Structural Eng./Earthquake Eng., JSCE, Vol. 13, No. 2, 163s—173s, 1996 October

ON LIMIT STATE DESIGN METHOD CONSIDERING SHEAR LAG PHENOMENON OF CORNER PARTS OF STEEL RIGID FRAMES

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This paper proposes a design method to estimate the shear lag phenomenon concerning L- and T-shaped corner parts of steel rigid frames with bi-symmetrical box cross section. Firstly, the fundamental beam model is explained to apply the beam theory to the beam and column members with corner part. Secondly, an approximate method is derived to calculate the additional normal stress due to shear lag through the stress analysis by using this beam model. Finally, a conventional method for ultimate strength of corner parts is proposed by the application of the ultimate strength of box cross section subjected to bending moment and shearing force.

Key Words: *corner parts of frame, limit state design, shear lag, ultimate strength*

1. INTRODUCTION

In the corner parts of steel rigid frames[1)], the axial force, bending moment and shearing force are transferred between beam and column members with the change of their direction. It is well known that the remarkable shear lag phenomenon occurs in the flange plates at the vicinity of corner parts and then the shearing stress becomes predominant in the panel zones enclosed by the flange and diaphragm plates.

When designing the corner parts of steel piers, the former phenomenon has been previously estimated by the numerical and experimental studies by Okumura and Ishizawa,[2),3)] while the latter was investigated by Beedle et al[4)]. Such a practical design method has already been adopted to apply to the design criterion of some Japanese Highway Administrations[5),6)]. However, the Japanese Specification for Highway Bridges[7)] has not yet developed the design method of corner parts.

The structural design method has recently been updated from the allowable stress design method to the limit state design one. Therefore, it is important to reexamine the shear lag phenomenon and ultimate strength of corner parts[8)-11)] corresponding to the improvement of structural detail of steel piers.

This paper presents a limit state design method of L- and T- type of corner parts with the box cross sections through the results of numerical and experimental research[12),13)]. Firstly, a beam model is predicated so that the physical properties of corner parts with the significant shearing force could easily be understood by applying the elementary beam theory. In this model, the normal forces applied to flange plates correlate well with the shear lag phenomenon. Secondly, an analytical method for shear lag of the beam model is developed and a conventional formula is proposed for evaluating the bending moment corresponding to additional normal stress due to shear lag in the vicinity of corner parts. This proposition is compared with the test results[12)] as well as the present design method[2),3),5),6)], which leads to a stress evaluating method for the serviceability limit state. Furthermore, an ultimate state limit of corner parts is proposed by investigating the influence of shear lag upon the ultimate strength of corner parts. Finally, a method for checking the buckling strength of stiffened flange plates in the vicinity of corner parts is discussed for further study.

This paper is translated into English from the Japanese paper, which originally appeared on J. Struct. Mech. Earthquake Eng., JSCE, No.455/ I -21, pp.95-104, 1992.10.

2. CORNER PARTS OF FRAMES AND STRUCTURAL ANALYTICAL MODELS

(1) Corner parts of frames in this study

This paper studies L- and T-type corner parts in which beam and column members are connected to each other at right angles. Noting that a T-type corner part can be separated into two L-type corner parts by the superposition of induced stress-resultants as shown in **Fig.1**, this paper is devoted to the analysis of the L-type corner part as shown in **Fig.2**.

The following assumptions are utilized;

1) Frames are treated as plane framed structures.
2) Stress concentration is negligible at the vicinity of corner parts.
3) As the longitudinal stiffeners are discontinuous at the vicinity of corner parts[9], the beam and column members are unstiffened and doubly symmetrical cross sections (See a-b and a-d in **Fig.2(c)**).

(2) Definition of corner parts in frames

This study defines the corner parts and another parts of beam and column members as follows;

1) Corner part is the region enclosed by the points a to d in **Fig.2(c)** and then the hatched area is termed panel zone.
2) Beam and column members are taken to be those at the vicinity of corner parts which have the cross sections a-d and a-b in **Fig.2(c)**, respectively.
3) The locations of cross sections a-b and a-d in **Fig.2(c)** correspond to the junctions of beam and column members.
4) Diaphragm plates indicate the plate elements a-d and a-b as shown by the broken lines in **Fig.2(c)**.

The notation of cross-sectional dimensions, properties and parameters is listed in **APPENDIX**.

(3)Analytical models of beam and column members with corner parts

It is assumed in the present design method[5),6)] of corner parts that the stress-resultants at the junction, a-b or a-d, of beam and column members can be represented by the flange normal forces F, thereafter referred to as flange forces, as shown in **Fig.3**.

Taking into consideration this mechanism as well as the close correlation with the test results[12)] mentioned later, the distribution of stress-resultants are characterized by means of the conventional beam theory.

Firstly, the beam of points 3-1-5 and column of points 4-2-6 including the corner part are rearranged by a beam model to which the elementary beam theory is applicable as shown in **Fig.4(a)**. This beam is assumed to be simply supported at the junctions j and j+2 of each flange plate of beam and column.

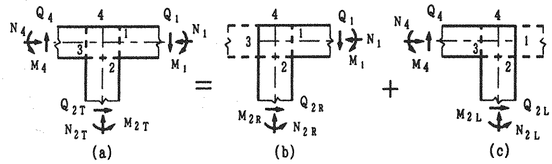


Fig.1 L- and T- type connections

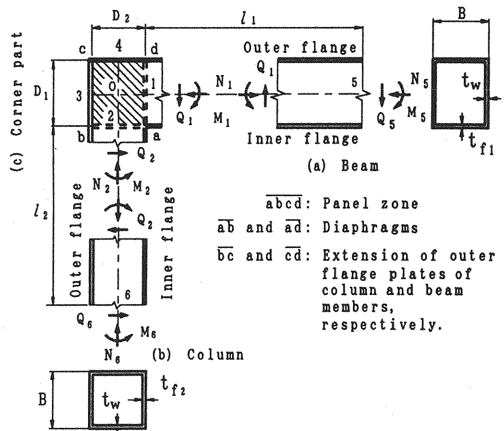


Fig.2 Stress-resultants applied to beam and column members at the vicinity of corner parts

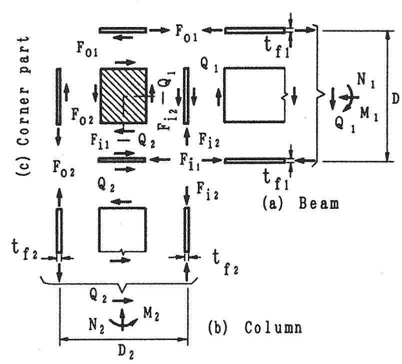


Fig.3 Induced flange normal forces of beam and column members at vicinity of corner parts as well as shearing forces at corner part

Secondly, the reactions F_{ok} and F_{ik} at the supports can be expressed in terms of stress-resultants N_k and M_k applied to the cross section at the junctions of beam and column as follows;

$$F_{ok} = \frac{M_k}{D_k} - \frac{N_k}{2}, \quad F_{ik} = \frac{M_k}{D_k} + \frac{N_k}{2} \qquad (1)_{a,b}$$

Noting that the normal stress equals zero at the end support j+2, the shearing force Q_k can be divided into uniformly distributed axial load q_{nk} and distributed moment q_{mk}.

$$q_{nk} = \frac{Q_k}{D_k}, \quad q_{mk} = -\frac{D_j}{2}\frac{Q_k}{D_k} \qquad (2)_{a,b}$$

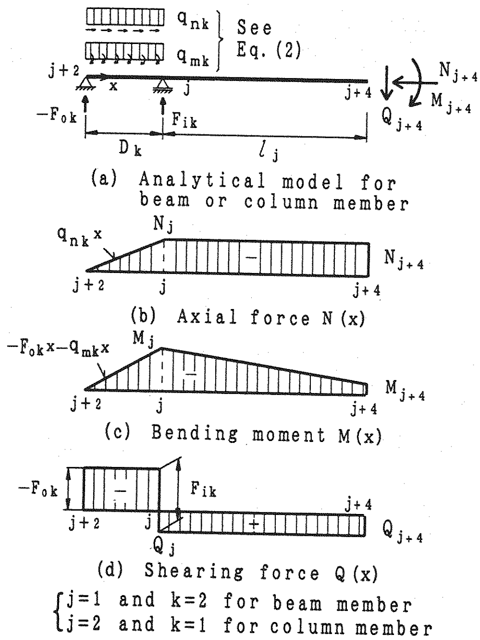


Fig.4 Analytical model and induced stress-resultants for beam and column members

In the above equations, subscripts j and k shall be set as j=1 and k=2 for beam members, being j=2 and k=1 for column members.

Figs.4(b), (c) and (d) show the diagrams of axial force, bending moment and shearing force of the beam model, respectively. This figure demonstrates that the maximum bending moment for checking the normal stress must be M_1 and M_2. Attention can also be drawn to a basic fact that shear lag phenomenon results from the remarkable change of shearing force at the inner flange plates.

3. SHEAR LAG ANALYSIS OF CORNER PARTS OF FRAMES

(1) Fundamental differential equations

The shear lag phenomenon of the beam model in **Fig.4**(a) is analyzed by using Reissner's hypothesis[13)] that the axial displacement $f(x)$ in a flange plate can be approximated by a 2nd order parabolic curve .

According to the displacements and stress-resultants as defined in **Fig.5**, the differential equations with respect to uniaxial displacement $u(x)$ and slope $\varphi(x)$ of the beam model and axial displacement $f(x)$ of flange plate due to shear lag are given as follows[13)]

$$EA_s\frac{du(x)}{dx} = N(x)$$

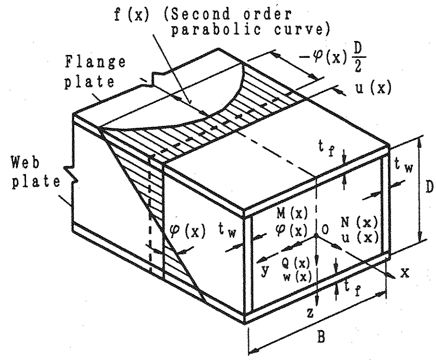


Fig.5 Definition of stress-resultants as well as displacements for analyzing shear lag of beam and column members with bi-symmetrical box cross section

$$EI_{ys}\frac{d\varphi(x)}{dx} - \frac{2}{3}EA_{fs}\frac{df(x)}{dx} = M(x)$$

$$-\frac{1}{2}A_{fs}D\frac{d^2\varphi(x)}{dx^2} + \frac{4}{5}A_{fs}\frac{d^2f(x)}{dx^2} - 8\frac{Gt_f}{EB}f(x) = 0 \qquad (3)_{a\text{-}c}$$

Since $u(x)$ can be solved independent of $f(x)$, the differential equation with respect to $f(x)$ results from Eqs.(3)b,c as follows;

$$\frac{d^2f(x)}{dx^2} - \alpha^2 f(x) = \frac{\beta D}{8EI_{ys}}Q(x) \qquad (4)$$

where A_{fs} and I_{fs} are the cross-sectional area and geometrical moment of inertia, respectively, and derived in accordance with the theory of elasticity[13)]

$$\left.\begin{aligned} A_s &= 2(A_{fs} + A_{ws}) \\ I_{ys} &= \frac{1}{2}B\bar{t}_f D^2 + \frac{1}{6}D^3\bar{t}_w \end{aligned}\right\} \qquad (5)_{a,b}$$

in which

$$\left.\begin{aligned} A_{fs} &= B\bar{t}_f, \quad A_{ws} = D\bar{t}_w \\ \bar{t}_f &= \frac{t_f}{(1-\mu^2)}, \quad \bar{t}_w = \frac{t_w}{(1-\mu^2)} \end{aligned}\right\} \qquad (6)_{a\text{-}d}$$

Furthermore, α, β and γ are the shear lag parameters given by;

$$\alpha = \frac{1}{B}\sqrt{\frac{\beta t_f}{(1+\mu)\bar{t}_f}} = \sqrt{\frac{10(1-\mu)}{B}}\sqrt{\frac{3+s}{1+2s}}$$

$$\beta = \frac{5}{1 - \dfrac{5}{12}\dfrac{B\bar{t}_f D^2}{I_{ys}}} = \frac{10(3+s)}{1+2s}$$

$$\gamma = \frac{1}{12}\frac{B\bar{t}_f D^2}{I_{ys}} = \frac{1}{2(3+s)} \qquad (7)_{a\text{-}c}$$

As is illustrated in **Fig.6**, the bending moment $m_s(x)$ corresponding to additional normal stress due to shear lag can be deduced from the solution of displacement $f(x)$ as follows;

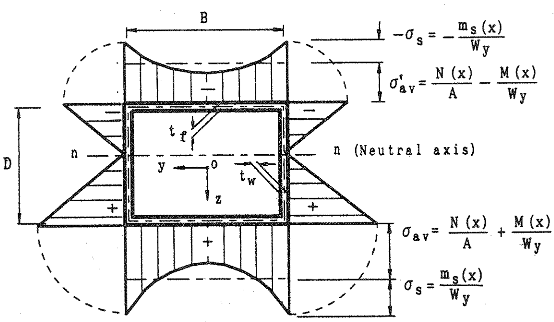


Fig.6 Normal stress distribution of box beam and column members including shear lag effect

$$m_s(x) = EI_{ys}\frac{8\gamma}{D}\frac{df(x)}{dx} \tag{8}$$

(2) General solution of additional bending moment $m_s(x)$ due to shear lag

The additional bending moment $m_s(x)$ is solved for the beam model. When $m_s(0)=0$ and $m_s(D+l)=0$ are set as the boundary conditions of $m_s(x)$, the solution of $m_s(x)$ can be obtained as follows;

i) For $0 \leqq x \leqq D$:

$$m_s(x) = m_{sp}\frac{2\sinh\alpha l}{\sinh\alpha(l+D)}\sinh\alpha x \tag{9}_a$$

ii) For $D<x \leqq D+l$:

$$m_s(x) = m_{sp}\frac{2\sinh\alpha D}{\sinh\alpha(l+D)}\sinh\alpha(D+l-x) \tag{9}_b$$

where m_{sp} is the additional bending moment at the support j of beam model and can be simplified as follows;

$$m_{sp} = \frac{F_i'\beta\gamma}{2\alpha} = \frac{1}{4}\sqrt{\frac{10}{1-\mu}}\frac{F_i'}{\sqrt{2s^2+7s+3}} \tag{10}$$

In Eqs.(9) and (10), l, D, s and F_i' shall be taken as;

$$l = l_j, \quad D = D_k, \quad s = s_j, \quad F_i' = F_{ik}' \tag{11}_{a\text{-}d}$$

Using the reaction F_{ik} at the support j, F'_{ik} can be rewritten by;

$$F_{ik}' = F_{ik} - q_{mk} = \frac{M_k}{D_k} + \frac{N_k}{2} + \frac{Q_k}{2}\frac{D_j}{D_k} \tag{12}$$

It should be noted that the uniform shearing force q_{mk} due to distributed moment q_{mk} would not contribute to the shear lag of flange plates, but F_{ik} is different from F'_{ik} by q_{mk}.

(3) Proposition for calculating normal stress

The solution of additional bending moment $m(x)$ leads directly to the following formulae for estimating the normal stress in the box cross section.

i) For upper(u) and lower(l) flange plates:

$$\sigma_{u,l} = \mp\frac{M(x)}{W_y} + \frac{N(x)}{A}$$

$$\mp\left[1-\frac{3+s}{2}\left(1-\left(\frac{y}{B/2}\right)^2\right)\right]\frac{m_s(x)}{W_s} \tag{13}_a$$

Setting $y=B/2$, this equation is simplified as follows.

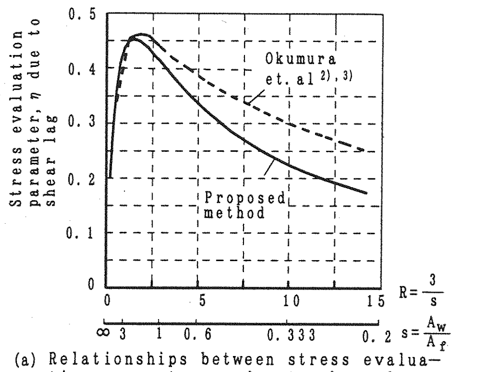
$$\sigma_{u,l} = \mp\frac{M(x)}{W_y} \mp \frac{m_s(x)}{W_y} + \frac{N(x)}{A} \tag{13}_b$$

ii) For web plates:

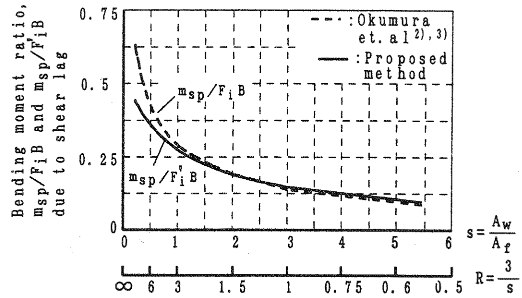
$$\sigma_w = \frac{M(x)}{I_y}z + \frac{m_s(x)}{I_y}z + \frac{N(x)}{A} \tag{14}$$

where I_y, W_y and A are the cross-sectional properties without considering Poisson's ratio μ, and are listed in **APPENDIX**.

The terms enclosed by the dotted line in Eqs.(13) and (14) correspond to the shear lag stress which can not be estimated by the elementary beam theory. **Fig.6** illustrates an example of the normal stress distribution in the box cross section.



(a) Relationships between stress evaluation parameter, η due to shear lag and parameter, R or s



(b) Relationships between bending moment, m_{sp} due to shear lag and parameter, s or R

Fig.7 Comparison of stress evaluation parameter and bending moment ratio due to shear lag of this parameter with Okumura's method[2),3)]

(4) Comparison of proposed method with present design method

Eq.(10) which gives an additional bending moment m_{sp} is compared with the present design method[3)].

Ref.3) represents the additional normal stress due to shear lag as follows;

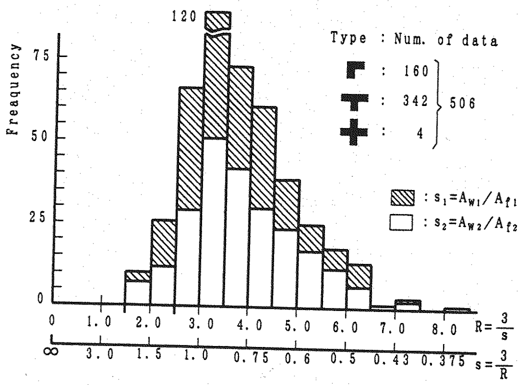


Fig.8 Histogram of shear lag parameter , R or s, of beam and column members in vicinity of corner parts[1]

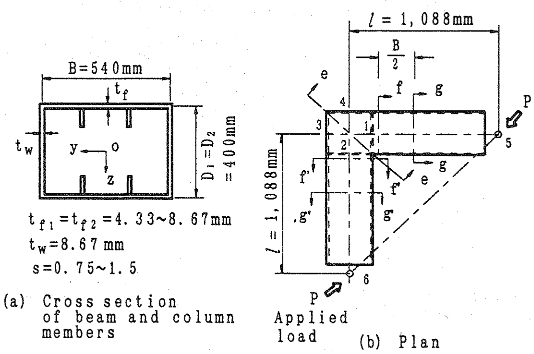


Fig.9 Test specimens in Ref.12)

$$\sigma_{sp}=\frac{B}{D}\frac{F_i}{2A_w}\eta \tag{15}$$

where F_i is the flange force in Eq.$(1)_b$ and η is specified in the form of a nomograph which is a function of the shear lag parameter R (=s/3).

Setting $F_i=F_i'$ and rearranging Eq.(10) in the same form as Eq.(15), η can be expressed by;

$$\eta=\frac{1}{2}\sqrt{\frac{10}{1-\mu}\frac{R}{(R+1)\sqrt{(R+1)(R+6)}}} \tag{16}$$

For the sake of comparison, Eq.(15) can also be represented by the additional moment m_{sp} as follows;

$$m_{sp}=\sigma_{sp}W_y=\frac{F_iB}{2}\frac{D}{D+t_f}\eta\left(\frac{1}{s}+\frac{1}{3}\right) \tag{17}$$

Fig.7 makes a comparison between Eq.(16) and η-R curve given in Ref.3) as well as the variation of m_{sp} obtained from Eqs.(10) and (17), in which m_{sp} estimated by Eqs.(10) and (17) is nondimensionalized by F_iB and $F_i'B$, respectively.

As is seen from this figure, the η - R curve obtained from Eq.(16) is somewhat larger than that of Ref.3) within the range of $R<1.5$ and smaller by 5 to 30 % within the range of $R>1.5$. The variation of m_{sp} obtained from Eq.(10) closely corresponds to that of Eq.(17) except in the range of $s<1.5$.

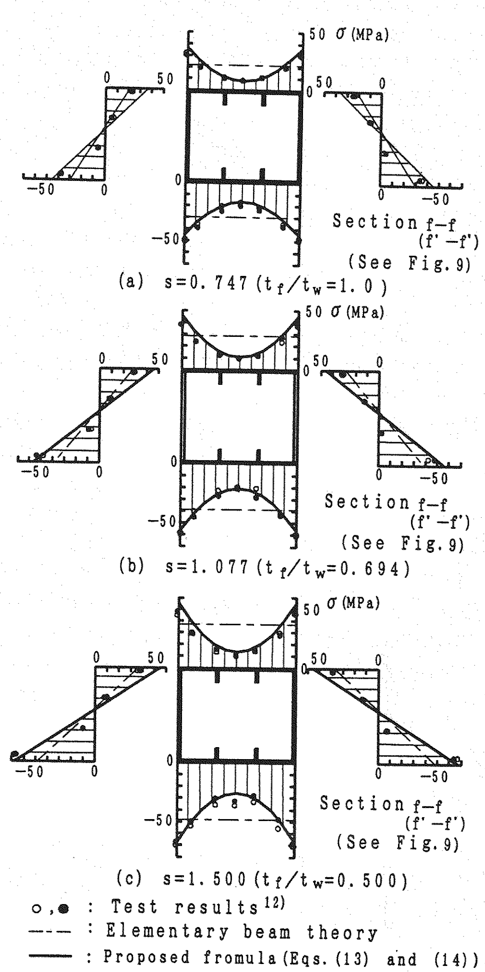


Fig.10 Comparison of normal stress distribution in box cross section by proposed method with test results[12] (P=100kN)

As noted previously, the flange forces F_i' and F_i differ in magnitude by q_{mk}. Taking into consideration that m_{sp} in practice is added to bending moment M, the differences between Eqs.(10) and (17) within the range of $s<1.5$ may not be so large. In order to elaborate on this point, the validity of the proposed method is discussed subsequently, in greater detail.

Fig.8 shows the variation of parameters R and s with reference to shear lag.

(5) Comparison of proposed method with test results[12]

Following the procedure described above, the stress evaluating method is compared with the test results[12]. **Fig.9** shows the detail of test specimens[12] with parameter s of 0.75 to 1.5.

In **Fig.10**, the normal stress distribution in the cross sections f-f and f'-f' by Eqs.(10) to (14) are plotted on the test results. The cross sections f-f and f'-f' are located at the distance of $B/2$ from the junctions of beam and column members.

As is seen from this figure, the normal stresses represented by the proposed method shows very good agreement with the corresponding test results. There is a slight difference between them only at the compressive flange plates along the edges of web plates.

Fig.11 gives the variation of normal stress in the flange plates along the panel zone and web plate with y of $B/2$ as well as at the location of a longitudinal stiffener with y of $B/6$.

The proposed method produces values which approximate quite well to the the test results even in the case of s=0.75. This fact suggests that the difference between the flange forces, F_i and F'_i, is negligible in estimating the normal stress. It is also significant that the normal stresses in the flange plates with the distance of $B/2$ from the junctions of beam and column members are almost equal to the values predicted by the elementary beam theory.

Therefore, the approximate method for calculating the shear lag stress by using Eqs.(10) to (14) will be useful to the practical design of corner parts.

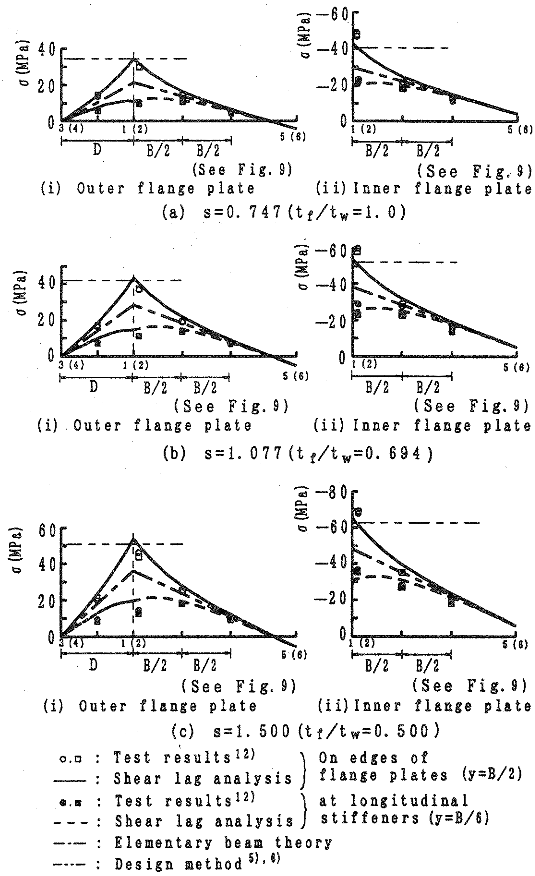


Fig.11 Comparison of normal stress distribution of flange plates by shear lag analysis with test results[12] (P=100kN)

4. SERVICEABILITY LIMIT STATE OF BEAM AND COLUMN MEMBERS AND PANEL ZONES

(1) Flange plates of beam and column members in the vicinity of corner parts

The normal stress of flange plates on the edges of web plates can be checked according to the following procedure.

At the locations where flange forces F_{ij} apply in the L-type corner parts in **Fig.2**, normal stresses can be checked by;

$$\frac{\nu\sigma_{moj}}{\sigma_y} \le 1, \quad \frac{\nu\sigma_{mij}}{\sigma_y} \le 1 \quad (j = 1,2) \qquad (18)_{a,b}$$

where ν : safety factor for serviceability limit state, σ_{moj} and σ_{mij} : maximum normal stresses of flange plates subjected to flange force F_{ij}.

Eq.$(19)_{a,b}$ are also applicable to flange and diaphragm plates which are extended in the corner parts.

(2) Web plates of beam and column members in the vicinity of corner parts

The combined effects stresses of normal and shearing stresses can be evaluated for the web plates of beam and column members as follows;

$$(\frac{\nu\sigma}{\sigma_y})^2 + (\frac{\nu\tau}{\tau_y})^2 \le 1 \qquad (19)$$

where σ: maximum normal stress applied to web plates of beam and column members (See Eq.(14)) and τ: average shearing stress in web plates of beam and column members. τ_y is the yield shearing stress given by;

$$\tau_y = \frac{\sigma_y}{\sqrt{3}} \qquad (20)$$

which is based on the yield criterion of Von Mises.

(3) Panel zones of corner parts

The serviceability limit state is evaluated for the panel zones with significant shearing force. Observing **Fig.3**(c), the average shearing stress τ_s of panel zones can be calculated by;

$$\tau_s = \frac{F_{oj}}{2D_k t_w} = \frac{F_{ik} - Q_j}{2D_j t_w} \qquad (21)$$

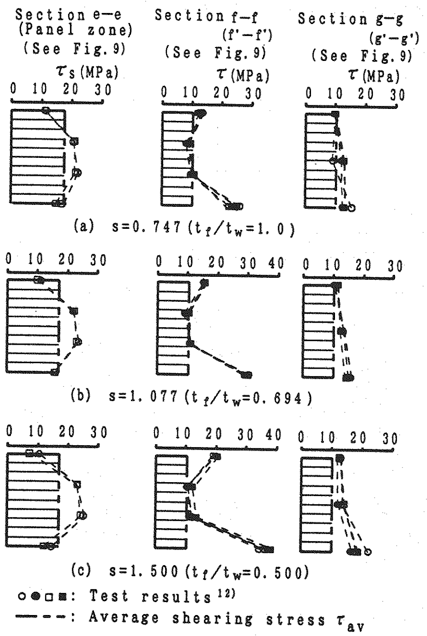


Fig.12 Comparison of average shearing stress of panel zones and web plates with test results[12] (P=100kN)

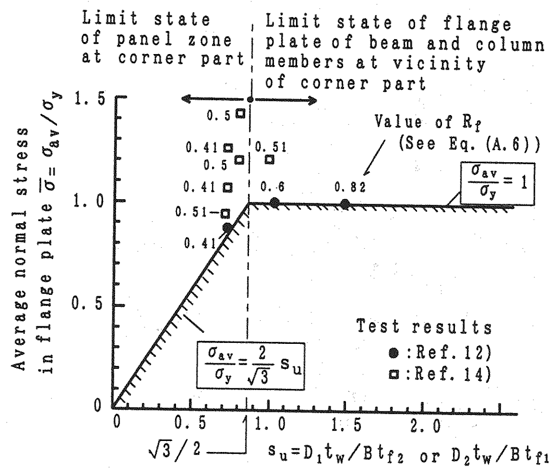


Fig.13 Upper bound curve of average normal stress of flange plates at vicinity of corner part due to shear strength of panel zone of corner part

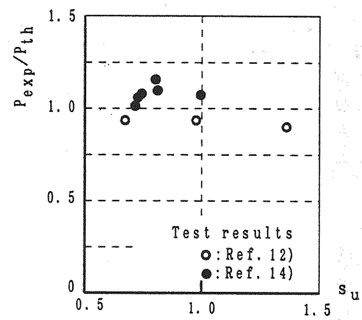


Fig.14 Comparison of theoretical ultimate strength, P_{th}, with test results, P_{exp}

In the same manner mentioned above, the serviceability limit state can be checked by;

$$\frac{\nu\tau_s}{\tau_y} \leq 1 \tag{22}$$

(4) Strengthened regions of corner parts

Test results[12] on the shearing stress in the panel zone and web plates of beam and column members are compared with the average shearing stress in **Fig.12**, which is arranged as functions of parameters, s and t_f/t_w.

This figure implies that the shearing stress in the panel zones becomes larger than the average stress with an increase in s. In addition, at the cross sections f-f and f'-f' in the beam and column members, the stress concentration appears due to the geometry of corner parts and becomes significant with an increase in s. Then, it should be considered that the test results clearly become larger than the average stress even in the cross sections g-g and g'-g' at distance of $B/2$ from the corner parts.

From the results mentioned above, the following observations are made with regard to the design of corner parts;

1) The shearing stress in the panel zones can be evaluated by Eq.(21). Accordingly, it is favorable to make the flange plate thickness t_f thick and keep s small.
2) In order to approximate the shearing stress of web plates of beam and column members by the average shearing stress, it is desirable to set the same conditions of t_f and s as indicated in Ref.1).
3) When calculating the normal stress in the cross section at the distance of $B/2$ from the corner part, the shear lag phenomenon is so small as to be negligible.
4) The fundamental concept of the present design method[5),6)], in which the corner parts must be strengthened over the range with the distance of $B/2$ from the junctions of beam and column members, is reasonable. It is, therefore, of great importance that the thickness of flange plates should be larger than that of panel zones and web plates.

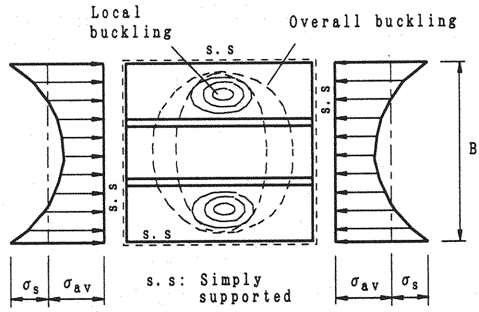


Fig.15 Local and overall buckling of compressive flange plate in beam and column members at vicinity of corner part as stiffened plate subjected to shear lag stress

5. ULTIMATE LIMIT STATE OF CORNER PARTS

(1)Ultimate strength of panel zones and flange-plates of beam and column members in the vicinity of corner parts

Before the proposition of an ultimate limit state for corner parts, the interaction between the shear lag phenomenon and the ultimate strength of corner parts is investigated.

Referring to the beam model in **Fig.4**(a) and Eq.(21), the ultimate shearing force applied to two panel zones can be obtained by;

$$Q_{yk} = 2\tau_y D_k t_w = \frac{2}{\sqrt{3}}\sigma_y D_k t_w \qquad (23)$$

When the average normal stress of flange plates including the additional normal stress due to shear lag is denoted as σ_{av}, the ultimate flange force F_{yj} can be expressed as;

$$F_{yj} = Bt_{fj}\sigma_{av} \qquad (24)$$

Noting that the upper bound of σ_{av} is yield stress σ_y, the equilibrium of Q_{yk} and F_{yj} leads to σ_{av} allotted by the flange plates as follows;

i) For the range of $0 \le s_{uj} \le \sqrt{3}/2$

$$\frac{\sigma_{av}}{\sigma_y} = \frac{2}{\sqrt{3}}\frac{D_k t_w}{Bt_{fj}} = \frac{2}{\sqrt{3}}s_{uj} \quad (s_{uj} \le \sqrt{3}/2) \qquad (25)_a$$

This range corresponds to the ultimate state of panel zones subjected to shearing force. Then, the average normal stress σ_{av} of flange plates is less than or equal to yield stress σ_y.

ii) For the range of $\sqrt{3}/2 \le s_{uj}$

$$\frac{\sigma_{av}}{\sigma_y} = 1.0 \qquad (\sqrt{3}/2 < s_{uj}) \qquad (25)_b$$

In this range, the flange plate reaches the ultimate state before yield of the panel zones.

The variation of σ_{av} with s_{uj} is shown in **Fig.13**. Test results of σ_{av}/σ_y by Refs.12) and 14) are also depicted in this figure. The test results are converted by;

$$\sigma_{av} = \frac{N}{A} \pm \frac{M}{W_y} \qquad (26)$$

where N and M are the axial force and bending moment at the junctions of beam and column members respectively. In Ref.12), these values are calculated from the maximum load applied to specimens. In Ref.14), in which tests are performed up to the range of considerably larger displacement, these are predicted by the reference values of critical loads where the displacement increases rapidly.

As is seen from this figure, the test results are almost located at the safety side of Eq.(25). Therefore, this equation is convenient to predict the lower bound of ultimate strength of corner parts.

(2) Checking method of ultimate strength of beam and column members and panel zones

a) Ultimate limit state of beam and column members in the vicinity of corner parts

In order to estimate the ultimate strength of beam and column members, the fully-plastic interaction curve of the box cross section subjected to shear and bending is developed by using the following assumptions;

i) The applied axial forces N_1 and N_2 are negligible.

ii) When the normal stress and shearing stress apply to the web plates of beam and column members, the yield criterion of Von Mises is available.

iii) The shearing force is transferred only to the web plates and the shearing stress is constant in the direction of their depth.

From these assumptions, the relationship between the ultimate shearing force νQ_j and bending moment νM_j can be obtained as;

$$\nu M_j \le M_{fpj} + M_{wpj}\sqrt{1-\Psi_j^2} \quad (\Psi_j \le 1) \qquad (27)$$

where

$$\left.\begin{aligned} \Psi_j &= \nu\frac{Q_j}{Q_{yj}} \\ M_{fpj} &= Bt_{fj}D_j\sigma_y,\quad M_{wpj} = \frac{D_j^2 t_{wj}}{2}\sigma_y \end{aligned}\right\} \qquad (28)_{a\text{-}c}$$

in which Q_{yj} : ultimate shearing force of beam and column members (See Eq.(23)) and ν: safety factor for ultimate limit state.

b) Ultimate limit state of panel zones of corner parts

In the same manner as **a)**, the ultimate limit state of panel zones can be checked by;

$$\nu\frac{F_{oj}}{Q_{yj}}=\nu\frac{F_{ij}-Q_k}{Q_{yk}}\leq 1 \quad (29)$$

where F_{oj} and F_{ik}: flange forces at the corner parts, Q_k: shearing force applied to beam and column members in the vicinity of corner parts.

Eq.(29) is almost equivalent to the checking method for connections proposed by Beedle[4], in which the ultimate strength of panel zones is decided by the yield criterion of Tresca but not Von Mises.

The strength evaluating method by Eqs.(27) and (29) is compared with test results[12),14)] in **Fig.14**. In this figure, P_{exp} is the maximum load obtained from the tests (See **Fig.9**) and P_{th} is the smaller value of the ultimate strengths predicted by Eqs.(27) and (29).

It can be seen from this figure that the proposed ultimate limit state of the corner parts is within the error of ±20% against the test results.

(3) Local buckling strength of compressive flange plates of beam and column members in the vicinity of corner parts

The flange plates of beam and column members at the vicinity of corner parts can be treated as the plates with the longitudinal stiffeners. **Fig.15** sketches the analytical model for such stiffened flange plates. The normal stress doesn't distribute uniformly but decreases at the centre of the plate due to shear lag as shown in this figure. Consequently, the plate buckling between the side stiffeners and edges of web plates should be checked. According to Ref.15), the whole stiffened plates shall also be checked for the overall buckling by using the average normal stress σ_{av}.

In the diaphragm plates, which are extended into the corner parts through the flange plates, the applied stress distribution is considerably different from that of other plate elements. The diaphragm plates are also generally reinforced by some stiffeners, because of a manhole for maintenance work at the center. Therefore, further work is required for the diaphragm plates to clarify the effects of these various factors on the buckling strength.

6. CONCLUSION

A limit state design method of corner parts of steel rigid frames is proposed in this paper. The main conclusions can be summarized as follows;

1) A beam model is presented for analyzing the beam and column members with the corner part. This model could be helpful to understanding the physical properties of corner parts by elementary beam theory.
2) Through the shear lag analysis of beam model, the additional bending moment is derived corresponding to the additional normal stress due to shear lag.
3) In order to calculate the additional bending moment at the corner parts, a conventional formula is proposed for practical design use. This formula needs three parameters, i.e., the ratio s of cross-sectional area of flange and web plates, flange force F_i' and width B of flange plate.
4) This proposition has good agreement with test results as well as present design method. Thus, a stress evaluating method is given for the serviceability limit state of corner parts.
5) Furthermore, an ultimate limit state of corner parts is proposed on the basis of the yield criterion of panel zones and flange plates of beam and column members.
6) For the stiffened flange plates in the vicinity of corner parts, a method for checking the local and overall buckling strength is described.
7) The induced forces of diaphragm plates in the corner parts may be much different from those of another members. More advanced study is necessary to evaluate the buckling strength of diaphragm plates.

ACKNOWLEDGMENTS : In conducting this study, we would like to thank Mr. S. Fukuoka et al., First Constructional Division of Hanshin Expressway Public Corporation for their great supports.

APPENDIX DEFINITION OF NOTATIONS

In the following notations, subscript j or k shall be set as j=1 and k=2 for beam members, and j=2 and k=1 for column members. The notations in the parentheses denote the generalized notation.

a) Cross-sectional dimensions

B : Width of flange plates of beam and column members

D_j (=D) : Depth of beam and column members

l_j (= l) : Length of beam and column members with corner part

t_{fj} (=t_f) : Plate thickness of flange plates

t_w : Thickness of web plates of beam and column members(=constant)

b) Cross-sectional properties

A_j (=A) : Cross-sectional area of beam and column members

A_{fj} $(=A_f)$: Cross-sectional area of flange plates of beam and column members

$$A_{fj} = Bt_{fj} \quad (A.1)$$

A_{wj} $(=A_w)$: Cross-sectional area of web plates of beam and column members

$$A_{wj} = D_j t_w \quad (A.2)$$

I_{yj} $(=I_y)$: Geometrical moment of inertia of beam and column members

$$I_{yj} = \frac{Bt_{fj}D_j^2}{2} + \frac{D_j^3 t_w}{6} \quad (A.3)$$

W_{yj} $(=W_y)$: Section modulus of beam and column members

$$W_{yj} = \frac{2I_{yj}}{D_j + t_{fj}} \quad (A.4)$$

c) Elastic modulus, stress and stress-resultants

E : Young's modulus (=2.06×10^{5}MPa)

G : Shear modulus of elasticity(=7.94×10^{4}MPa)

μ : Poisson's ratio(=0.3)

F_{ij} : Flange force at the junctions of beam and column members (See **Fig.3**(a) and Eq.(1)b)

F_{ij} : Flange force for estimating additional moment due to shear lag

F_{oj} : Shearing force applied to panel zones

m_s : Additional bending moment due to shear lag

m_{sp} : Additional bending moment at the junctions of beam and column members

N, Q and M : Uniaxial force, shearing force and bending moment

N_j, Q_j and M_j : Uniaxial force, shearing force and bending moment applied to the junctions of beam and column members

Q_{yj} : Ultimate shearing force of web plates of corner parts (See Eq.(22))

σ_{sp} : Additional normal stress due to shear lag which occurs at the junctions of beam and column members.

σ_y : Yield point of steel materials

τ : Average shearing stress of web plates in the beam and column members

τ_s : Average shearing stress of panel zones of corner parts

d) Parameters

R_j $(=R)$: Shear lag parameter

$$R_j = 3A_{fjj}/A_{wj} \quad (A.5)$$

R_f : Nondimensionalized plate slenderness ratio of flange plates of beam and column members

$$R_f = \frac{B}{(n+1)t_f}\sqrt{\frac{12(1-\mu^2)}{k\pi^2}}\sqrt{\frac{\sigma_y}{E}} \quad (A.6)$$

where n : number of longitudinal stiffeners and k : buckling coefficient of simply supported plate (=4.0)

s_j $(=s)$: Parameter with respect to cross-sectional area ratio of flange and web plates

$$s_j = A_{wj}/A_{fj} \quad (A.7)$$

s_{uj} $(=s_u)$: Ratio of cross-sectional areas of flange and web plates represented by

$$s_{uj} = D_k t_w / Bt_{fj} \quad (A.8)$$

η : Shear lag parameter (See **Fig.7**(a))

ν : Safety factor for serviceability state or ultimate states

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