Structural Eng./Earthquake Eng., JSCE, Vol. 14, No. 1, 63s—72s, 1997 April

CONFIDENCE REGION OF IDENTIFIED PARAMETERS DUE TO ERRORS IN PRESCRIBED PARAMETERS

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In identifying structural parameters, errors in prescribed model parameters influence the value of identified parameters. This paper proposes a formula to estimate the confidence region of identified parameters based on the propagation law of errors and the sensitivity of identified parameter with respect to the model parameter errors. Numerical experiments with a simple model proved that the estimated values were in good agreement with the results of identification analysis, thus verifying the validity of the formula. Using prior information about unknown parameters is thought to effectively minimize the effects of prescribed parameter errors in the identification. Therefore, this study also formulated and theoretically verified a method of estimating the confidence region of identified parameters based on prior information. In this second approach, numerical experiments again demonstrated the validity of the method proposed.

Key Words : *confidence region, sensitivity, prescribed parameter error, identified parameter, Bayesian estimation*

1. INTRODUCTION

Evaluating structural parameters is an effective means of assessing dynamic behaviors of the ground and structures in seismic engineering. A number of studies have been conducted to acquire valuable knowledge and have proposed various approaches for this purpose. See references 1) to 7) for example. Although methods of identifying structural parameters have improved through these efforts, the parameter identification greatly relies on accuracy of measurement. Thus, further advancement in measurement technology is required. Numerous uncertainties are involved in structural identification problems, which inevitably influences identified results. Both input signals and output responses of a given structure will contain some observation noise. Parameter values given as known are not free from errors. Past studies have proven that observation noise can be eliminated to some extent by using a Kalman filter[8)], dynamic programming filter[9)], and other filters. On the other hand, the effects of prescribed parameter errors have not been studied much, except for the study by Koh and See[10)]. Applying the extended Kalman filter to a structural system with an error in one of the masses, unknown parameters are identified. They demonstrated that the converged values differ from their true values and advocated the importance of introducing system noise. To compensate for the discrepancy, they proposed the concept of system noise. Yoshida and Hoshiya[11)] proposed a method of inverse analysis which took into consideration uncertainties of known conditions for a static problem, and discussed the effects of known and unknown conditions, including the degree of certainty of prior information. In a study to identify dynamic structural parameters, the authors[12)] proposed a method of evaluating the effects of model parameter error and observation noise on the results of identified unknown parameters, along with their sensitivity with respect to errors, and confirmed the validity of the proposed approach through numerical experiments.

This paper deals with the effects of errors in prescribed parameters on the identified parameters based

This paper is translated into English from the Japanese paper, which originally appeared on J. Struct. Mech. Earthquake Eng., JSCE, No.537/I-35, pp.267-275, 1996. 4.

on the probability theory, and proposes a method of evaluating these effects as a confidence region of identified parameters, using the sensitivity of unknown parameters with respect to the model parameter errors[12]. In past studies, the confidence regions of identified parameters were evaluated with respect to observation noise based on actual identified results. In contrast, this method evaluates the confidence region of the parameters to be identified without actually conducting identification.

This paper presents a method to evaluate the influence of errors in prescribed parameters on identified parameter values in the form of confidence region. The validity of the method is confirmed through numerical experiments with a simple model. Furthermore, similar formulation is made to evaluate the confidence region of parameters identified by a probability based approach and its validity is verified by numerical experiments.

2. CONFIDENCE REGION OF IDENTIFIED RESULTS

(1) Theoretical Formula

Eq. (1) is the equation of motion in a linear multi-degree-of-freedom system under a seismic load.

$$M\ddot{z}(t) + C\dot{z}(t) + Kz(t) = -M\mathbf{1}\ddot{y}_0(t) \qquad (1)$$

where M, C and K are mass, damping and stiffness matrices, while $\ddot{z}(t)$, $\dot{z}(t)$ and $z(t)$ are relative acceleration, relative velocity and relative displacement vector, respectively. The symbol **1** is a vector whose components are all 1.0. $\ddot{y}_0(t)$ is a scalar quantity which denotes the seismic motion (acceleration) of the ground surface. Unknown model parameters to be identified are expressed as $X = \{X_1, X_2, \ldots, X_M\}^T$ and prescribed parameters as $Y = \{Y_1, Y_2, \ldots, Y_L\}^T$. As the observed value, a time history of acceleration is used. If the observed value $\ddot{u}_i(t)$ at the observation point i, its corresponding analysis value $\ddot{z}_i(t)$ and observation error $\epsilon_i(t)$ are known, the following relationship exists.

$$\ddot{u}_i(t) = \ddot{z}_i(t) + \epsilon_i(t) \quad , \quad i \in A \qquad (2)$$

When the least square method with Eq. (2) is utilized, the evaluation function can be defined as

$$J(X, Y) = \frac{1}{2}\int_{t0}^{t1} \sum_{i \in A} w_i \{\ddot{u}_i - \ddot{z}_i(X, Y)\}^2 dt \qquad (3)$$

where t_0–t_1 is the analysis time interval. w_i is a weight coefficient, the value of which depends on the degrees of importance and reliability of measured data[13].

Let the prescribed model parameter value Y_ℓ deviates from its true value $\overline{Y}_\ell$ by ΔY_ℓ. If ΔY_ℓ is sufficiently small, its effect on the values of identified parameters remains small and thus can be written as

$$X = \overline{X} + \frac{\partial X}{\partial Y_\ell}\Delta Y_\ell \qquad (4)$$

where $\overline{X}$ is the mean value or the true value of X. When the variance and covariance of the prescribed parameter are given, Eq. (4) and the propagation law of errors[14] give the variance-covariance matrix of identified results as

$$\Sigma_{XX} = \Lambda_{XY}^T \Sigma_{YY} \Lambda_{XY} \qquad (5)$$

Σ_{YY} signifies the variance-covariance matrix of the prescribed parameter, which is expressed as

$$\Sigma_{YY} = \begin{bmatrix} \sigma_{Y1Y1} & \sigma_{Y1Y2} & \cdots & \sigma_{Y1YL} \\ \sigma_{Y2Y1} & \sigma_{Y2Y2} & & \vdots \\ \vdots & & \cdots & \\ \sigma_{YLY1} & \cdots & & \sigma_{YLYL} \end{bmatrix} \qquad (6)$$

Λ_{XY} is the sensitivity of the identified parameter with respect to model parameter errors.

$$\Lambda_{XY} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1M} \\ \lambda_{21} & \lambda_{22} & & \vdots \\ \vdots & & \cdots & \\ \lambda_{L1} & \cdots & & \lambda_{LM} \end{bmatrix} = \begin{bmatrix} \frac{\partial X_1}{\partial Y_1} & \frac{\partial X_2}{\partial Y_1} & \cdots & \frac{\partial X_M}{\partial Y_1} \\ \frac{\partial X_1}{\partial Y_2} & \frac{\partial X_2}{\partial Y_2} & & \vdots \\ \vdots & & \cdots & \\ \frac{\partial X_1}{\partial Y_L} & \cdots & & \frac{\partial X_M}{\partial Y_L} \end{bmatrix} \qquad (7)$$

The element of Eq. (7) can be calculated from the following sensitivity equation[12].

$$\sum_{j=1}^{M}\left\{\int_{t0}^{t1} \sum_{i \in A} w_i \frac{\partial \ddot{z}_i}{\partial X_j}\frac{\partial \ddot{z}_i}{\partial X_k} dt\right\}\lambda_{j\ell} = -\int_{t0}^{t1} \sum_{i \in A} w_i \frac{\partial \ddot{z}_i}{\partial Y_\ell}\frac{\partial \ddot{z}_i}{\partial X_k} dt$$

$$\begin{pmatrix} \ell = 1, \ldots, L \\ k = 1, \ldots, M \end{pmatrix} \qquad (8)$$

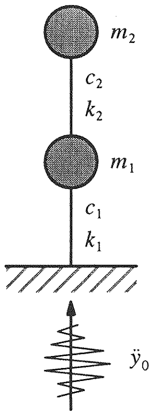
When the distribution type of identified results are assumed to be normal distribution, the probability density function is expressed as

$$P(X) = \frac{1}{(2\pi)^{\frac{M}{2}} |\Sigma_{XX}|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(X-\overline{X})^T \Sigma_{XX}^{-1} (X-\overline{X})\right\} \qquad (9)$$

When the confidence level is $1-e$, therefore, the confidence region of identified results is given as[14]

$$(X-\overline{X})^T \Sigma_{XX}^{-1} (X-\overline{X}) < \varkappa_{e,M}^2 \qquad (10)$$

where $\varkappa_{e,M}^2$ is the limit value at the confidence level of $1-e$ and M degrees of freedom. Eq. (10) describes the inside of a probability ellipsoid. This equation gives an ellipse for M=2, ellipsoid for M=3, and hyper-



$$\overline{m}_1 = \frac{50}{9.8}\ (\mathrm{tf \cdot sec^2/m})$$

$$\overline{m}_2 = \frac{50}{9.8}\ (\mathrm{tf \cdot sec^2/m})$$

$$\overline{c}_1 = 60\ \ (\mathrm{tf \cdot sec/m})$$

$$\overline{c}_2 = 50\ \ (\mathrm{tf \cdot sec/m})$$

$$\overline{k}_1 = 4000\,(\mathrm{tf/m})$$

$$\overline{k}_2 = 3000\,(\mathrm{tf/m})$$

Fig. 1 Analysis Model

ellipsoid for M≧4.

In actual problems, $\overline{X}$ in Eq. (10) is a parameter which should be identified using the population mean $\overline{Y}$ of prescribed parameters. Although $\overline{Y}$ is unknown in actuality, it is possible to evaluate the error in its estimated value by the Bootstrap method[15] or other statistical methods. This makes the proposed theory applicable to actual problems.

(2) Numerical Example

This theory was verified as explained below, through numerical experiments with the two degree-of-freedom system shown in **Fig. 1**.

The input wave was the El Centro wave (NS component) of the Imperial Valley earthquake in 1940, with maximum acceleration adjusted to 300 gal. Time history responses were analyzed using the values of structural parameters in **Fig. 1**. The results were adopted as the observed data in this study.

Prescribed structural parameters were masses m_1 and m_2. Unknown parameters were damping coefficients c_1 and c_2 and stiffness k_1 and k_2. First, the effects of prescribed masses on the identified results were discussed, based on the following assumptions: masses m_1 and m_2 are normally distributed with errors, some statistical characteristics, and mean values $\overline{m}_1$ and $\overline{m}_2$, respectively; $\overline{c}_1$ and $\overline{c}_2$, and $\overline{k}_1$ and $\overline{k}_2$ are the identified results that agree with true values when masses are $\overline{m}_1$ and $\overline{m}_2$; and the weight coefficient is w_i=1. To confirm the validity of the theoretical formula, structural parameters were actually identified by the modified Marquardt method[16] with 1,000 pairs of normally distributed random numbers as the masses with mean values $\overline{m}_1$ and $\overline{m}_2$, respectively, for comparison with theoretical values. The masses are assumed not to be correlated and to have a coefficient of variation COV=0.05.

The histograms in **Fig. 2** show the distributions of prescribed masses and identified results. The curves show the probability density function of normal distribution. **Fig. 2** confirms that masses generated by random numbers have the specified mean value and standard deviation. Due to the dispersion of prescribed masses, the identified results disperse with a coefficient of variation of 0.03 to 0.07 under a normal distribution. **Fig. 3** compares the identified and theoretical values. The probability ellipse in **Fig. 3** represents the confidence limit with two degrees of freedom and a confidence level of 95%. Small circles in the figure indicates identified results. Since there are four identified parameters, their confidence region makes a hyper-ellipsoid with four degrees of freedom, which cannot be graphically represented. Therefore, six pairs of two different parameters are composed, for each of which a probability distribution with two degrees of freedom is drawn in **Fig. 3**. The analysis proves that the identified results vary widely according to the prescribed parameter errors. The distributions of the two damping coefficients ($c_1 - c_2$) are only slightly correlated with the identified results. Correlations between other parameters are more or less positive. **Fig. 3** proves that the identified results are in good agreement with probability ellipses. The numerator of the fraction framed in **Fig. 3** shows the number of identified parameters that fall in the probability ellipse. The ratio of the number of identified parameters that are in the probability ellipse to the total number at the confidence level of 95% is near 950/1,000 in each case. This proves that the identified parameters are in good agreement with the theory. **Table 1** shows the sensitivity of identified parameters with respect to mass errors which was used to calculate the probability ellipse. **Table 2** compares the values of variance-covariance and coefficients of correlation of identified parameters with those estimated by Eq. (5). They are also in good agreement. Trace(Σ_{XX}) in **Table 2** describes the traces of the variance-covariance matrices. Coefficients of correlation of different parameters show that the two damping coefficients are only slightly correlated, while other parameters are correlated to some extent. The above evidence proves that the effects of prescribed parameter errors on identified parameters can be evaluated using a confidence region expressed as a probability ellipse.

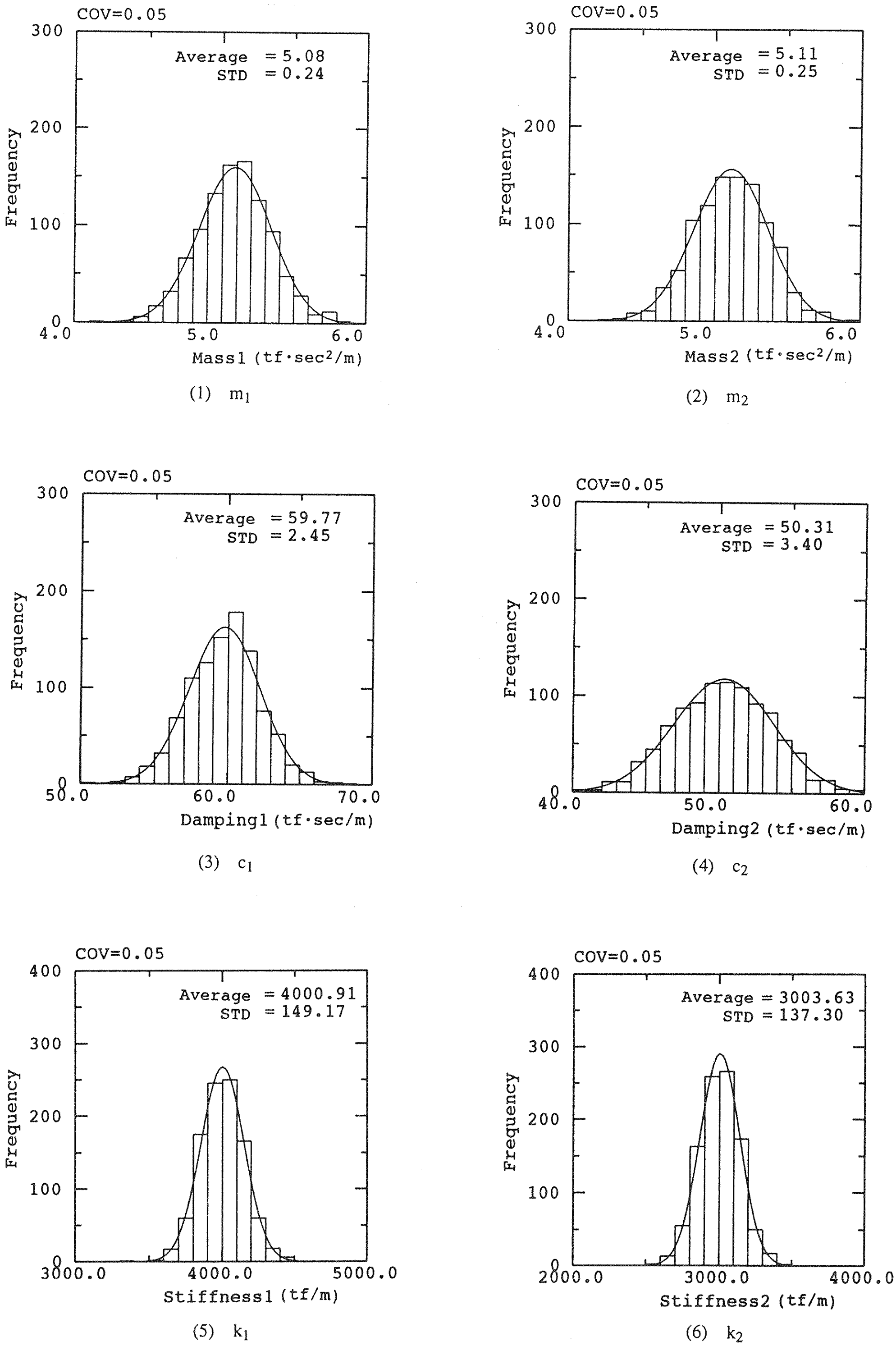


Fig.2 Distribution of prescribed parameters and identified parameters

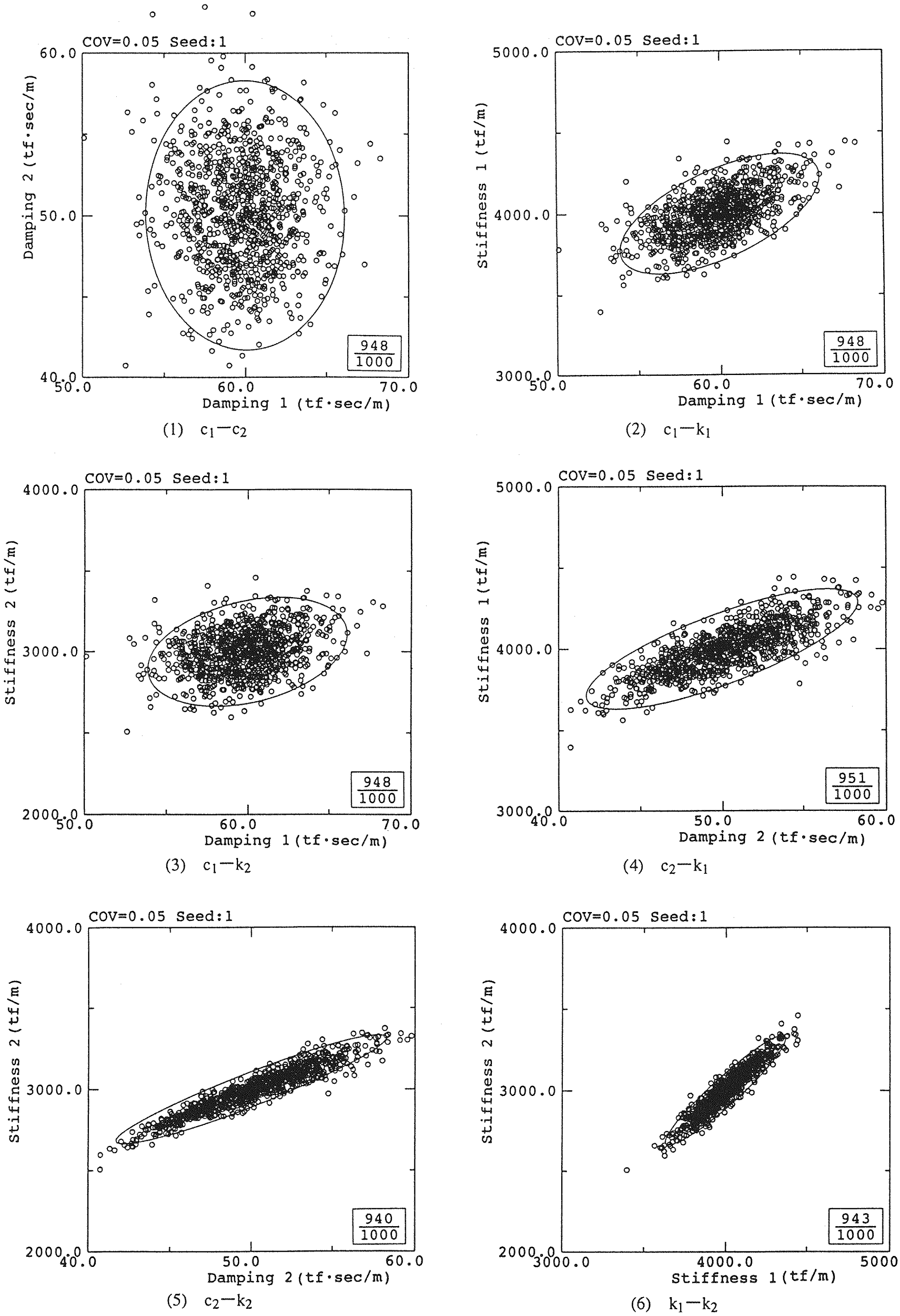


Fig.3 Confidence region of identified results

Table 1 Sensitivities of identified parameters with respect to mass errors

Y_ℓ	$\frac{\partial c_1}{\partial Y_\ell}$	$\frac{\partial c_2}{\partial Y_\ell}$	$\frac{\partial k_1}{\partial Y_\ell}$	$\frac{\partial k_2}{\partial Y_\ell}$
m_1	9.56	−3.14	248.26	51.18
m_2	2.20	12.94	535.74	536.82

Units $\frac{\partial c_i}{\partial m_j}$: (1/sec) , $\frac{\partial k_i}{\partial m_j}$: (1/sec^2)

Table 2 Comparison of variance-covariances and correlation coefficients

		Estimated	Identified
Variance-covariance	σ_{c1c1}	6.3	6.0
	σ_{c1c2}	−0.1	−0.2
	σ_{c1k1}	231.1	219.9
	σ_{c1k2}	108.7	103.1
	σ_{c2c2}	11.5	11.6
	σ_{c2k1}	400.4	399.4
	σ_{c2k2}	441.6	441.5
	σ_{k1k1}	22689.1	22240.6
	σ_{k1k2}	19542.7	19334.1
	σ_{k2k2}	18924.1	18846.2
	trace(Σ_{XX})	41631.0	41104.4
Correlation coefficient	ρ_{c1c2}	−0.0120	−0.0192
	ρ_{c1k1}	0.6131	0.6018
	ρ_{c1k2}	0.3156	0.3065
	ρ_{c2k1}	0.7826	0.7869
	ρ_{c2k2}	0.9450	0.9448
	ρ_{k1k2}	0.9431	0.9444

3. REDUCTION OF THE CONFIDENCE REGION BASED ON PRIOR INFORMATION

In parameter identification, there exists the problem of compromising reciprocal requirements for model resolution and estimation errors. According to available study results[17),18)], this drawback is eliminated if a solution based on the minimum variance criterion is obtained by applying prior information. This approach may effectively identify parameters with high precision when prescribed parameters have errors, such as in the present case. As discussed above with respect to an example problem, the effects of prescribed parameter errors can be evaluated as the confidence region of identified parameters. Therefore, the theory presented in this paper will be able to evaluate the precision of identified results, which is improved by a solution based on the minimum variance criterion obtained by applying prior information. When the problem is non-linear, this solution is the one obtained by the Bayesian estimation method[18)], which is an approach based on the probability theory. Therefore, the sensitivity of identified parameters with respect to model errors is not applicable as it is, because the evaluation function discussed there is different from the one to be applied to this case. First, therefore, a formula will be introduced below to determine the sensitivity of identified parameters with respect to model errors; this sensitivity can be treated with Bayesian estimation. Next, the confidence region of identified parameters affected by prescribed parameter errors will be introduced by applying the sensitivity thus obtained to an example problem similar to the one above.

(1) Formulation of Sensitivity Equation

As prior information, it is assumed that identified parameters are normally distributed with the following mean value and variance-covariance.

$$E[X] = \overline{X} \quad , \quad E\left[(X-\overline{X})(X-\overline{X})^T\right] = M \tag{11}$$

where $E[\cdot]$ is the expectation. When the statistical characteristics of the observation error $\epsilon_i(t)$ in Eq. (2) are expressed as Eq. (12), Eq. (13) gives the evaluation function by Bayesian estimation.

$$E[\epsilon] = 0 \;, \quad E\left[\epsilon\epsilon^T\right] = R \tag{12}$$

$$J = \frac{1}{2}\int_{t0}^{t1} \{\ddot{u}-\ddot{z}(X)\}^T R^{-1}\{\ddot{u}-\ddot{z}(X)\}dt + \frac{1}{2}(X-\overline{X})^T M^{-1}(X-\overline{X}) \tag{13}$$

When a model parameter Y_ℓ whose true value is $\overline{Y}_\ell$ has an error of ΔY_ℓ, substituting Eq. (4) into Eq. (13) gives

$$\begin{aligned} J = \frac{1}{2}\int_{t0}^{t1} &\left\{\ddot{u}-\ddot{z}\left(\overline{X} + \frac{\partial X}{\partial Y_\ell}\Delta Y_\ell, \overline{Y}_\ell + \Delta Y_\ell\right)\right\}^T R^{-1} \\ &\cdot \left\{\ddot{u}-\ddot{z}\left(\overline{X} + \frac{\partial X}{\partial Y_\ell}\Delta Y_\ell, \overline{Y}_\ell + \Delta Y_\ell\right)\right\}dt \\ &+ \frac{1}{2}\left\{\left(\overline{X} + \frac{\partial X}{\partial Y_\ell}\Delta Y_\ell\right)-\overline{X}\right\}^T M^{-1}\left\{\left(\overline{X} + \frac{\partial X}{\partial Y_\ell}\Delta Y_\ell\right)-\overline{X}\right\} \end{aligned} \tag{14}$$

where $\ddot{\bar{u}}$ is an observed value which doesn't contain observation noise. Taylor expansion approximates Eq. (14) as

$$J = \frac{1}{2}\int_{t0}^{t1} \left\{\ddot{\bar{u}}-\ddot{z}(\overline{X})-H_X\lambda_\ell \Delta Y_\ell - H_Y \Delta Y_\ell\right\}^T R^{-1} \cdot \left\{\ddot{\bar{u}}-\ddot{z}(\overline{X})-H_X\lambda_\ell \Delta Y_\ell - H_Y \Delta Y_\ell\right\} dt + \frac{1}{2}\left\{(\overline{X}+\lambda_\ell \Delta Y_\ell)-\overline{X}\right\}^T M^{-1}\left\{(\overline{X}+\lambda_\ell \Delta Y_\ell)-\overline{X}\right\} \quad (15)$$

where

$$H_X = \begin{bmatrix} \frac{\partial \ddot{z}_1}{\partial X_1} & \frac{\partial \ddot{z}_1}{\partial X_2} & \cdots & \frac{\partial \ddot{z}_1}{\partial X_M} \\ \frac{\partial \ddot{z}_2}{\partial X_1} & \frac{\partial \ddot{z}_2}{\partial X_2} & & \vdots \\ \vdots & & \cdots & \\ \frac{\partial \ddot{z}_N}{\partial X_1} & \cdots & & \frac{\partial \ddot{z}_N}{\partial X_M} \end{bmatrix},$$

$$\lambda_\ell = \begin{Bmatrix} \frac{\partial X_1}{\partial Y_\ell} \\ \frac{\partial X_2}{\partial Y_\ell} \\ \vdots \\ \frac{\partial X_M}{\partial Y_\ell} \end{Bmatrix}, \quad H_Y = \begin{Bmatrix} \frac{\partial \ddot{z}_1}{\partial Y_\ell} \\ \frac{\partial \ddot{z}_2}{\partial Y_\ell} \\ \vdots \\ \frac{\partial \ddot{z}_N}{\partial Y_\ell} \end{Bmatrix} \quad (16)$$

$$(\ell = 1,\ldots,L)$$

and λ_ℓ is the sensitivity vector of identified parameters with respect to model parameter errors. After manipulation using the identity $\ddot{\bar{u}}=\ddot{z}(\overline{X})$, Eq. (15) is reduced to

$$J = \frac{1}{2}(\Delta Y_\ell)^2 \int_{t0}^{t1} \left(-H_X\lambda_\ell - H_Y\right)^T R^{-1}\left(-H_X\lambda_\ell - H_Y\right)dt + \frac{1}{2}(\Delta Y_\ell)^2 \lambda_\ell^T M^{-1}\lambda_\ell \quad (17)$$

The condition to minimize Eq. (17) irrespective of the prescribed parameter errors ΔY_ℓ gives

$$dJ = d\lambda_\ell^T \left\{ -\int_{t0}^{t1} H_X^T R^{-1}(-H_X\lambda_\ell - H_Y)dt + M^{-1}\lambda_\ell \right\} = 0 \quad (18)$$

Eq. (18) gives the sensitivity equation as

$$\left\{\int_{t0}^{t1} H_X^T R^{-1} H_X\, dt + M^{-1}\right\}\lambda_\ell = -\int_{t0}^{t1} H_X^T R^{-1} H_Y\, dt \quad (\ell = 1,\ldots,L) \quad (19)$$

The solution of the simultaneous Eq. (19) gives the sensitivity. The propagation law of errors in Eq. (5) determines the variance-covariance matrix of identified parameters.

(2) Numerical Example

The theory was verified with the two degree-of-freedom system and prescribed masses used in the example problem in **2.(2)** through numerical experiments.

To adjust conditions to those in **2.(2)**, the components of variance-covariance matrix of observation errors were set as $R_{ii} = 1.0, (i = 1,2)$ and $R_{ij} = 0.0$, $(i \neq j)$. As prior information, mean values of both the damping coefficient and stiffness along with their COV's = 0.005 are assigned. Such a confidence level with a COV as small as 0.005 doesn't normally necessitate identification analysis. The reasons for setting the prior information this way are as follows: First, R^{-1} and M^{-1} in Eq. (18) can be regarded as weighted coefficients to show the reliability of the observation values and prior information, or parameters to determine which term (first or second) on the right side of Eq. (13) should be weighted more heavily, or to adjust the balance between them. Namely, they only have a relative significance. (For such cases, the extended Bayesian estimation[19] multiplies the second term on the right side of Eq. (13) by a scalar quantity.) In this case, the variances of all observed values were set to 1.0 to adjust the conditions to those of the previous example. To make the second term on the right side of Eq. (13) significant, therefore, it is necessary to make the variance of the identified parameter small, as mentioned above. In this paper, this value is used only to verify the theory. In actual problems, however, an appropriate value should be adopted based on the covariance matrix of observation errors.

Fig. 4 compares the identified and theoretical values. The probability ellipse in the figure represents the confidence region, obtained from Eq. (10), for two degrees of freedom with a confidence level of 95%. The sensitivity of identified parameters with respect to mass errors were obtained by the sensitivity equation (19). Small circles plotted in the figure indicate the results from identification. Application of prior information gave a small variance for identified results for all parameters. This suggests that the variance of identified parameters given as prior information worked as a constraining condition. The damping coefficients $(c_1 - c_2)$, which were only slightly correlated when prior information was not given, are now positively correlated due to the prior information. Coefficients of correlation between other parameters also increased in comparison with the case without using prior information. The figure shows that the distribution of identified parameters and the theoretical

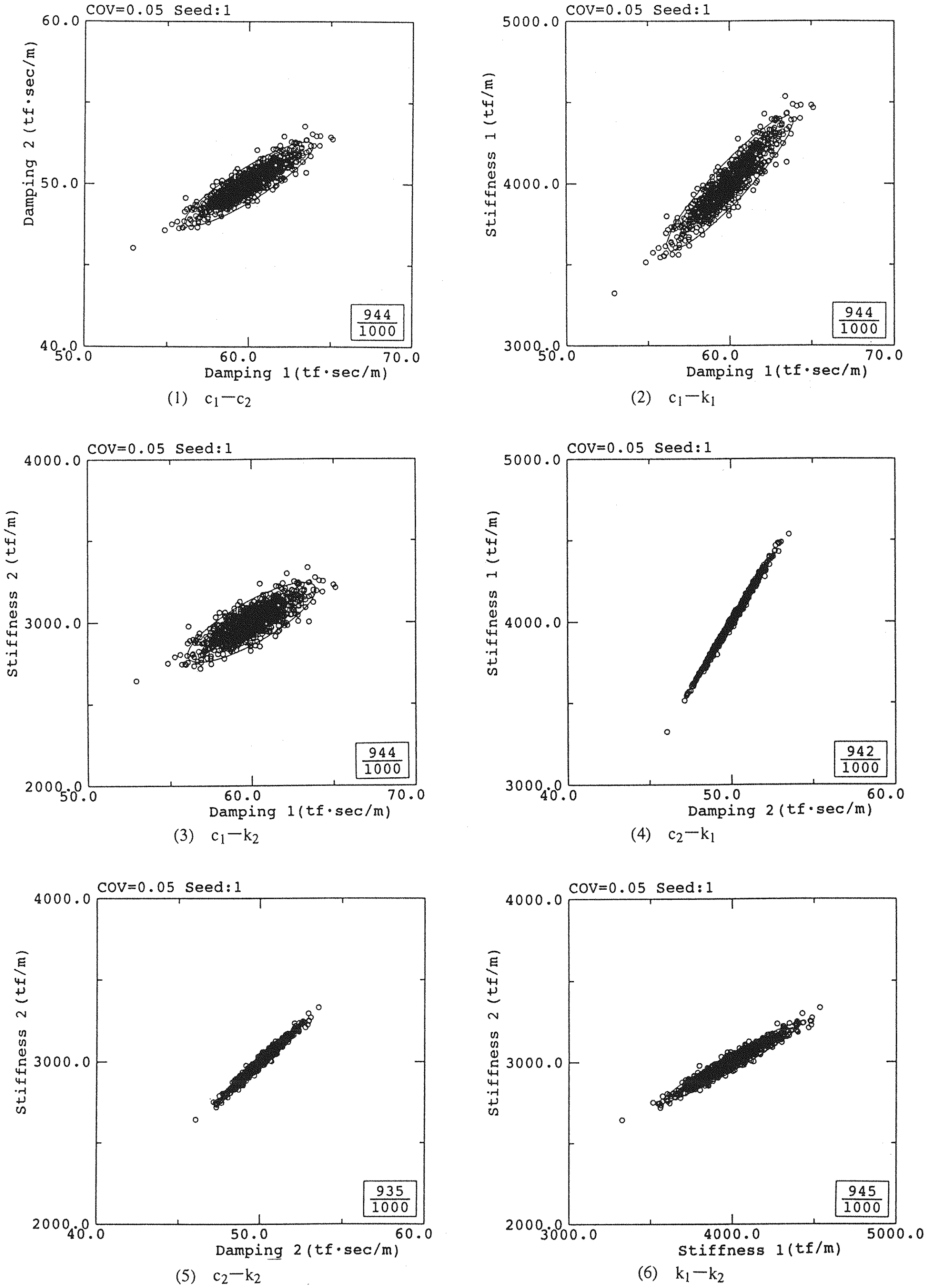


Fig.4 Confidence region of identified results with prior information

Table 3 Sensitivities of identified parameters with respect to mass errors

Y_ℓ	$\frac{\partial c_1}{\partial Y_\ell}$	$\frac{\partial c_2}{\partial Y_\ell}$	$\frac{\partial k_1}{\partial Y_\ell}$	$\frac{\partial k_2}{\partial Y_\ell}$
m_1	4.24	1.15	237.33	46.46
m_2	4.65	3.99	628.93	384.11

Units $\frac{\partial c_i}{\partial m_j}$: (1/sec) , $\frac{\partial k_i}{\partial m_j}$: (1/sec²)

Table 4 Comparison of variance-covariances and correlation coefficients

		Estimated	Identified
Variance-covariance	σ_{c1c1}	2.6	2.5
	σ_{c1c2}	1.5	1.5
	σ_{c1k1}	255.8	250.7
	σ_{c1k2}	129.0	127.0
	σ_{c2c2}	1.1	1.1
	σ_{c2k1}	181.3	178.9
	σ_{c2k2}	103.3	102.3
	σ_{k1k1}	29406.9	28895.5
	σ_{k1k2}	16438.7	16242.9
	σ_{k2k2}	9742.0	9671.4
	trace(Σ_{XX})	39152.6	38570.5
Correlation coefficient	ρ_{c1c2}	0.8965	0.8969
	ρ_{c1k1}	0.9290	0.9250
	ρ_{c1k2}	0.8142	0.8100
	ρ_{c2k1}	0.9968	0.9969
	ρ_{c2k2}	0.9872	0.9852
	ρ_{k1k2}	0.9712	0.9716

probability ellipse are in good agreement. The numerator of the fraction framed in the figure shows the number of identified parameters that fall in the probability ellipse. The ratio of the number of identified parameters that are in the probability ellipse to the total number at a confidence level of 95 % is near 950/1,000 in each case. This proves that the identified parameters are in good agreement with the theory. **Table 3** shows the sensitivities of identified parameters with respect to mass errors, which is smaller than those in **Table 1** in most cases. This also shows that the variance of identified results is smaller when prior information is applied. **Table 4** compares the variances, covariances, and coefficients of correlation between identified and theoretical values, and demonstrates good agreement between them.

The variances of identified parameters except k_1 are smaller when prior information is used. The variance of k_1 has slightly increased, presumably because the prior information on k_1 (variance), which was not small enough, didn't work as a weight. The traces of variance-covariance matrices of identified parameters with and without prior information were compared. Those with prior information were smaller. This demonstrated that the solution obtained is based on the minimum variance criterion that minimizes total variance. The coefficients of correlation, which are all near 1.0, quantitatively show the high degrees of correlation between parameters.

4. CONCLUSIONS

This paper has proposed a method of evaluating the confidence region of identified parameters under the influence of errors in prescribed parameters, and verified the theory through numerical experiments. Conclusions regarding the formulated theory and numerical experiments are summarized as follows.

(1) The confidence region of identified parameters under the influence of prescribed parameter errors can be evaluated as a probability ellipse.

(2) In order to minimize the effects of prescribed parameter errors on identified parameters, a theoretical formula was introduced to estimate the confidence region of identified parameters when an evaluation function was formulated based on prior information.

(3) The confidence region of identified parameters estimated by the method proposed in this paper satisfactorily agrees with the distribution of actual identified results.

(4) Identification that takes prior information into consideration reduces the variance of identified parameters generated by prescribed parameter errors.

(5) When prescribed parameters have errors, prior information enhances the correlation between identified parameters.

This study dealt only with the effects of prescribed parameter errors. In actual problems, however, observed values contain observation noise, which should be taken into consideration when evaluating the confidence region of identified parameters. The proposed method can be expanded to evaluate observation noise as well.

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