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THE STABILITY DESIGN OF TOWERS OF CABLE-SUPPORTED BRIDGES

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Recently, cable-supported bridges in Japan tend to have longer spans. Bridges with a span of 2500 meters are now in a planning stage. Towers of cable-supported bridges have been designed, however, based on each public corporation's own design codes. Consequently, there exist no established common design codes for the towers of cable-supported bridges. The authors have conducted a study of Japanese stability design procedures concerning towers for long-span bridges in order to establish a set of new procedures in the future. This paper presents some characteristics of the practical stability designs and problems concerning tower design codes.

***Key Words** : stability design, suspension bridges, cable-stayed bridges, towers, steel structures*

1. INTRODUCTION

Towers of suspension bridges can be classified into the truss, portal, and combined types. In general, the weight of steel tower becomes lightest in the truss type bridges if design conditions are identical. Due to considerations of the artistic nature of external views as well as efforts to construct bridges as historic monuments, many portal type bridges have been constructed throughout the world.

In the past, the most economical span for a cable-stayed bridge was considered to range between 100 to 350 meters. Recently, however, spans of cable-stayed bridges have increased greatly. The Tatara Bridge built by Honshu-Shikoku Bridge Authority has a span of 890 meters. In these long-span cable-stayed bridges, the ratio of the tower height to the span length is larger as compared to that of suspension bridges. This characteristic greatly influences the overall view of the bridges.

In the past, towers of cable-supported bridges were designed based on the Standard Specifications for Highway Bridges and their Comments[1]. However, at present, towers are designed based upon each authority's own codes[2),3]. Here, designs are carried out on the basis of allowable stress design method centering around the effective buckling length.

Design engineers have pointed out the irrationality in the concept of buckling length as well as uncertainty in the safety verification formula based on the finite displacement theory[4)−7]. Due to these weaknesses, the current method is not necessarily adequate for future tower designs.

As seen in recent codes such as BS5400[8], DIN18800[9], and Design Code for Steel Structures of the Japan Society of Civil Engineers[10], the fundamental concept is changing from allowable stress design to the limit state design where the maximum load-carrying capacity of structural members become critical.

In Japan, several long-span cable-supported bridges to be built in the 21st Century with a span of 1000 to 2500 meters are now under study. The authors have conducted detailed studies of the tower of some domestic and overseas cable-supported bridges[11),12]. The present report discusses the characteristics of the existing buckling design of towers and underlying policies in order to evaluate difficulties in the present design procedures.

2. BRIDGES INVESTIGATED

In the past, suspension bridges with an effective span of 400 to 500 meters or greater were considered to be of a long span. Now, this definition is blurred because there are many bridges

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Table 1 Towers of suspension bridges

Identification	Main span	Tower height	tower type	Date opened
Akashi-Kaikyo (HSBA)	1990	283	Truss	U.C.
Minami-Bisan-Seto (HSBA)	1100	180	Truss	1988
Kurushima-III (HSBA)	1030	178	4-story	U.C.
Kurushima-II (HSBA)	1020	165	4-story	U.C.
Kita-Bisan-Seto (HSBA)	990	169	Truss	1988
Shimotsui-Seto (HSBA)	940	141	3-story	1988
Ohnaruto (HSBA)	876	126	Truss	1985
In-No-Shima (HSBA)	770	136	Truss	1983
Hakucho (HDB)	720	120	4-story	U.C.
Kanmon (JHPC)	610	134	Truss	1973
Kurushima-I (HSBA)	600	144	3-story	U.C.
Rainbow (MEPC)	570	117	3-story	1994
Ohshima (HSBA)	560	90	2-story	1985
Hirado (NP)	465	78	3-story	1977

HSBA : Honshu-Shikoku Bridge Authority
JHPC : Japan Highway Public Corporation
MEPC : Metropolitan Expressway Public Corporation
HDB : Hokkaido Development Board
NP : Nagasaki Prefecture
U.C. : Under construction
Unit : m

with a span exceeding 500 meters. The authors counted over 50 suspension bridges in the world where their effective spans are larger than 500 meters. **Table 1** shows 14 suspension bridges with steel towers in Japan the authors studied.

Today, over sixty cable-stayed bridges with maximum spans exceeding 200 meters are under construction in the world. There are thirteen cable-stayed bridges with a span larger than 400 meters demonstrating the trend of increased spans[12]. **Table 2** shows twenty-two Japanese cable-stayed bridges with steel towers covering both ordinary and composite cable-stayed types (including those under construction).

The current study by the authors utilized reference materials[6], and supplementary numerical calculations were carried out as required.

3. BUCKLING DESIGN METHOD OF TOWER

Towers of cable-supported bridges are generally flexible. A tower receives the vertical force, horizontal displacement, unbalanced cable force, and the reactive forces to wind and seismic loads in addition to the dead load of the tower. In other words, the structural members of a tower receive biaxial bending moments and axial compressive forces simultaneously. Therefore, a tower must be designed to ensure sufficient buckling stability of the main structures, structural members, and stiffening plates.

Design Codes of Japan Society of Highway Bridges are commonly adopted in tower designs. However, the JSHB Codes do not cover special cases where bridge spans exceeding 200 meters. Instead, individual authorities and municipalities evaluate their trial designs through own design methods.

Buckling design methods for towers can now be classified into the following four types:

Method 1 is a general design method based on the JSHB Codes as shown in **Fig.1-a**.

Method 2 is a method of designing towers of cable-stayed bridges where verification of the ultimate strength of the tower as a whole is carried out subsequent to the decision concerning the cross section based on Method 1, as shown in **Fig.1-b**.

Method 3 is a standard design method applicable to the towers of suspension bridges. Here, the method of evaluating factors concerning buckling design is different from Method 1. Method 3 was applied for the Honshu-Shikoku Bridges as shown in **Fig.1-c**. History of HSBA Codes is shown in **Table 3**.

Method 4 is a new design method standardized by Hanshin Expressway Public Corporation for cable-stayed bridges as shown in **Fig.1-d**. Method 4 makes extensive use of nonlinear analyses.

Table 2 Towers of cable-stayed bridges

Identification	Main span	Tower height	tower type	Date opened
Tatara (HSBA)	890	220	Inv.-Y	U.C.
Nagoya-port-Chuo (JHPC)	590	190	A	U.C.
Tsurumi-Tsubasa (MEPC)	510	182	Inv.-Y	1994
Iguchi (HSBA)	490	123	A	1991
Higashi-Kobe (HEPC)	485	147	H	1993
Yokohama-Bei (MEPC)	460	172	H	1989
Iwaguro-Jima (HSBA)	420	152	H	1988
Hitsuishi-Jima (HSBA)	420	143	H	1988
Nagoya-port-Higashi (JHPC)	410	125	A	U.C.
Nagoya-port-Nishi (JHPC)	405	122	A	1985
Yamatogawa (HEPC)	355	70	Single	1982
Tenpozan (HEPC)	350	161	A	1989
Suehiro (TP)	250	48	Single	1975
Kamome (OC)	240	48	Single	1975
Yasaka (MC)	240	47	A	1987
Kemi-1 (WP)	239	94.7	A	1993
Sugawara-Shirokita (OC)	238	46	Single	1989
Rokko (KC)	220	57	H	1976
Katsushika-Harp (MEPC)	220	68	Single	1987
Toyosato (OC)	216	35	Portal	1970
Onomichi (HSBA)	215	39	Portal	1968
Torikai-Niwaji (OP)	200	49	Single	1987

HEPC : Hanshin Expressway Public Corporation
TP : Tokushima Prefecture
OC : Osaka City
MC : Ministry of Construction
WP : Wakayama Prefecture
KC : Kobe City
OP : Osaka Prefecture
U.C. : Under construction
Unit : m

4. FACTORS IN EXISTING BUCKLING DESIGNS

Tables 4,5,6 and **7** show a summary of design factors mentioned in the above 3. with respect to the cable-supported bridges including structural analyses, stress and stability checks, effective buckling length, and load-carrying capacity checks. In **Table 4**, with respect to the Honshu-Shikoku suspension bridges, only the methods for Akashi-Kaikyo Bridge and Shimotsui-Seto Bridge are shown as representative examples.

(1) Structural Analysis

a) Suspension Bridges

The structural analysis of the tower is calculated for both the bridge axis (out-of-plane structure) and the direction perpendicular to the bridge axis (in-plane structure). Linear analysis is applied to all towers designed based on JSHB Codes.

Table 3 shows a history of HSBA Codes. 1965 HSBA Codes utilized a linear theory as far as the structural analysis in the direction of the bridge axis was concerned. 1972 and 1980 HSBA Codes introduced a finite displacement theory which considers the equilibrium state after deformation in order to take account of the P–Δ effect.

With respect to the in-plane structural analysis, 1965 and 1972 Codes were based on a linear theory. In 1980 Codes, the theoretical basis was changed to the finite displacement theory. In the recent design of towers, finite displacement analyses are performed based on 3-dimensional models, and the sectional forces are confirmed.

b) Cable-Stayed Bridges

Structural analysis of the towers of cable-stayed bridges mainly utilizes linear analysis for each plane structures. However, in the detailed designs of recent bridges, 3-dimensional models have also been introduced. Introduction of 3-D models was necessary to understand the mechanical behavior of 3-dimensional structures such as A-type towers.

Furthermore, some bridges utilize finite displacement analysis or 3-D finite displacement

Table 3 History of HSBA Code of Tower Construction for Suspension Bridges

	1965 code[13]	1972 code[14]	1980 code[15], 1988 code[2]
Structural analysis (longitudinal axis)	linear theory, plane model (top-hinged, base-fixed)	nonlinear theory, plane model (top-hinged, base-fixed)	nonlinear theory, plane model (top-hinged, base-fixed)
Structural analysis (transverse axis)	linear theory, plane model (base-fixed)	linear theory, plane model (base-fixed)	nonlinear theory, plane model (base-fixed)
Rigidity required	Horizontal reactive force on tower top ; $F \geq 0$		
Stress check	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_a$	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_{ca0}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_{cal}$
Stability check		$\frac{\sigma_c}{\sigma_{cax}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1 - \sigma_c/\sigma_{eax})} \leq 1$	$\frac{\sigma_c}{\sigma_{cax}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1 - \sigma_c/\sigma_{eax})} \leq 1$
Buckling length	Truss : panel length	Truss type : panel length, Portal type : alignment charts	Truss and Portal type : E_f method, σ_{eax} is panel length
Reduction factor to equivalent moment		Truss type : Austin formula Portal type : $c_x = 0.85$	Truss and portal type : Austin formula
Check of overall buckling			$\frac{\ell_e}{r_x} < 0.7 \frac{H}{r_y}$
Check of ultimate strength		Considering	
Stress and stability check of beam		When axial force is tensile, $\sigma_t + \sigma_{bt} \leq \sigma_{ta}$ When axial force is compressive, $\frac{\sigma_c}{\sigma_{ca}} + \frac{\sigma_{bc}}{\sigma_{ba0}} \leq 1$	When axial force is tensile, $\sigma_t + \sigma_{bt} \leq \sigma_{ta}$ When axial force is compressive, $\frac{\sigma_c}{\sigma_{ca}} + \frac{\sigma_{bc}}{\sigma_{bag}(1 - \sigma_c/\sigma_{ea})} \leq 1$ $\sigma_c + \frac{\sigma_{bc}}{1 - \sigma_c/\sigma_{ea}} \leq \sigma_{cal}$
Note	σ_c, σ_t : compressive and tensile stress, σ_{bx}, σ_{by} : bending compressive stress about x- y-axis, σ_{bc}, σ_{bt} : bending compressive and tensile stress, σ_{bag} : allowable bending stress, $\sigma_{ea}, \sigma_{eax}$: allowable buckling stress, $\sigma_a, \sigma_{cax}, \sigma_{ca0}$: allowable compressive stress, σ_{cal} : local buckling stress, c_x : reduction factors about x-axis, r_x, r_y : radius of gyration about x-, y-axis		

analysis to calculate sectional forces while considering the effect of sag of cables so that accurate judgements can be made with respect to the impact of nonlinearity even though their fundamental structural analyses are of linear nature.

Finite displacement theory was adopted to structural analysis of the Honshu-Shikoku Bridges. This was done to take the effect of nonlinearity of long-span bridges into consideration and simultaneously harmonize the approach with the method of designing the towers of suspension bridges. In these bridges, the accuracy of the in-plane analyses is confirmed separately through 3-dimensional finite displacement analyses considering the effect of sag of cables.

A linear analysis is performed for the structural analyses of HEPC bridges. Here, a correction factor r which indicates the effect of nonlinearity of the structure is introduced. This factor is determined by the ratio of the section force obtained by elasto-plastic finite displacement analysis with 1.7 times load of the design load, to the section force obtained by linear analysis.

(2) Stress and Stability Check

a) Suspension Bridges

With respect to the bending buckling in the axial direction of bridges, 1965 Codes required bending rigidity which satisfies the relationship $F = 0$ for the horizontal reactive force of the cable system at the top of the tower, following the proposal by Birdsall[16]. Because of the design method in the $F < 0$ region by Klöppel[17] and of subsequent experimental findings that the actual ultimate strength at completed stage is different from calculated strength using the method given by Birdsall[10], the condition for required rigidity was eliminated in 1972 Codes resulting in more flexible design approaches. **Fig.2** shows the distribution of F values obtained by actual investigation.

Stress checks conforming to 1965 Codes seem to have been conducted on the basis of the JSHB Codes. The stress checks to satisfy 1980 Codes use the condition that the ultimate strength does not exceed the local buckling of stiffening plates.

In the bridge axis, 1972 Codes no longer require

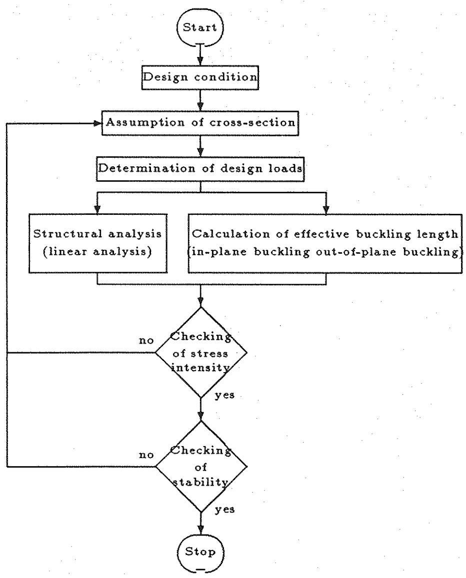


Fig.1-a Method 1

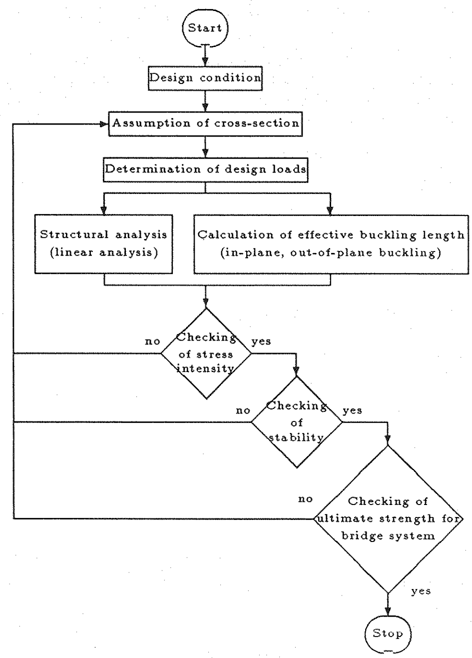


Fig.1-b Method 2

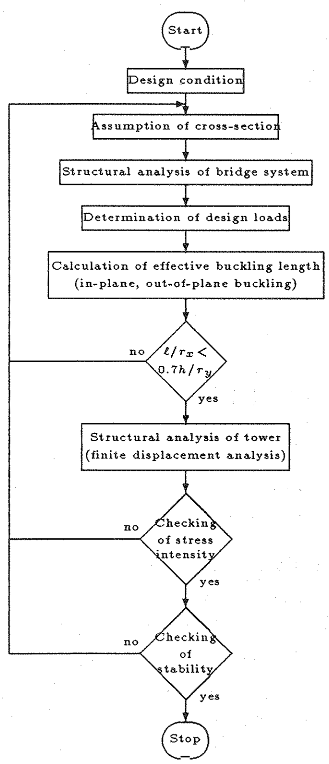


Fig.1-c Method 3

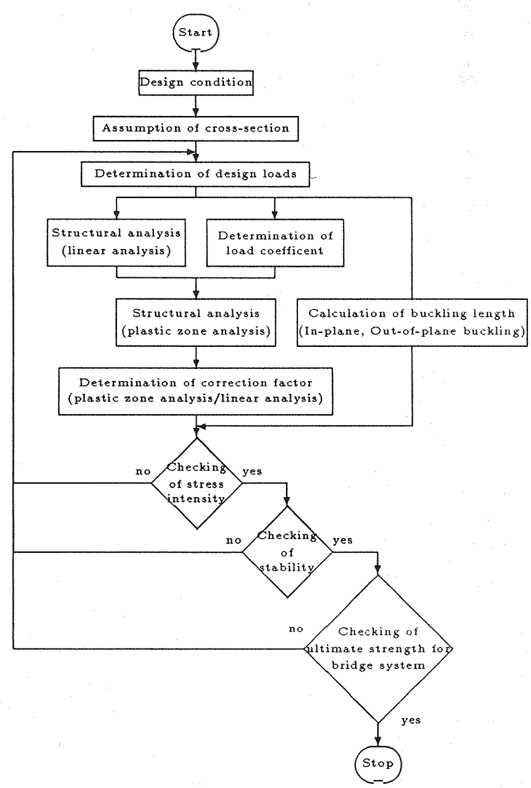
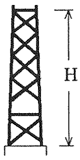
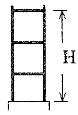
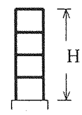
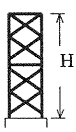
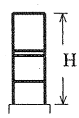
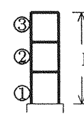


Fig.1-d Method 4

Fig.1 Design procedures of towers for cable-stayed bridges

Table 4 Comparison of Design Factors of Tower for Suspension Bridges

Bridge	Akashi-Kaikyo(U.C.)	Shimotsui-Seto(1988)	Hakucho(U.C.)
Tower type and span length	Truss, 283m 1990m	Portal, 141m 940m	Portal, 120m 720m
Structural analysis	nonlinear theory, plane model	nonlinear theory, plane model	nonlinear theory, plane model
Stress check	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_{cal}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_{cal}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_{cal}$
Stability check	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \le 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \le 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \le 1$
Buckling length	E_f method	E_f method	E_f method
Check of overall buckling	$\frac{\ell_e}{r_x} < 0.7\frac{H}{r_y}$	$\frac{\ell_e}{r_x} < 0.7\frac{H}{r_y}$	$\frac{\ell_e}{r_x} < 0.7\frac{H}{r_y}$
			

Bridge	Kanmon(1973)	Rainbow(1994)	Hirado(1977)
Tower type and span length	Truss, 134m 712m	Portal, 117m 570m	Portal, 78m 465m
Structural analysis	linear theory, plane model	nonlinear theory, plane model	linear theory, plane model
Stress check	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_a$	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_{cal}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_a$
Stability check		$\frac{\sigma_c}{\sigma_{cax}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \le 1$	$\frac{\sigma_c}{\sigma_{cax}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y \sigma_{by}}{\sigma_{ba0}} \le 1$
Buckling length	Table values of JSHB	E_f method	① : 1.5H, ②,③ : 1.9H
Check of overall buckling		$\frac{\ell_e}{r_x} < 0.7\frac{H}{r_y}$	$\frac{\ell_e}{r_x} < 0.7\frac{H}{r_y}$
			
Note	r_x, r_y : radius of gyration about x-, y-axis, ℓ_e : effective buckling length		

stability checks. In the direction of transverse axis, 1972 Codes utilizes a linear interaction equation concerning the beam and the column under uniaxial moment and the compressive force.

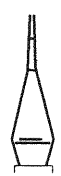
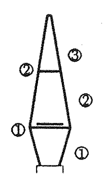
In 1972 Codes, the buckling length, an important factor in evaluating the allowable axial compressive stress, is calculated by an alignment charts or table-values as shown in **Table 3**. 1980 Codes, on the other hand, uses the effective tangential modulus method[2] (E_f method) taking inelasticity of column into account. Furthermore, the length of member is adopted as the buckling length of the Euler buckling stress σ_{eax} in the (1-$\sigma_{eax}/\sigma_{eax}$) term of stability equation, while taking the effect of the bending moment amplification into consideration.

1972 Codes use a constant value of 0.85 as the reduction factor to determine the equivalent uniform moment in order to carry out structural analysis based on the linear theory and especially applicable for portal type towers. 1980 Codes, however, utilize the Austin formula for both portal and truss type towers.

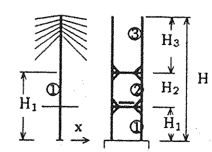
In the beam design, the section forces of members are calculated through finite displacement analysis. The JSHB verification method is also employed since these members are subjected to axial force as well as uniaxial bending moment.

Table 5 Comparison of Design Factors of Tower for Cable-stayed Bridges

Bridge	Tatara (U.C.)	Nagoya-port (U.C.)	Tsurumi-Tsubasa (1995)	iguchi (1994)	Higashi-Kobe (1993)	Yokohama-Bei (1989)
Tower, cable type and span length	Opposite-Y, 220m, 2-plane, Fan, 890m	A, 190m, 2-plane, Fan, 590m A, 125m, 2-plane, Fan, 410m	A, 182m, 2-plane, Fan, 510m	A, 123m, 2-plane, Fan, 490m	H, 147m, 2-plane, Harp, 485m	H, 172m, 2-plane, Fan, 460m
Structural analysis	nonlinear theory, plane model	linear theory, 3-dim. model	nonlinear theory, plane model	nonlinear theory, plane model	nonlinear theory, plane model	linear theory, bridge system, 3-dim. model
Stress check	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_{cal}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_{cal}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_{cal}$	$\sigma_c + \sigma_{bx} + \sigma_{by} \leq \sigma_{cal}$	$\gamma_n \sigma_c + \gamma_x \sigma_{bx} + \gamma_y \sigma_{by} \leq \sigma_{cal}$	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eax}}$ $\leq \sigma_{cal}$
Stability check	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \leq 1$	$\frac{\sigma_c}{\sigma_{cax}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \leq 1$ $\frac{\sigma_c}{\sigma_{cay}} + \frac{c_y \sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \leq 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \leq 1$	$\frac{\sigma_c}{\sigma_{cax}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \leq 1$ $\frac{\sigma_c}{\sigma_{cay}} + \frac{c_y \sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \leq 1$	$\frac{\gamma_n \sigma_c}{\sigma_{ca}} + \frac{\gamma_x \sigma_{bx}}{\sigma_{ba0}} + \frac{\gamma_y \sigma_{by}}{\sigma_{bagy}} \leq 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})}$ $+ \frac{c_y \sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \leq 1$
Buckling length	Out-of-plane buckling : E_f method In-plane buckling : E_f method	Out-of-plane buckling : Elastic-eigenvalue analysis, ① 115.7m ② 126.0m In-plane buckling : Elastic-eigen value analysis, ① 93.0m ② 89.0m ③ 86.0m	Out-of-plane buckling : E_f method In-plane buckling : E_f method	Out-of-plane buckling : E_f method In-plane buckling : E_f method σ_{eax} : panel length	Out-of-plane buckling : Elastic-eigen value analysis, ① $2H_1^*$ In-plane buckling : E_f method ① $1.49H_1$ ② $1.13H_2$ ③ $1.67H_3$	Out-of-plane buckling : ① $2.1H_1$ ② Elastic-eigenvalue analysis, ③ 0 In-plane buckling : ① $1.2H_1$ ② $1.2H_2$ ③ $2.1H_3$
Check of ultimate strength	Plastic zone analysis Bridge system, 3-dim. model	Plastic zone analysis Bridge system, 3-dim. model	Plastic zone analysis Bridge system, 3-dim. model	Plastic zone analysis Bridge system, 3-dim. model $\sigma < \sigma_y$	Plastic zone analysis Bridge system, 3-dim. model $\alpha < \alpha_{req}$	Elastic finite displacement analysis Bridge system, plane model
Note	σ_c : axial stress, σ_{bx}, σ_{by} : bending stress about x-, y-axis, σ_{cal} : allowable local buckling stress, σ_{ca} : allowable compressive stress, c_x : reduction factor about x-axis, σ_{ba0} : allowable bending stress, σ_{eax} : elastic buckling stress about x-axis	The accuracy of effective length(in-plane buckling) is confirmed by E_f method and mode-estimation method c_y : reduction factor about y-axis, σ_{bagy} : allowable bending stress about y-axis, σ_{eay} : elastic buckling stress about y-axis		σ_y : yield stress	Plastic zone analysis is considered residual stress, local buckling, and initial deflection; H/1500. H_1^*=distance from tower-base to inflection point of deformation mode $\gamma = \frac{\text{force obtained by plastic zone analysis}}{\text{force obtained by linear analysis}}$	Effective length is determined by JSHB, considering elastic eigen-value analysis.





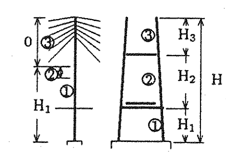


Table 6 Comparison of Design Factors of Tower for Cable-stayed Bridges

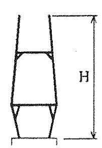
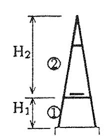
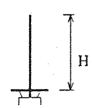
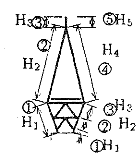
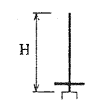
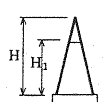
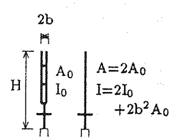
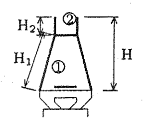
Bridge	Iwaguro, Hitsuishi-jima (1988)	Nagoya-port-Nishi (1985)	Yamatogawa (1982)	Tenpozan (1989)	Suehiro (1975)	Kamome (1975)
Tower, cable type and span length	H, 152m, 2-plane, Fan, 420m H, 143m, 2-plane, Fan, 420m	A, 122m, 2-plane, Fan, 405m	Single, 70m, 1-plane, Harp, 355m	A, 161m, 2-plane, Fan, 350m	Single, 48m, 1-plane, Fan, 250m	Single, 48m, 1-plane, Fan, 240m
Structural analysis	nonlinear theory, plane model	linear theory, bridge system, 3-dim. model	linear theory, plane model	linear theory, bridge system, 3-dim. model	linear theory, plane model	linear theory, plane model
Stress check	$\sigma_c + \sigma_{bx} + \sigma_{by} \le \sigma_{cal}$	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eax}} \le \sigma_{cal}$	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eax}} \le \sigma_{cal}$	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eax}} \le \sigma_{cal}$		$\sigma_c \le \sigma_{ca},\ \sigma_b \le \sigma_{ba}$
Stability check	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} \le 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y \sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \le 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y \sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \le 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x \sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y \sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \le 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{\sigma_{by}}{\sigma_{ba}} \le 1$	$\frac{\sigma_c}{\sigma_{cax}} + \frac{\sigma_{bx} + \sigma_{by}}{\sigma_{ba}} \le 1$
Buckling length	Out-of-plane buckling : E_f method In-plane buckling : E_f method σ_{eax} : panel length	Out-of-plane buckling : ① 2.0H1, ② 1.0H₂ In-plane buckling : Determinig from experimental results of ultimate strength	Out-of-plane buckling : 1.0H In-plane buckling : 0.7H	Out-of-plane buckling : ① $1.08H_1$ ② $1.05H_2$ ③ $2.00H_3$ In-plane buckling : ① $1.00H_1$ ② $0.80H_2$ ③ $0.85H_3$ ④ $0.70H_4$ ⑤ $2.00H_5$	Out-of-plane buckling : 2.0H In-plane buckling : 2.0H	Out-of-plane buckling : 1.0H In-plane buckling : 2.0H
Check of ultimate strength			Plastic zone analysis Bridge system, 3-dim. model $\alpha < \alpha_{req}$	Plastic zone analysis Bridge system, 3-dim. model $\alpha < \alpha_{req}$		
Note	The accuracy of structural analysis is confirmed by 3-dimensional finite displacement analysis.	The accuracy of structural analysis is confirmed by 3-dimensional finite displacement analysis. Effective length is confirmed by eigenvalue analysis.	Effective length is determined by JSHB, considering trapezoid distributed axial force.	Effective length is determined by elastic eigenvalue analysis, subjected load ; D+α L (out-of-plane) α(D+L+PS) (in-plane) Plastic zone analysis is considered as follows ; perfect elasto-plastic, Von Mieses, Plandtl-reuss, pure torsion, residual stress ; SM400 : $\sigma_{rt} = 0.9\sigma_y, \sigma_{rc} = 0.4\sigma_y$ SM580 : $\sigma_{rt} = 0.9\sigma_y, \sigma_{rc} = 0.2\sigma_y$ initial imperfection ; 10cm at tower top (out-of-plane) H/1000, sine wave (in-plane)	σ_{ba} : allowable bending stress	
						

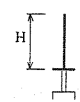
Table 7 Comparison of Design Factors of Tower for Cable-stayed Bridges

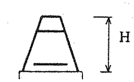
Bridge	Yasaka (1987)	sugawara (1989)	Rokko (1976)	Katsushika-Harp (1987)	Toyosato (1970)	Onomichi (1968)
Tower,cable type and span length	A, 47m, 2-plane, Fan, 240m	Single, 46m, 1-plane, Fan, 238m	H, 57m, 2-plane, Fan, 220m	Single, 68m, 1-plane, Fan, 220m	Portal, 35m, 1-plane, Fan, 216m	Portal, 39m, 2-plane, Conv., 215m
Structural analysis	linear theory, plane model	linear theory, plane model	linear theory, plane model	linear theory, bridge system, 3-dim. model	linear theory, plane model	linear theory, plane model
Stress check	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eay}} \leq \sigma_{cal}$	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eay}} \leq \sigma_{cal}$	$\sigma_{c1} + \sigma_{c2} < \sigma_{ca}$ (tower top)	$\sigma_c + \frac{\sigma_{bx}}{1-\sigma_c/\sigma_{eax}} + \frac{\sigma_{by}}{1-\sigma_c/\sigma_{eay}} \leq \sigma_{cal}$	$\sigma_c + \sigma_{bx} \leq \sigma_{ax}$ $\sigma_c + \sigma_{by} \leq \sigma_{ay}$	$\sigma_c + \sigma_{by} \leq \sigma_{ay}$
Stability check	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x\sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y\sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \leq 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x\sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y\sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \leq 1$	$\frac{\sigma_c}{\sigma_{cax}} + \frac{\sigma_{bx}+\sigma_{by}}{\sigma_{ba}} \leq 1$	$\frac{\sigma_c}{\sigma_{ca}} + \frac{c_x\sigma_{bx}}{\sigma_{ba0}(1-\sigma_c/\sigma_{eax})} + \frac{c_y\sigma_{by}}{\sigma_{bagy}(1-\sigma_c/\sigma_{eay})} \leq 1$		
Buckling length	Out-of-plane buckling : 1.0H In-plane buckling : $1.0H_1$	Out-of-plane buckling : 1.0H In-plane buckling : Considering of overall buckling and buckling of member .	Out-of-plane buckling : 1.0H In-plane buckling : ① $1.5H_1$ ② $2.2H_2$	Out-of-plane buckling : 1.0H In-plane buckling : 1.0H	Out-of-plane buckling : 1.0H In-plane buckling : 1.0H	Out-of-plane buckling : 1.0H In-plane buckling : 1.0H
Check of ultimate strength				Bridge system, 3-dim. model $\sigma < 1.7\sigma_{cal}$		
Note	H_1=distance from tower-base to the center of gravity of cables	Effective length of member is determined by JSHB. Effective length of frame is calculated by elastic eigenvalue analysis for single model and portal model, $\ell_e = \frac{\sqrt{\alpha}}{\sqrt{\alpha'}}\ell'_e$ α=eigenvalue of portal model α', ℓ'_e=eigenvalue and effective length of single model.	The accuracy of structural analysis is confirmed by 2-order theory. Effective length is confirmed by elastic eigenvalue analysis. σ_{c1} : initial axial stress, σ_{c2} : axial stress of panel point	Effective length of in-plane buckling is confirmed by elastic eigenvalue analysis.	σ_{ax}, σ_{ay} : allowable stress about x-, y-axis	

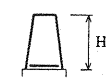












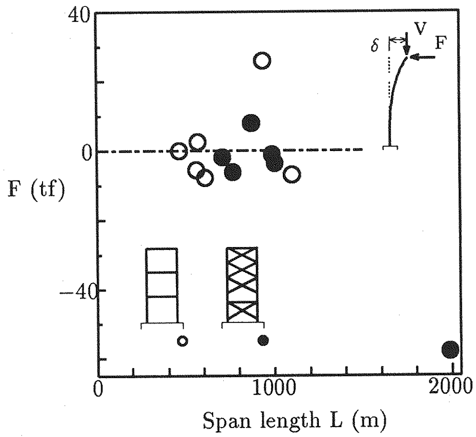


Fig. 2 Relationship between horizontal force and main span length

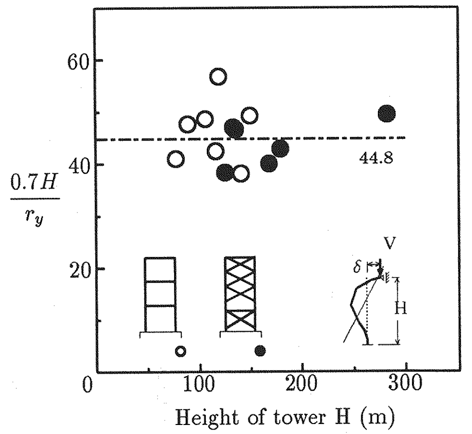


Fig. 3 Effective buckling length of bridge's axial direction

b) Cable-Stayed Bridges

In many cases in the past, designs of towers of many cable-stayed bridges were carried out utilizing the JSHB method. This method was adopted because the effect of the nonlinearity of towers was not very great, the method of verification was relatively simple, and also conceptual matching was possible between the design of towers and that of other structures.

Since the section force of the Honshu-Shikoku Bridge can be calculated through finite displacement analysis, a verification equation in the HSBA Codes is utilized for the main towers of suspension bridges[2]. In this equation, the term of the bending moment amplification is omitted from the stress-check equation. Linear equations are utilized involving the uniaxial moment exclusively in the transverse direction to determine the ultimate strength interaction. Stability of the biaxial bending moment is checked by adding a linear equation involving the uniaxial bending moment in the longitudinal direction.

Furthermore, the stress and stability of the HEPC bridges (under construction) are checked utilizing the linear equation for biaxial bending while taking the correction factor described in (1) into consideration and eliminating the effect of the bending moment amplification .

(3) Effective Buckling Length

a) Suspension Bridges

As described in (2), the effective buckling length of the tower in the bridge axis (out-of-plane buckling) takes a value between 2.0H and 0.7H depending on how the critical condition with respect to the horizontal reactive force of cable is defined. When the buckling length 0.7H at $F = -\infty$ (hinged at the tower top and fixed at the base) is considered to be the apparent buckling length, the relationship between the slenderness ratio of $0.7H/r_y$ and the height of the tower is shown in **Fig.3**. Here, r_y is the radius of gyration of the average cross section for the total height of the tower.

Generally, the slenderness ratio of columns is distributed in the range between 38.2 and 56.8, and the average slenderness ratio is $0.7H/r_y = 44.8$. No slenderness ratio difference is observed between truss and portal types.

The method for determining the buckling length in the transverse direction (in-plane buckling) differs depending upon whether the design method is based on the small displacement theory or the finite displacement theory. In the case of the small displacement method, the buckling length is obtained by the approximation method[1),2)] which takes into account the beam rigidity with respect to the columns of interest, and generally the longest value is selected. The buckling length of the truss type is often governed by the opening length of the frame. Conversely, when the structural analysis is based on the finite displacement theory, the buckling length for the entire frame system is obtained by the E_f method, which takes account of the development of a non-elastic region in the column from 1980 Codes as shown in **Table 3**.

Now, the buckling length ℓ_e was calculated by the above-mentioned approximation method, the E_f method and the elastic eigenvalue method[19),20)]. The relationships between the

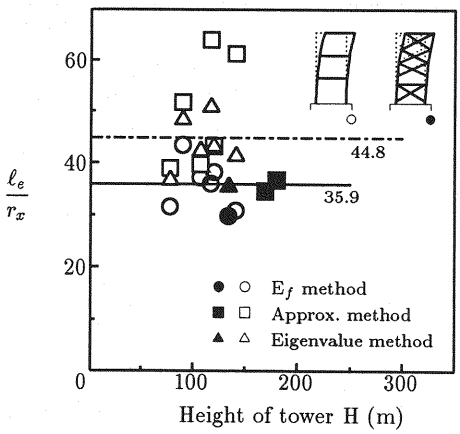


Fig. 4 Effective buckling length of transverse direction

slenderness ratio ℓ_e/r_x and the height of the tower is shown in **Fig.4**.

The buckling length obtained by the E_f method represents the smallest value, and the average slenderness ratio ℓ_e/r_x is 35.9. The main reason why the buckling length becomes shortest in this method is the increase in the apparent rotational restriction of the joint-edge of the column in considering the elastic rigidity of the beam together with the inelasticity of the column's rigidity.

b) Cable-Stayed Bridges

The buckling length of a single type tower bridges is determined as a column model widely ranging from 2.0H for in-plane buckling where the base is fixed and the top is free to 0.7H where the base is fixed but the top is hinged. A base-and-top hinged column model with 1.0H is often adopted for out-of-plane buckling while taking the cable pull-back effect into consideration. With the portal type, 1.0H is used for both in-plane and out-of-plane buckling.

In the case of A-shape and H-shape towers, one of the following methods of determining the buckling length can be used as shown in **Tables 5, 6** and **7**.

1) Method of using the values in the table by JSHB Codes as the buckling length of in-plane and out-of-plane buckling.
2) Method of determining the buckling length for both in-plane and out-of-plane buckling by the E_f method.
3) Method of adopting the value in the table prepared by JSHB Codes for out-of-plane buckling, and determining the buckling length for in-plane buckling with reference to experimental results on the ultimate strength.
4) Method of determining out-of-plane buckling by elastic eigenvalue analysis and determining in-plane buckling by the E_f method.

Because the distribution of the axial compressive force is complicated due to the actual cable configuration, a method to take the normal stress distribution into account is adopted in the design by HEPC. Also, there are many bridges for which eigenvalue analysis of the entire structural system is performed and the accuracy is checked against the values tabulated by JSHB Codes.

(4) Verification of Ultimate Strength

a) Suspension Bridges

In terms of checking of buckling and ultimate strength for overall systems of structures, 1972 Codes required sufficient safety with respect to the critical load. 1980 Codes, however, required through the condition of satisfying ℓ_e/r_x <0.7H/r_y, that the cross section is designed such that the overall buckling load in the transverse direction exceeded the out-of-plane buckling load.

b) Cable-Stayed Bridges

The structural conformation of the towers of cable-stayed bridges are not simple, and the conditions of stress are complex. Therefore, as discussed in 3., in recent bridges, the design method 2 tends to be adopted rather than method 1. 1980 Codes introduced a check for load-carrying capacity based on nonlinear analysis of a 3-dimensional model of an entire structure. Through detailed examinations, however, the authors pointed out the fact that verification methods differ from bridge to bridge. Specifically, the following check conditions are adopted.

(a) The critical stress, σ, as determined by elastic finite displacement analysis, satisfies the allowable stress, $1.7\sigma_{cal}$, which takes account of the local buckling of the component plate.
(b) The ultimate strength σ, as determined by elasto-plastic finite displacement analysis, satisfies the yield stress σ_y .
(c) The load coefficient α at the ultimate state, as determined by elasto-plastic finite displacement analysis, satisfies the required load factor α_{req}.

The verification (a) is carried out by elastic finite displacement analysis under a load multiplied by the load coefficient α. The verifications (b) and (c) are carried out by elasto-plastic finite displacement analysis taking into account the column's initial imperfection under a factored design

Table 8 Load combination and load factor

Bridges	Load combination and load factor	Factor required
Iguchi	1.30(D+PS+T+SD)+α(i+1)L	α_{req} =2.20
Higashi-kobe	1.10(D+PS)+1.00(SD+E+T)+α W	α_{req} =1.54
	1.10(D+PS)+1.00(SD+E+T+L)+α EQ	α_{req} =1.54
Yamatogawa	1.70(D+PS)+α(i+L)	α_{req} =1.70
	1.00(D+PS+T)+α((i+L)/2+EQ)	α_{req} =1.26
	1.26(D+PS)+α W	α_{req} =1.26
Tenpozan	1.30(D+PS)+α(i+L)	α_{req} =2.17
	1.30(D+PS+T)+α(i+L)	α_{req} =1.30
	1.25(D+PS+T)+α W	α_{req} =1.25
	1.30(D+PS+T)+α(EQ+L)	α_{req} =1.30
Katushika-harp	1.70(D+PS+E)+α*(i+L)	σ_{cr} =1.7σ_{cal}
	1.36(D+PS+E)+α*(i+L+W)	

D : Dead load, L : Live load, T : Thermal force change, W : Wind load,
i : Impact, E : Erection error, PS : Prestress force, EQ : Earthquake load,
SD : Displacement of supports, α : Load factor, α* =1.7/incremental coefficent of σ_{ca}

load. Existing combination of loads and load factors applied varies from bridge to bridge. Example combinations of loads and factors are shown in **Table 8**.

5. ISSUES OF CURRENT DESIGN METHODS

Characteristics of the buckling design for towers have been described and discussed. However, the authors pointed out that there still exist several unsolved issues in the design method. Those issues are summarized as follows.

1) Uncertainty exists in making a decision to select an appropriate set of analytical models for structural analysis. In current analytical models, various combinations, either of plane and space structures or of partial and the entire structures, are applied depending on the bridge type.
2) Nonlinear analysis is being introduced in determining section force as well as in checking the ultimate strength. The level of nonlinearity, however, has not been defined on the basis of finite displacement theory in the existing codes. Consequently, the reliabilities of nonlinearity of strain, stress-strain relationship, equilibrium equations and strain-displacement relationship, etc, are unclear. Some checks of precision of solutions are required including checks of calculation errors, especially in inelastic finite displacement analysis. This requirement does not deny the necessity of developing a general-purpose software.
3) The stability formulas used by JSHB Codes are empirical linear equations based on small displacement analysis. Accordingly, from the standpoint of obtaining section forces based on nonlinear analyses, the current linear verification formulas or their improved versions still have questionable validity.
4) The buckling length varies depending on loading and boundary conditions. Consequently, it is fairly difficult to conclusively determine the buckling length of complex towers with tapered columns. The elastic eigenvalue analyses and E_f method recently adopted have some contradictions in that the buckling length increases greatly in the member in the region of small axial forces. Although procedures of rectifying this difficulty are being studied[4),5)], in reality, the buckling length is determined by considering the conditions of buckling modes and cable forces acting on the tower, and by comparing the solutions obtained by various eigenvalue calculations. In the circumstances, the evaluation is left up to the judgment of designers.
5) There are some technical difficulties in checks for the ultimate strength of the entire structure as to whether or not the following effects are taken into consideration;
   - the effects of basic factors [material nonlinearity (tangent modulus, residual stress, postbuckling behavior and strength as well as load hysteresis), restrictions on edges, nature of loads, characteristics of joints, etc.],
   - the effects of biaxial bending moment,
   - and effects of local buckling.

In Japan, nonlinear analyses considering these factors are not used widely enough.

6) At present, there are no common codes for the combination of loads and the load factors. In many cases, these vary depending upon the bridge.

6. CONCLUSIONS

The details and history of buckling design for towers were investigated for some of the major Japanese long-span cable-supported bridges. Useful design information was obtained through design policies and verification procedures.

Mechanical characteristics of cable-supported bridges must be considered adequately in the design of towers. It is time for us to reconsider the traditional methods for evaluating the ultimate strength by buckling length. Bearing in mind the availability of powerful computers today, new design methods to go beyond conventional design frameworks and giving more freedom in verifying safety should be discussed from both theoretical and practical angles.

Recent proposals of new frame structural design methods reflect the above awareness as they do not utilize the buckling length and yet give consideration to the effect of deformation[21),22),23),24)]. Much can be expected of future research in this area. The authors hope that the results in this report will be useful in establishing a unified design method for frame structures including the towers in cable-supported bridges.

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