k^2 \cos^2 \theta} \\ &= (1+k)\sigma_1 \end{aligned} \quad (16)$$

Table 1 Material property

FC*	FT*	E*	μ
1000	1000	7.00E+04	0.3

* : unit (kgf/cm²)**Table 2** Loading condition and case label

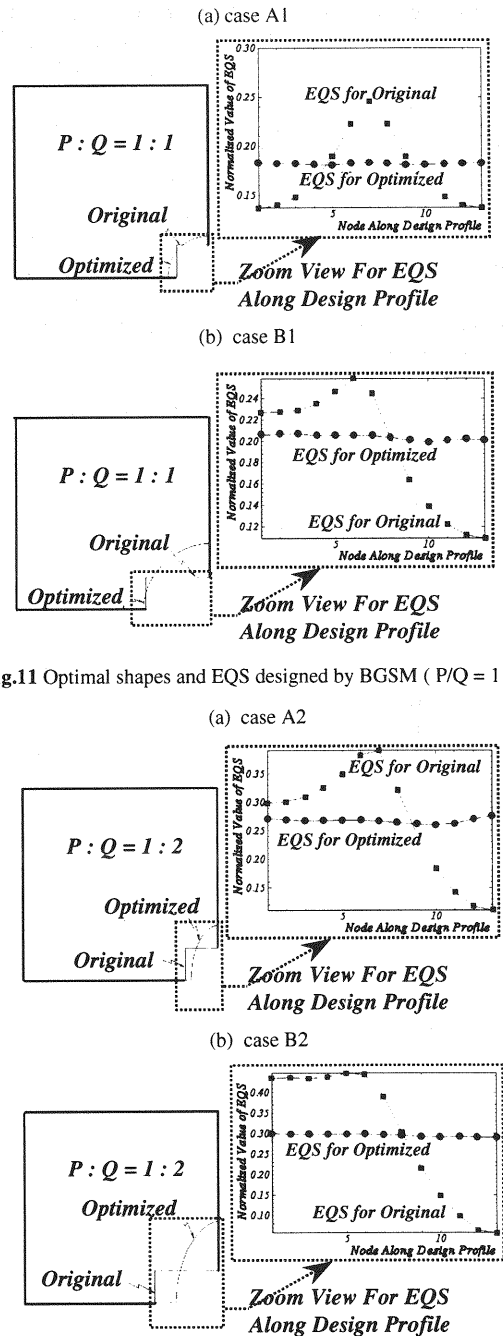
P : Q	A	B
1 : 1	A1	B1
1 : 2	A2	B2

Thus, provided $\sigma_2/\sigma_1 = b/a = k$, the tangential stress will be constant along the edge. According to Baud's postulation, the shape with uniform strength distribution is considered as optimum. Furthermore, to verify the availability of BGSM, different initial shapes of holes including (a) square and (b) rectangle were used as numerical examples shown in Fig.10. Due to symmetry, only one quarter of these structural models were analyzed. The material property and loading condition with case label were given in Table 1 and 2 respectively. Due to ductile material used in these cases, von Mises stress was selected as the equivalent stress (abbr. as EQS). By applying BGSM, the optimized shape for different initial shapes under the equal biaxial stress ratio (i.e. $P/Q = 1$) and EQS along the design profile before and after optimization was given in Fig.11. In addition, the axis ratio of the design profile for the results of BGSM and analytical solution was shown in Table 3. For the unequal biaxial stress ratio (i.e. $P/Q = 0.5$), the optimized shapes and EQS distribution was shown in Fig.12 while Table 4 offered the comparison between the results of BGSM and the analytical solution.

By reviewing the definition given in Eq.(3), the reason causing the variation of the optimized shape for different initial shape can be understood because the reference stress is defined as the mean stress along the design profile rather than a fixed value. However, the axis ratio in Table 3 and 4 for different initial shape under $P/Q = 1$ and 0.5 showed the good agreement between analytical result and the numerical result given by BGSM. In addition, the equivalent stress along the design profile could be quite uniform for all the cases shown in Fig.11 and 12. Thus, the correctness of BGSM could be verified.

(2) Verification on applicability of BGSM

After the correctness of BGSM was verified, the applicability of BGSM was also necessary to be checked. Based on our experience to use BGSM for designing structures, its applicability could be recognized acceptable due to success on many cases. Here, with the limitation of space, one example for illustrating its comprehensive applicability - numerical analysis on the adaptive growth of tree by BGSM was depicted as follows. In accordance with

**Fig.11** Optimal shapes and EQS designed by BGSM ($P/Q = 1$)**Fig.12** Optimal shapes and EQS designed by BGSM ($P/Q = 0.5$)

The knowledge on the biological growth of trees, the direction of adaptive growth of trees depends on which type of the constituent material of trees - compressive wood or tensile wood. By considering this, the material used in this example was assumed to be compressive wood (i.e. brittle material). Thus, maximum principal stress is used as the equivalent stress. By applying the bending force on the branch

of the tree, the isostress diagrams of original and optimized tree were shown in Fig.13(a) and (b) individually. The design profile was symbolized as Γ . The objective of the optimization was to reduce stress concentration. After numerical simulation of BGSM, through Fig.13(a)-(c), what can be observed is , by comparing the value of maximum principal stress in the stress level before and after optimization, the stress concentration along Γ after optimization is effectively reduced. As a result, the applicability of BGSM for numerical simulation on biological adaptation process is verified.

5. COMPARISON ON COMPUTATION EFFICIENCY

(1) Optimality criteria method based on sensitivity analysis

In order to evaluate BGSM, one optimality-criterion approach using sensitivity analysis (abbr. as SA) was also proposed by the authors. The features of this method were explained here. The objective function for this method is the same as shown in Eq.(2a) up to (2c). Here, von Mises stress was used for the equivalent stress. Based on the definition of von Mises stress, the equivalent stress is the functional of the stress tensor and the stress tensor is the function of nodal coordinates, as expressed in Eq.(17) for the plane stress case.

$$\begin{aligned}\sigma_{eq} &= \sqrt{2/2}(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\sigma_{xy}^2)^{1/2} \\ &= f(\sigma_{ij}) = f(\sigma_x, \sigma_y, \sigma_{xy}) \\ &= f(\sigma_x(x, y), \sigma_y(x, y), \sigma_{xy}(x, y))\end{aligned}\quad (17)$$

By Taylor's first-order expansion at mean stress σ_{ref} , the equivalent stresses at each evaluation point can be approximated as below

$$\sigma_{eq} = \sigma_{ref} + \frac{\partial f}{\partial \sigma_{ij}} \delta \sigma_{ij} + \Xi(\epsilon) \quad (18)$$

where $\Xi(\epsilon)$: the error functional and σ_{ij} : the components of stress tensor. As it is shown, the sensitivity coefficients of stresses regarding the design variable is essential. The sensitivity analysis in this paper adopted the semi-analytical method by a finite difference method¹⁶⁾. σ_{ij} is differentiated with respect to the design variables as follow

$$\delta \sigma_{ij} = \frac{\partial \sigma_{ij}}{\partial X_j} \delta_j \quad (19)$$

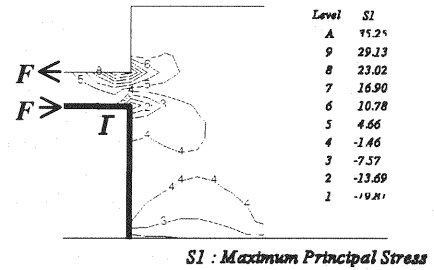
where j : j^{th} design point; X_j : coordinate of design points (i.e. x_i and y_i , $i=1..n$) and δ_j : perturbation of coordinates for j^{th} design point. By substituting Eq.(19) into Eq.(18), Eq.(18) can be rearranged as

Table 3 Comparison on Ra and Rb ($P/Q = 1$)

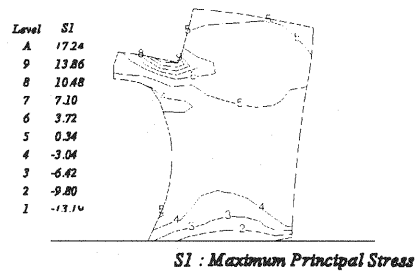
P : Q = 1 : 1				
	Ra	Rb	Rb/Ra	
A	Ori	5.00	5.00	1.00
	Opm	6.38	6.38	1.00
B	Ori	10.00	5.00	0.50
	Opm	10.40	10.29	0.99
Analytical			1.00	

Table 4 Comparison on Ra and Rb ($P/Q = 0.5$)

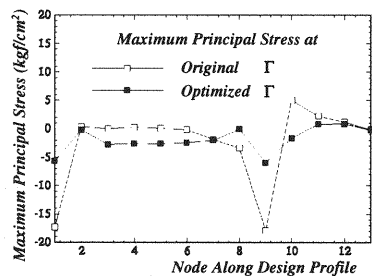
P : Q = 1 : 2				
	Ra	Rb	Rb/Ra	
A	Ori	5.00	5.00	1.00
	Opm	4.17	8.54	2.05
B	Ori	10.00	5.00	0.50
	Opm	6.66	12.67	1.90
Analytical			2.00	



(a) isostress diagram at original shape



(b) isostress diagram at optimized shape



(c) distribution of maximum principal stress along design profile

Fig.13 Sketch of numerical simulation by BGSM for adaptive growth of trees

$$\begin{aligned}\sigma_{eq} &= \sigma_{ref} + \left(\frac{\partial f}{\partial \sigma_x} \frac{\partial \sigma_x}{\partial x_j} + \frac{\partial f}{\partial \sigma_y} \frac{\partial \sigma_y}{\partial x_j} + \frac{\partial f}{\partial \sigma_{xy}} \frac{\partial \sigma_{xy}}{\partial x_j} \right) \delta x_j + \\ &\quad \left(\frac{\partial f}{\partial \sigma_x} \frac{\partial \sigma_x}{\partial y_j} + \frac{\partial f}{\partial \sigma_y} \frac{\partial \sigma_y}{\partial y_j} + \frac{\partial f}{\partial \sigma_{xy}} \frac{\partial \sigma_{xy}}{\partial y_j} \right) \delta y_j + \Xi(\epsilon) \\ &= \sigma_{ref} + \alpha_j \delta x_j + \beta_j \delta y_j + \Xi(\epsilon)\end{aligned}\quad (20)$$

Eq.(20) can be rearranged in the form of matrix as shown in Eq.(21). As a matter of fact, the right

side of Eq.(21) can be approximated by multiplying one constant (ϕ) for diminishing the effect of error functional.

$$\alpha_j \partial x_j + \beta_j \partial y_j = \sigma_{eq} - \sigma_{ref} - \exists(\epsilon) \quad (21)$$

$$= \phi [\sigma_{eq} - \sigma_{ref}]$$

In Eq.(21), the order of coefficient matrix $[\alpha_j, \beta_j]$ is $n * 2n$. As a result, the solution of $(\delta x, \delta y)$ cannot be solved by only n equations because only n design points are selected along the design profile. Hence, some extra information related to $(\delta x, \delta y)$ needs to be introduced into Eq.(21). For solving that, the updating vector for each design point is assumed to be in the direction of the bisector of the angle defined by the idea presented in Fig.14. Then, the relationship for $(\delta x, \delta y)$ can be established as follow

$$\delta y / \delta x = \frac{x_{i+1} - x_i^*}{y_i^* - y_{i+1}} \quad (22)$$

By substituting Eq.(22) into Eq.(21), the order of coefficient matrix $[\alpha_j, \beta_j]$ becomes $n * n$. Then, after obtaining the derivatives of stress regarding design variables and using the relationship given in Eq.(22), the solution for the updating vector can be obtained. With the updating vectors, the shape of the design structure will be changed by the addition of these updating vectors with the present coordinates of design points. This process will be repeated until Eq.(2a) up to (2c) can be satisfied.

(2) Observation on comparative result

As the purpose of research stated in Chapter 1, one example for comparison was offered here. The structural FEM model with its dimension and material property was pointed out in Fig.15. The equivalent stress considered in this case was von Mises stress. Fig.16 indicated the optimized shapes obtained from the two methods with little difference, which indirectly verified the correctness of SA. In addition, the ratio of area difference to original area, converged value of objective function, the ratio of strain energy difference to initial strain energy, the ratio of maximum EQS at optimum to initial maximum EQS obtained by BGSM and SA were tabulated in Table 5. From these data, the correspondence of the optimized result generated by BGSM and SA could be acknowledged. However, on the other hand, the computation cost spent by the two methods was quite different. For the FEM model of this numerical experiment, in which there were 153 nodes and 128 elements, the programs were carried

Table 5 Comparison on the optimized results by SA and BGSM

	$\frac{A_{optm} - A_{ori}}{A_{ori}}$	Val _{obj}	$\frac{Se_{optm} - Se_{ori}}{Se_{ori}}$	$\frac{EQS_{(max)optm}}{EQS_{(max)ori}}$	Time comp (each loop)
SA	1.0028	0.56	0.9844	0.5363	198 sec
BGS	1.0056	0.57	0.9756	0.5427	1.2 sec

A:Area, Val: Value of Objective Function, Se: Strain Energy

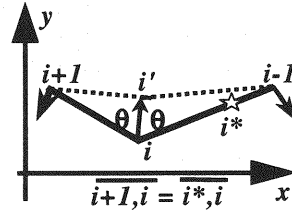


Fig.14 Local shifting of node i to i'

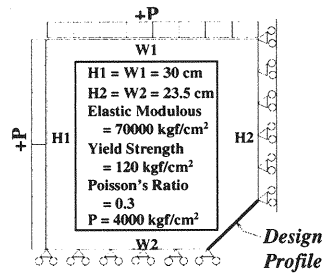


Fig.15 FEM model of design structure

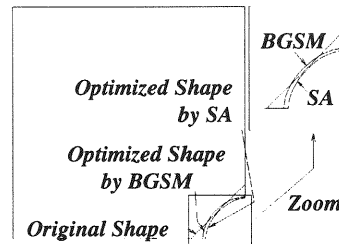


Fig.16 Comparison on shape of design structure

out on SUN Sparc station 10. For BGSM, one iterative step only took 1.2 sec while about 198 sec spent by SA; that is, only 98.4 sec was needed for BGSM to find out the optimum (82 iterative loops) while about 17820 sec (about 4.95 hr) needed by SA. As a result, though the similar optimal solutions could be achieved by the two methods, the high efficiency of BGSM for saving a lot of computation cost was obviously confirmed.

6. CONCLUDING REMARKS

The following conclusions can be extracted from this study:

- (1) By observing the algorithm of BGSM, the

gradientless optimization technique by simulating the biological adaptation made this method very simple without special coding procedure, unlike the conventional approach for the need to code their special algorithm.

(2) The availability of BGSM was proved from the viewpoint of correctness and applicability by (a) finding the conformity between the numerical result obtained by BGSM and the analytical result and (b) applying this method on many design structures successfully for the acceptable achievement on the design objective.

(3) With the sophisticated definition on the biological growth strain, the applicability of BGSM could be extended from the use on ductile material to brittle material, which is used to be considered by setting different level of stress constraint in mathematical programming approach. From the example given in this paper, the present definition of biological growth strain for brittle material showed its possibility; however, the pertinence of such definition needs further confirmation.

(4) By the comparative example given in this paper, the excellence of BGSM on improving computation efficiency could be verified for its ignorance of gradient calculation, which is regarded as an indispensable part in the conventional structural optimization technique.

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