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CREEP ANALYSIS AND TORSIONAL VIBRATION ANALYSIS OF CABLE-STAYED BRIDGES WITH TWO EDGE COMPOSITE GIRDERS

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In the cable-stayed bridges with two edge composite girders, due to creep in the concrete slab, stress redistribution occurs not only within the composite girders themselves but also between girders, cables, and towers. In order to conduct creep analysis for this particular type of bridge structure, two methods will be presented and applied to a roughly designed bridge, the results of which will be compared. Due to the relatively low natural frequency caused by the low torsional rigidity of the open girder section, precise torsional vibration analysis will also be crucial in considering the aerodynamic stability. Two vibration analysis methods will be introduced and applied to the hypothetical bridge and in turn, compared.

Key Words : *cable-stayed bridges, concrete-steel composite girder, creep analysis, torsional vibration analysis*

1. INTRODUCTION

Cable-stayed bridges of a new type of structure, where a composite bridge girder consisting of steel girders and concrete slab is used, have recently been constructed in Canada, the United States, and China[1),2),3)]. Completed in October 1993, the longest bridge of this type is the Yangpu Bridge over the Huang Pu River in Shanghai with a main span of 602m. Although this type of bridge has yet to be realized in Japan, its structural behaviour and feasibility are under investigation, as it is expected to be one of the bridge structure types to be further developed in the future[4)].

Although there are several types of composite bridge girder sections, the section with two edge I girders, as used in the Annacis Bridge in Vancouver, is widely used. As such, the focus of this paper will be on cable-stayed bridges with two edge composite I girders.

Characteristics that distinguish this type of bridge structure from steel or concrete cable-stayed structures, include the use of a steel-concrete composite bridge girder with an open section and its low torsional rigidity. Although open section girders have been used in steel or concrete cable-stayed bridges, there are only a handful that span over 300m.

Estimating the creep effect of the concrete slab with enough precision for practical purposes and rigorously conducting torsional vibration analysis are two important issues to be carefully considered in the design of cable-stayed bridges with two edge composite girders. Although, as mentioned before, a number of this type of bridge structure exist, no detailed information on the analysis procedures of creep and torsional vibration has been presented so far.

Due to the fact that stress redistribution occurs not only within the composite bridge girder itself but also between the girder, the cables, and the tower, creep analysis of the cable-stayed bridge structures with a composite girder is rather involved. Precise analysis methods to estimate this complex stress redistribution caused by creep will be presented and discussed in this paper.

Since a composite bridge girder with two edge steel girders has a Π-shaped open section, its torsional rigidity--mainly warping torsional rigidity,

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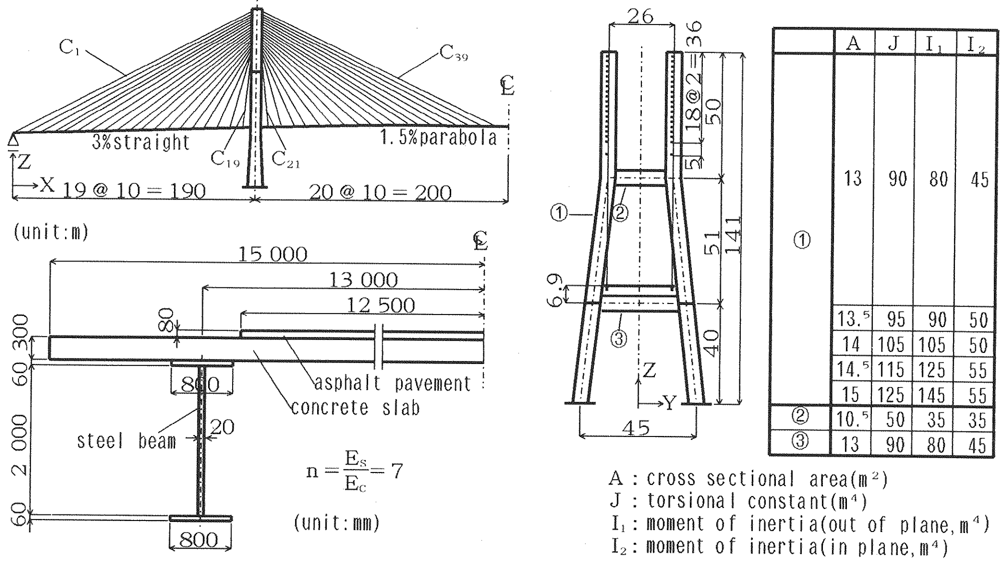


Fig. 1 Cable-stayed bridge with two edge composite girders used as a calculation model

Table 1 Cross sectional areas(A) and initial axial forces(T) of cables

No.	A	T
1	0.0175	13780
2	0.0075	7790
3		7800
4		7800
5		7110
6		7110
7		7120
8	0.0060	6120
9		6130
10		6130
11		5330
12		5340
13		5340
14	0.0050	4430
15		4430
16		4420
17		3710
18		3690
19		3660
20	0.0050	3700
21~23	0.0050	3630
24~26		4310
27~29	0.0060	5190
30~32		5980
33~35	0.0075	6960
36~38		7640
39	0.0150	13520

(unit: m², kN)

not St. Venant's torsional rigidity, which in this case is negligible--is much smaller than that of a box girder. This, in turn, causes low natural frequency in the bridge structure, often requiring detailed investigation of aerodynamic stability. As is the case, torsional vibration analysis becomes crucial; this proves to be somewhat difficult as the effect of warping needs to be thoroughly taken into account. Similar difficulty has been observed in the study of aerodynamic stability of the Helgeland Bridge (a concrete cable-stayed bridge with a main span of 423m)[5]. This particular issue will be fully discussed in this paper.

For the purposes of this paper, a cable-stayed bridge with two edge composite girders has been roughly designed. Using this particular bridge as a calculation model, creep analysis and torsional vibration analysis--two methods for each type of analysis--will be conducted, the results being compared and discussed.

The following will first describe the calculation model bridge, then, creep analysis, and finally, torsional vibration analysis.

2. CABLE-STAYED BRIDGE USED AS A CALCULATION MODEL

The cable-stayed bridge used as a calculation model in this paper is a bridge having a composite bridge girder as represented by the Annacis Bridge in Vancouver and the Nanpu Bridge in Shanghai. Using these bridges and one being investigated in Japan[4] as a reference, the configuration of the cable-stayed bridge is determined as shown in **Fig.1**.

This bridge is a three span continuous cable-stayed bridge with a main span of 400m and side spans of 190m. It is symmetric about the center of the main span. Cables are arranged in two planes and connected to the composite girder at the centroids of the steel sections. The girder moves freely in the longitudinal direction without any rigid constraints. It is fixed in the vertical and transverse directions at both end piers and is rigidly connected only in the transverse direction to the towers.

Table 2 Dead loads, masses and polar moments of mass of the girder

	dead loads (kN/m)	mass (t/m)	polar mom. of mass ($t \cdot m^2$ /m)
pave.	45	4.6	240
slab	221	22.5	1690
h. rail	28	2.9	650
steel	69	7.0	1190
total	363	37.0	3770

The geometry of the girder cross section and cross sectional properties of the tower are also shown in **Fig.1**. The cross sectional areas of the cables are summarized in **Table 1**. Young's modulus for the slab and tower concrete is assumed to be E_C=29.4 kN/mm^2. That of the steel beams and cables is E_S=206.0 kN/mm^2.

The magnitude of dead loads and masses of the girder needed for creep or vibration analysis is shown in **Table 2**, where polar moments of mass are related to the center of the mass.

3. CREEP ANALYSIS

(1) Analysis methods

As previously mentioned, in cable-stayed bridges with a composite girder composed of concrete slab and steel girders, stress redistribution due to the creep of the concrete slab occurs not only within the composite girder itself, but also between the girder, the cables and the tower. This condition makes creep analysis difficult to carry out.

The cable-stayed bridge under consideration is assumed to be a plane frame structure to which the two analysis methods--method A and method B--will be applied. In both methods full composite action without any slip is assumed between the concrete slab and the steel girders.

a) Method A

As shown in **Fig.2**, the composite girder is represented by two members--one of concrete and one of steel--that are connected by rigid bars placed between them. In this idealized model, the frame structure consists only of concrete and steel members so that the analysis method developed by the author et al.[6] can be applied. It is difficult to say

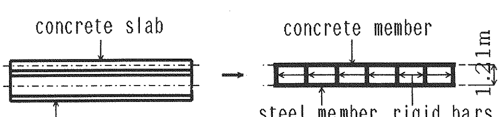


Fig. 2 Composite girder idealized by two members (method A)

generally how to determine the spacing of the rigid bars. For the following analysis, the rigid bars are placed every 2.5m to achieve almost the same stress condition in the girder as the one to be obtained from method B under dead load.

b) Method B

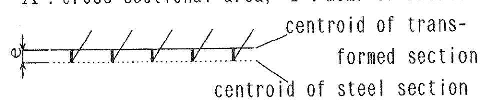
This method is based on the concept of the transformed composite cross section. Detailed analysis procedure of this method is presented in 7). The properties of the transformed cross section needed for the analysis are summarized in **Table 3**.

Table 3 Properties of the transformed composite cross section(method B)

	$\phi = 0$	$\phi = 1.0$	$\phi = 2.0$
n	7	14	21
e (m)	1.00	0.85	0.74
A (m^2)	1.56	0.92	0.70
I (m^4)	0.57	0.51	0.48

ϕ : creep coeficient, $n = E_s (1+\phi) / E_c$

A : cross sectional area, I : mom. of inertia



(2) Analysis results

Creep analysis was conducted using either method A or method B. The analysis focused on stress redistribution due to creep in the completed structure, but not on one during the erection stage.

In both methods cross sectional forces due to dead load, which were needed for the analysis, were preliminarily calculated. Static indeterminate forces like cable forces were deliberately adjusted to achieve smoother bending moment distribution in the girder, with the added condition that the tower did not undergo any bending moments. Prescribing the initial stress state under dead load in such a manner, however, does not mean that dead load is applied to the completed structure and then cable forces are adjusted in the construction of cable-stayed bridges. In actuality, cantilever erection methods are usually used. Even in the cantilever erection, however, erection proceeds so as to realize the prescribed stress state in the completed structure[8]. Thus, determining the initial stress state as described above is adequate at least for the analysis involved in this particular study.

Cable forces determined in such a manner, which are the same for both methods A and B, are summarized in **Table 1**. The dead loads acting on the girder are shown in **Table 2**. The dead loads (own weight) acting on the tower were calculated

Table 4 Displacements and reaction changes due to creep

		before creep		after creep has taken place				
		$\phi=0$		$\phi=1$		$\phi=2$		
		method A	method B	method A	method B	method A		method B
						$\phi_v=0$	$\phi_v=0.4$	
displ.	δ_{x1}			0.051	0.047	0.094	0.089	0.082
	δ_{z5}			0.019	0.018	0.035	0.033	0.031
	δ_{z2}			0.072	0.066	0.131	0.123	0.114
	δ_{x4}			0.040	0.037	0.073	0.069	0.063
reaction	R_{z1}	3500	3500	3390	3400	3300	3320	3330
	R_{x2}	0	0	230	210	430	400	370
	M_{y2}	39500	39200	36900	36800	34800	35000	35100
	R_{x3}	0	0	230	210	430	400	370
	R_{z3}	237220	237220	237120	237130	237030	237040	237060
	M_{y3}	0	0	34300	31500	62800	59200	54600

(unit: m, kN)

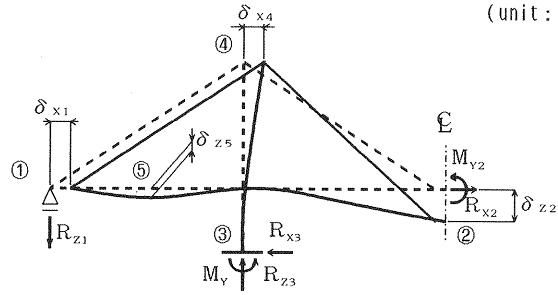
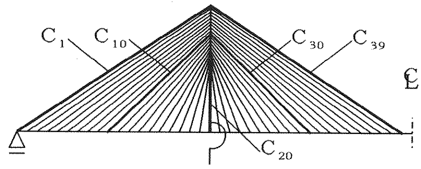


Table 5 Changes of cable forces due to creep

	before creep		after creep has taken place				
	$\phi=0$		$\phi=1$		$\phi=2$		
	method A	method B	method A	method B	method A		method B
					$\phi_v=0$	$\phi_v=0.4$	
C_1	13780	13780	13430	13460	13140	13180	13220
C_{10}	6130	6130	6160	6160	6180	6180	6170
C_{20}	3700	3700	3700	3700	3710	3710	3710
C_{30}	5980	5980	5990	5990	6000	6000	6000
C_{39}	13520	13520	13390	13400	13290	13300	13320

(unit: kN)



by multiplying the cross sectional areas by the unit weight of 24.5 kN/m^3.

Two values were assumed for creep coefficient ϕ ; $\phi=1$ and 2. In general, prefabricated concrete panels are used for the slab of this type of cable-stayed bridge, being placed on steel girders several months after the concrete casting. The two values for ϕ were determined considering this actual construction process. Creep in the concrete tower, which should be taken into account in the actual design, was not considered here because it was desirable to know only the effect of the creep in the concrete slab. The results of the creep analyses are shown in figures and tables. It is noted here that the analysis by method A was conducted stepwise using the small creep increment of $\Delta\phi=0.5$, and

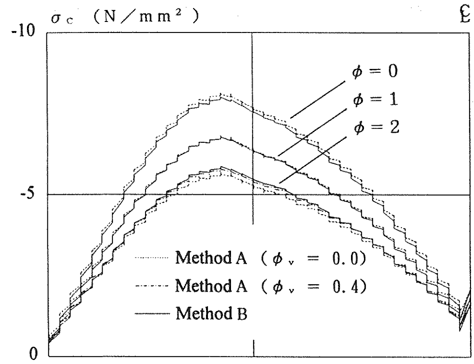


Fig. 3 Stresses in the concrete slab

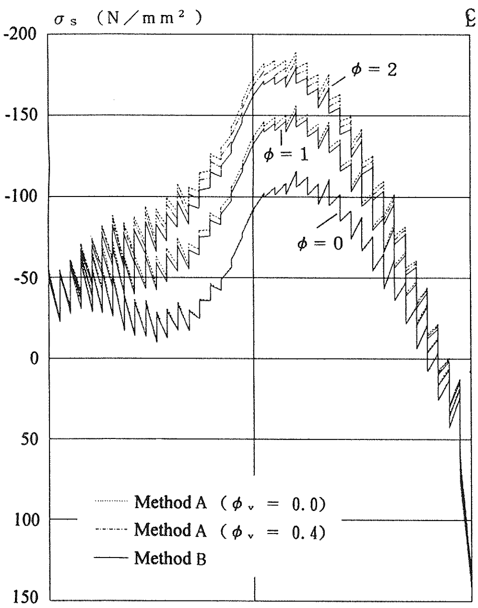


Fig. 4 Stresses at the lower flange of the steel beam

that recovery creep due to delayed elastic strain (ϕ_v=0.4) was additionally considered in the analysis for ϕ=2.

a)Displacements and reactions

Displacements at several key points and reaction changes due to creep are summarized in **Table 4**. As already pointed out[7], it is evident that method B underestimates the effect of creep. This tendency is highly noticeable in the case of ϕ=2, but the difference between the results of method B and method A with the effect of recovery creep included, is relatively small. This can be explained by the fact that method B evaluates the effect of the recovery creep at its maximum.

b)Cable forces

Cable forces before and after creep has taken place are shown in **Table 5**. Again, the effect of

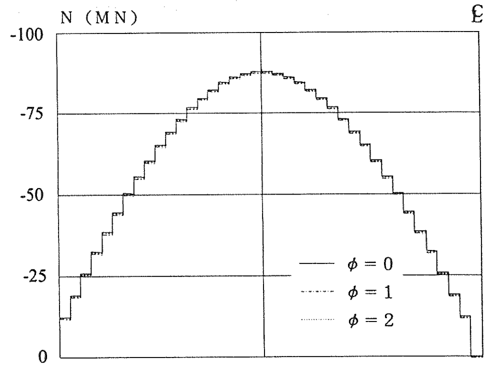


Fig. 5 Axial forces N in the composite girder

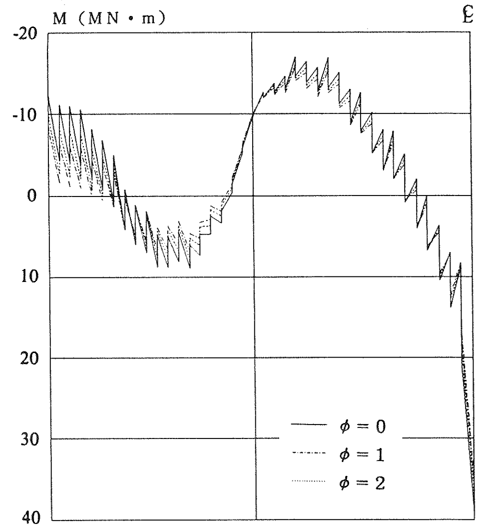


Fig. 6 Bending moments M in the composite girder

creep is clearly underestimated by method B. It is noted that changes of cable forces due to creep are rather small in the cable-stayed bridge under consideration.

c)Stresses in the girder

Stresses in the girder are plotted in **Figs.3** and **4**. The former shows the normal stresses σ_c at the centroid of the concrete slab, while the latter illustrates the normal stresses σ_s at the lower flange of the steel girder. The stress states in the initial condition (ϕ=0) are almost the same in both methods, which may indirectly justify the idealization as shown in **Fig.2**. Stress transfer from the concrete slab to the steel girder within the composite girder due to creep is clearly observed in **Figs.3** and **4**. The difference between the results of method A and method B is not remarkable. In particular, the difference between the results of method A with the effect of recovery creep included and method B is small, even when ϕ=2.

d) Axial forces and bending moments in the composite girder

Figs.5 and **6** show, respectively, axial forces and bending moments in the composite girder obtained by method B. Corresponding figures for method A can not be illustrated because the composite girder was idealized by two members, as shown in **Fig.2**. It is obvious from **Figs.5** and **6** that cross sectional forces of the composite girder change little due to creep, which indicates that very small stress redistribution occurs between the girder, the cables, and the tower. This is in accordance with the fact that, as previously mentioned, cable forces change little due to creep. It is concluded that stress redistribution occurs primarily within the composite girder and only secondarily between the structural members in the case of the cable-stayed bridge under consideration.

(3) Comparison of the analysis methods

From the results of the creep analyses applied to the model cable-stayed bridge, the following can be said.

Although method B evaluates the effect of recovery creep at its maximum and consequently underestimates the effect of creep, the results obtained are not so different from those obtained by method A, especially when the effect of the recovery creep is considered.

In method A some error could result from the idealized situation where the composite girder is assumed to be composed of two members, one of concrete and one of steel. However, this error is small in the bridge under consideration. The error could be sufficiently reduced, if the number of the rigid bars connecting the two members were increased. This would lead, however, to the increase in the degree of freedom, which would be undesirable from the computational point of view.

In general, method A has a wider application possibility than method B;for instance, only method A is applicable, when the evaluation of the effect of the recovery creep is required or when the effect of reinforcements and/or the prestressed wires arranged in the concrete slab has to be taken into account.

On the other hand method B has an advantage in that the computation task required is relatively small. In addition, the concept of the transformed composite cross section is familiar to design engineers and is also used in typical structural analyses for other loads such as dead and live loads.

For the practical purposes of the actual design, stresses in the composite girder may be roughly calculated, at least in the preliminary stage by using the cross sectional forces due to dead loads, assumed to remain unchanged during creep process, and the properties of the transformed cross section, dependent on the creep coefficient ϕ.

Although method A is more desirable because it is more refined, method B may be applied in many cases without any significant error. The error included in method B can become outstanding, however, if the value of the creep coefficient ϕ is large or the rigidity of the steel girder is relatively large in comparison with that of the concrete slab[7].

4. TORSIONAL VIBRATION ANALYSIS

(1) Analysis methods

The Π-shaped, open section of the girder is one of the characteristics of the cable-stayed bridges considered in this paper. In this case, warping torsional rigidity, not St. Venant's, constitutes the torsional stiffness of the girder. Refined torsional vibration analysis considering such torsion characteristics is important regarding aerodynamic stability because of the rather low natural frequency of this type of cable-stayed bridge. In this paper the following two analysis methods--method I and II--were applied to the bridge shown in **Fig.1**.

a) Method I

This method utilizes an analysis program commonly available for the vibration analysis of space frame structures. Since, in general, only St. Venant's torsion is considered in such programs, it can not be directly applied to the bridge under consideration. As such, the following modification is required. Namely, the girder structure is assumed to be made up of three members as shown in **Fig.7**. One of the three members is placed along the shear center of the composite girder (beam 1), while two of them are arranged along the centroids of the steel girders(beams 2). Beam 1 and beams 2 are connected by rigid bars placed every 10m. Beam 1 has a rigid connection and beams 2 are pin-connected to the rigid bars.

Beam 1 is assumed to have St. Venant's torsional rigidity G_SJ (negligible in actuality) and horizontal flexural rigidity E_SI_l (I_l=142m^4) of the composite

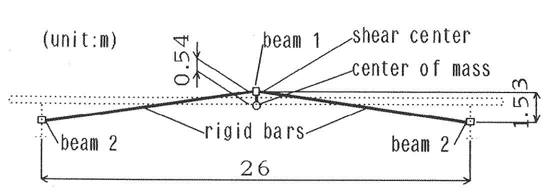


Fig. 7 Idealization of the composite girder by three beams (method I)

girder. Only vertical flexural rigidity $E_S I_2$ is given to the beams 2, where the moment of inertia I_2 is not the actual one but is one determined according to the following equation to estimate the warping torsional rigidity of the girder correctly:

$$I_2 = \frac{2J\omega}{a^2} = \frac{2 \times 85}{26^2} = 0.25\, m^4 \qquad (1)$$

where $J\omega$ is the warping torsional constant of the girder, and a is the distance between beams 2. The value of I_2 calculated by equation (1) is found to be equal to that obtained under the assumption that half of the concrete slab width(about 7.5m per one steel girder) is effective. It should be noted here that the effective width is one third for the deck without side overhangs[10]. The cross sectional properties mentioned above are related to the cross section transformed to the steel.

Masses in the horizontal direction and polar moments of mass are provided along the center of the girder. Rigid bars are arranged between the shear center and the center of the mass of the girder every 10m.

There is nothing particular about the cables and the tower. It is noted only that the masses were taken into account in both horizontal directions for the tower.

In general, the torsion of the girder is coupled with bending in the horizontal plane but not with bending in the vertical plane, provided that the cross section of the girder is symmetric. This is the case of the bridge that is under consideration. Therefore, displacements and rotations at all points existing in X-Z plane were constrained in the vertical plane to prevent flexural motion in the plane.

b) Method II

This method may be categorized as one for the vibration analysis methods of space frame structures. It was developed specifically to be applied to the cable-stayed bridges like those considered here. In this method the cables and the tower are treated in the same manner as in method I. There is, however, a difference in the treatment of the girder.

The bridge girder is idealized as one bar subjected to both torsion and transverse flexure. The member axis coincides with the shear center of the girder. Displacements of the girder are represented by the angle of twist θ, angle of twist per unit length θ', transverse deflection v and slope v′(**Fig.8**). The stiffness equation regarding the transverse flexure is the same as the one used in usual displacement methods. A stiffness equation regarding torsion is derived by expressing angle of twist θ approximately to a cubic polynomial[9].

It is important in this analysis method to consider boundary conditions at the supports and the connection condition between the girder and the tower correctly. Since this is achieved in the same manner as stated in 8), detailed explanation has been excluded at this time.

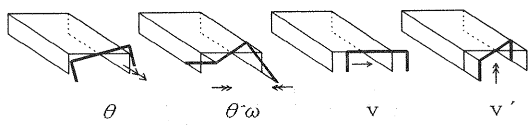


Fig. 8 Displacements considered in analysis(method II)

The masses of the girder are provided at the shear center. Thus, not only do the masses in the transverse direction, and polar moments of mass exist, but their coupled term exist as well, the effect of which should be considered in the analysis. Polar moment of mass about the shear center is 3780 tm^2/m, and the coupled term is 20 tm/m for the bridge girder under consideration. An analysis, in which the coupled term was neglected, was also conducted to understand its effect.

(2) Analysis results

Since only modes symmetric about the center of the main span were considered, half of the structure was used for the analysis. Natural frequencies obtained from the analysis, up to the 10^{th} mode, are summarized in **Table 6**. It is clear that the natural frequencies calculated by method I and method II agree almost perfectly. No significant difference is observed in the vibration modes as shown in **Fig.9**. Torsion of the girder is predominant in the first mode, and horizontal flexure in the second mode. Natural frequencies of the first and the second modes become closer when the effect of the coupled term of the mass is taken into account. This effect is, however, not significant if the distance between the center of the mass and the shear center is small as in the girder under consideration.

(3) Comparison of the analysis methods

No significant difference is observed between the results of method I and II, which indicates, indirectly, that the two analysis methods applied yield correct solution. Since method I is based on an analysis method typically used for space frame structures, there is no need to develop a new program. It is not desirable, however, to apply this method with some modifications to the specific problem discussed in this paper since its degree of freedom is much larger than that of method II. On the other hand, method II has been developed

Table 6 Natural frequencies

mode no.	1	2	3	4	5	6	7	8	9	10
method Ⅰ	0.367	0.412	0.673	0.739	0.885	1.07	1.16	1.23	1.42	1.59
method Ⅱ	0.366	0.411	0.672	0.736	0.881	1.07	1.14	1.23	1.42	1.58
	0.358	0.418	0.674	0.736	0.883	1.07	1.14	1.22	1.42	1.57

Note:Values in lower row in method II are for the case, in which coupled term of mass is neglected. (unit:Hz.)

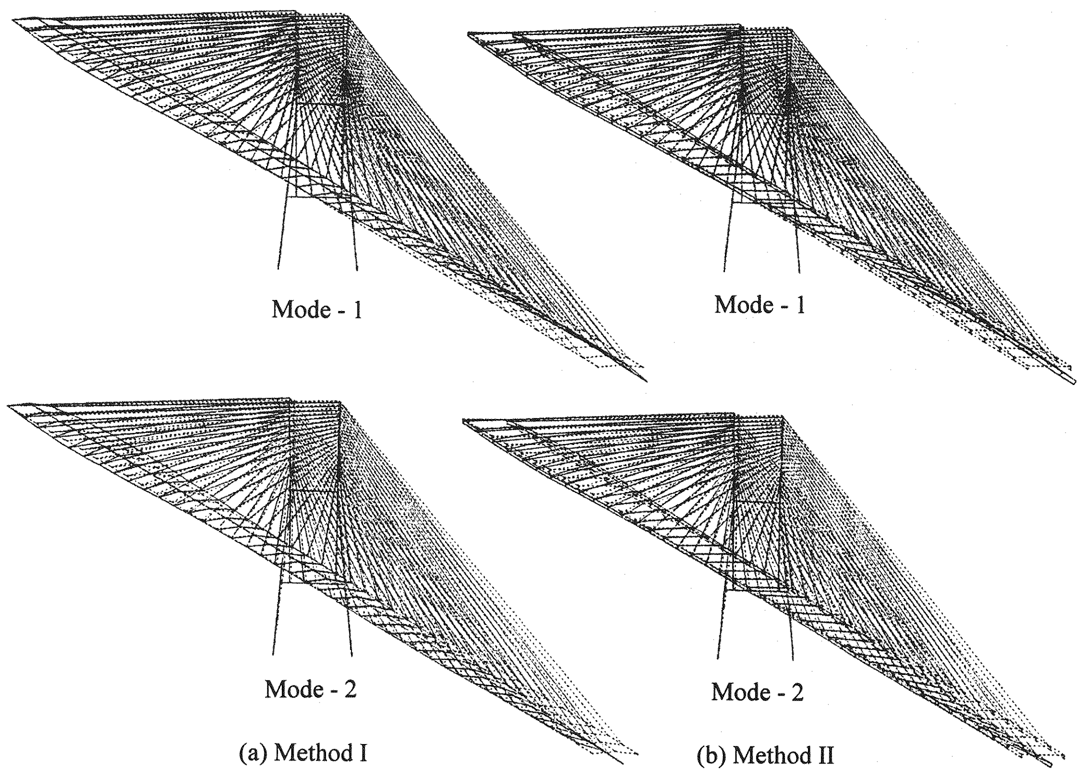


Fig. 9 Vibration modes

specifically for the problem considered and is, therefore, very effective. Also the calculation time needed is much less than that of method I.

5. CONCLUSIONS

In this paper, creep analysis and torsional vibration analysis, which are significant to the design of the cable-stayed bridges with two edge composite girders, were discussed. The construction of such bridges have been increasing in recent years. Two methods were presented for each analysis, applied to a roughly designed cable-stayed bridge of this type and compared. The results obtained in this study are summarized as follows:

1) Creep analysis, in which the composite bridge girder is idealized by a concrete and a steel member, yields sufficiently precise results but requires more computation time because of the increase in the degree of freedom.

2) Creep analysis based on the concept of the transformed cross section underestimates the effect of creep, but the error associated is not large, particularly when compared with the creep analysis that considers the effect of the recovery creep.

3) A design process which only takes into account stress redistribution within the composite girder, while cross sectional forces are assumed to be unchanged due to creep, is possible at the preliminary stage. This is especially desirable, provided that more refined creep analysis such as the one mentioned in 1) is carried out at the final stage.

4) It is possible to conduct torsional vibration analysis of sufficient precision by applying the

analysis method commonly used in the practice for the space frame structures. However, the computation time required is long because of the increase in the degree of freedom.

5) The torsional vibration analysis developed specifically for the cable-stayed bridges considered here is of sufficient precision, and computation time is short.

6) If the reference is placed at the shear center, the coupled term of the mass related to torsion and horizontal flexure results because the shear center generally does not coincide with the center of the mass of the girder. The effect of this coupled term is, however, small in the cable-stayed bridge considered here. The natural frequencies of the first torsional mode and the second horizontal mode become closer due to the coupled term.

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