Structural Eng./Earthquake Eng., JSCE, Vol. 13, No. 2, 131s—135s, 1996 October
(J. Struct. Mech. Earthquake Eng., JSCE, No. 549/ I —37)

EXPECTED SEISMIC DAMAGE DURING NARROW-BAND STRUCTURAL RESPONSES

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Ductility has been a popular damage measuring index, though it fails to account for the role played by repeated inelastic excursions of different amplitudes. A typical earthquake response time history is characterized by the repetitive cycles with different orders of peaks. In such a case, damage not only depends on the maximum response and its magnitude beyond the yield level but also on the higher order peaks. Thus, the additional parameters of interest are the total number of peaks, and the number as well as the extent of non-linear excursions. To obtain the expected damage corresponding to a response process, a model has been considered which includes the uncertainties both in terms of the amplitudes of the peaks and of their ordered occurences. Under the assumption of a narrow-banded response, the effects of the system parameters and the response duration on damage and ductility have been studied.

Key Words : *cumulative damage, peak statistics, response duration, ductility.*

1. INTRODUCTION

The current practice of reducing the linear design earthquake forces due to economic reasons is based on the energy dissipation through non-linear behaviour during the severe ground shaking. The structures are designed such that they possess sufficient ductility to withstand a few inelastic excursions without collapse. Thus, ductility demand of a structure has been a popular measuring index of the damage potential of the earthquake. However, two structures with the same ductility demand may suffer different degrees of damages, when the inelastic excursions in their seismic responses differ in number and ordered amplitudes. A damage index based on the damage accumulation from different ordered inelastic excursions should be able to provide the additional information on the structural damage.

Jeong and Iwan[1)] studied the effect of duration on the damage of the structures. They considered each response peak to contribute to the damage entirely independent of the other peaks. Park et al.[2)] proposed a cumulative damage indicator based on the linear combination of plastic displacement and energy dissipation capacity under monotonic loading. This model, popularly known as the Park-Ang's model, is a calibrated model and has been widely used in the recent years despite some deficiencies (see Reference 3)). Based on this model, Fajfar[4)] proposed an equivalent ductility factor to account for the low cycle fatigue. His work did not account for the differences in the ordered peak amplitudes, and the presence of various frequencies with randomly varying phases and amplitudes in the time history of response. Considering the ductility to be a random variable, Vanmarcke[5)], and Vanmarcke and Veneziano[6)] formulated the probabilistic ductility ratio. Basu and Gupta[7)] incorporated the parameter of number of non-linear excursions in their formulation of this ratio. This parameter, though ignored by earlier researchers, has direct link with the damage accumulation from repetitive excursions.

Villaverde[8)] investigated the factors responsible for the collapse of upper floors of a large number of buildings in the Michoacan Earthquake of September 19, 1985, and attributed the cause of this damage to the combined effect of large accelerations with long durations i.e. with greater number of zero crossings. The work of Anderson and Bertero[9)] also emphasized the importance of the number of non-linear excursions and total number of peaks in the damage of structures. Basu and Gupta[10)] proposed a model based on the order statistics to evaluate the damage accumulation from several non-linear excursions in the response duration, and associated the ductility with the total damage. They considered the total number of peaks and the number of non-linear excursions as parameters. The present paper uses their formulation to study damage particularly in case of the narrow-banded responses, as this case permits establishing a direct link between the structural damage and the parameters like period of the structure and response duration. Mason-Coffin relationship for plastic strain and low cyclic failure life has been adopted here. It is possible to extend the results of this study to assess the cumulative damage caused by several moderate to small size events, not just by a single large size event alone.

2. DISTRIBUTION OF MAXIMA

Let the stationary, zero mean, Gaussian response of a structure be assumed as narrow-band, and be denoted by $X(t)$, which may be represented by

$$X(t)=\sum_{n} C_n \cos(\omega_n t+\phi_n) \qquad (1)$$

where, ω_n are the circular frequencies, ϕ_n are the random phases uniformly distributed between 0 and 2π, and C_n are the amplitudes related to the energy spectrum, $S(\omega)$ of $X(t)$ by the following relation

$$\sum_{\omega n=\omega}^{\omega+d\omega} \frac{1}{2} C_n^{\,2}=S(\omega)\,d\omega. \qquad (2)$$

The probability distribution of maxima of $X(t)$ depends on the root-mean-square (r.m.s.) value of $X(t)$, a_{rms} and on a parameter ε, which is a measure of the width of the energy spectrum, $S(\omega)$. These are represented as

$$a_{rms}=m_0^{1/2} \qquad (3)$$

and

$$\varepsilon=\left[\frac{m_0 m_4-m_2^2}{m_0 m_4}\right]^{1/2} \qquad (4)$$

where, in general, the n^{th} moment, m_n of the energy spectrum is defined by

$$m_n=\int_0^\infty \omega^n S(\omega)\,d\omega\,;\; n=0,1,2... \qquad (5)$$

For a narrow-banded response, $\varepsilon=0.0$, and in this case, the probability density function of the maxima is as in Rayleigh distribution (see Reference 11)) i.e.

$$p(\eta)=\eta e^{-\eta^2/2}\;,\;\eta\geq 0 \qquad (6)$$

where, η is the peak amplitude normalized with respect to a_{rms}. The probability of exceedance of η is given by

$$P(\eta)=\int_\eta^\infty p(u)\,du. \qquad (7)$$

Let there be a total of n peaks in the duration, T. Then, n is given by

$$n=\frac{T}{2\pi}\sqrt{\frac{m_4}{m_2}}. \qquad (8)$$

Further, the p.d.f. of the i^{th} order peak in the total of n peaks (assumed statistically independent) may be given by (see Reference 12))

$$p_{(i)}(\eta)=\frac{n!}{(n-i)!\,i!}[P(\eta)]^{i-1}[1-P(\eta)]^{n-i}p(\eta). \qquad (9)$$

3. CUMULATIVE EXPECTED DAMAGE

Consider a structure which is subjected to a strong ground motion. The repeated application of large strains may cause failure of the members according to the relation

$$N\mu^s=C \qquad (10)$$

where, N is the number of cycles to failure at a constant ductility amplitude μ, and s and C are the positive empirical constants. This relationship corresponds to the Mason-Coffin relationship between the plastic strain and low cycle failure life. The rule proposed by Palmgren and Miner (see Reference 13)) may now be used to calculate the cumulative damage by summing up each incremental damage. This simple approach, though approximate, reflects the physical behaviour of the problem reasonably well. Moreover, our emphasis here is on considering that damage model which takes into account the entire history of response.

Let the n number of (normalized) ordered peaks in $X(t)$ be denoted by $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$. Also, let the normalized response yield level be l. The probability distribution of the largest peak and the damage caused due to a single cyclic application of the largest peak amplitude, $X_{(1)}=x_{(1)}$ normalized with respect to the yield level, l can be obtained from Equations (9) and (10) respectively (see Reference 4) for details). These two expressions are combined to give the expected damage due to the largest peak as

$$E(D_1)=\int_0^\infty \frac{x_{(1)}^s}{Cl^s} n[1-P(x_{(1)})]^{n-1}p(x_{(1)})\,dx_{(1)}$$

$$=\frac{E(x_{(1)}^s)}{Cl^s}. \qquad (11)$$

It may be noted that this may vary between 0 for "no damage" situation to 1 for the "complete collapse" situation. From this expression, we define the equivalent amplitude for the first peak, say x_{1eq}, as that peak amplitude which would cause the damage, $E(D_1)$. Hence, we can write,

$$x_{1eq}=[E(x_{(1)}^s)]^{1/s}. \qquad (12)$$

In general, to find the expected damage due to the excursion corresponding to the i^{th} order peak, we must account for its dependence on the $i-1$ peaks above it. Thus, the conditional order statistics must be used to find the conditional expected damage due to the i^{th} peak, $X_{(i)}$ on the condition that $X_{(1)}$, $X_{(2)}$, ..., $X_{(i-1)}$ are known. Thus, we can write (see Reference 10))

$$E(D_i|x_{(1)}, \ldots, x_{(i-1)}) = \int_0^{x_{(i-1)eq}} \frac{x_{(i)}^s}{Cl^s}(n-i+1)$$

$$p(x_{(i)}) \frac{[1-P(x_{(i)})]^{n-i}}{[1-P(x_{(i-1)eq})]^{n-i+1}} dx_{(i)}. \quad (13)$$

The corresponding equivalent amplitude level for the i^{th} peak is given by

$$x_{ieq} = [E(D_i|x_{(1)}, \ldots, x_{(i-1)})]^{1/s}. \quad (14)$$

The conditional damage has the Markov property (see Reference 10)) that damage due to the i^{th} order peak excursion is dependent only on the next lower order peak i.e. on the $(i-1)^{th}$ order of peak. This is written as

$$E(D_i|x_{(1)}, \ldots, x_{(i-1)}) = E(D_i|x_{(i-1)}). \quad (15)$$

If there are k excursions beyond the yield level, l, the cumulative damage for the k excursions may be represented as

$$E(D_k) = \sum_{i=1}^{k} E(D_i|x_{(i-1)}). \quad (16)$$

On averaging over all the possible excursions, we obtain the expected cumulative damage with total n peaks and a specified yield level, l as

$$E(D) = \sum_{k=1}^{n} \frac{n!}{(n-k)!k!}[P(l)]^k[1-P(l)]^{n-k}E(D_k). \quad (17)$$

The corresponding ductility level is obtained as

$$\mu = \frac{x_{1eq}}{l}. \quad (18)$$

For a narrow-band process, n can be related to the dynamic properties of the system i.e. natural time period, T_n and the damping ratio, ξ as (see Equation (8)),

$$n = \sqrt{1-\xi^2}\left(\frac{T}{T_n}\right). \quad (19)$$

For low damping systems, $n \simeq T/T_n$. This expression enables us to obtain the damage directly in terms of the system and ground motion parameters.

In a ductile system, non-linear behavior is exhibited which may require the adjustment of damping and natural frequency of the system (see Reference 14)) before the above formulation is applied. Further, it should be noted that this formulation does not include the contributions of the low-frequency drift to the structural damage due to the assumption of zero mean process.

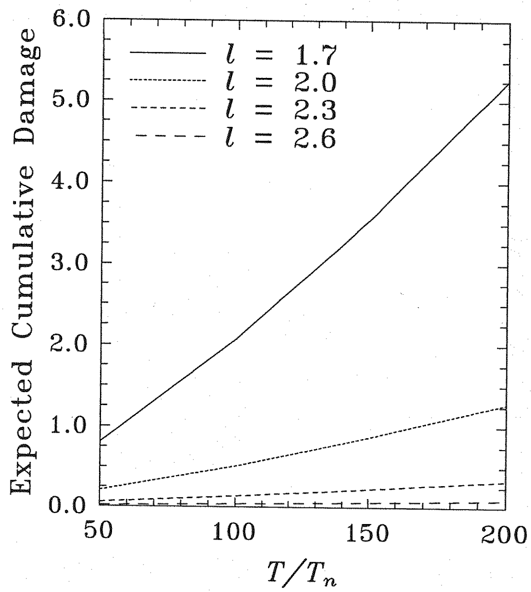

Fig.1 Expected Cumulative Damage, $E(D)$ versus T/T_n for Concrete.

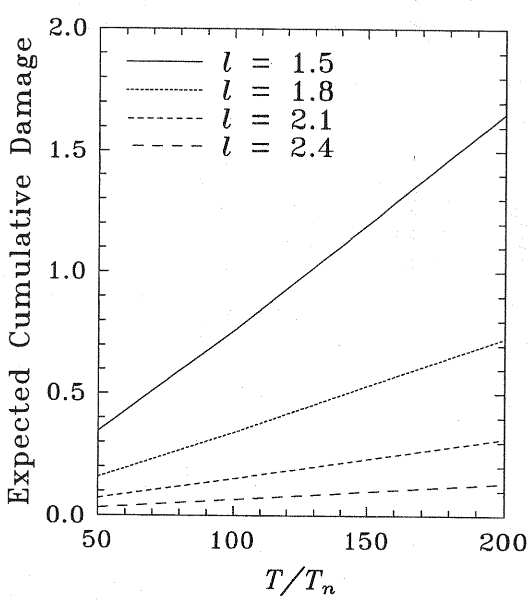

Fig.2 Expected Cumulative Damage, $E(D)$ versus T/T_n for Steel.

4. RESULTS AND DISCUSSION

The numerical results on expected damage and ductility have been calculated with the parameters in Equation (10) assumed to be as $s=6$, $C=416$ for concrete, and $s=2$, $C=167$ for steel structures respectively (as reported by Yamada[15)]).

The variation in expected cumulative damage,

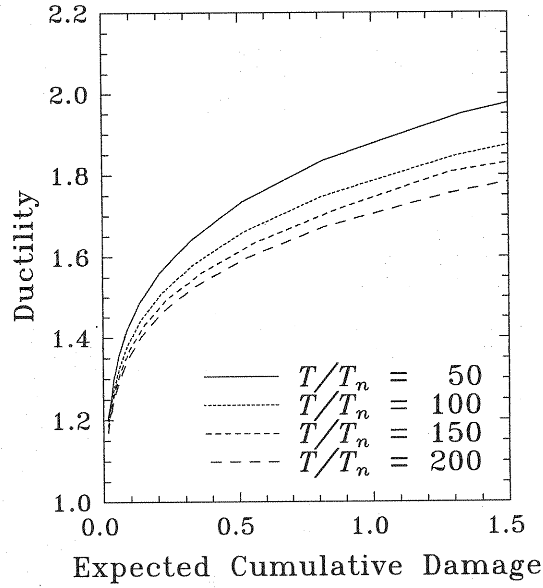

Fig.3 Variation in Ductility, μ with Expected Cumulative Damage, $E(D)$ for Concrete.

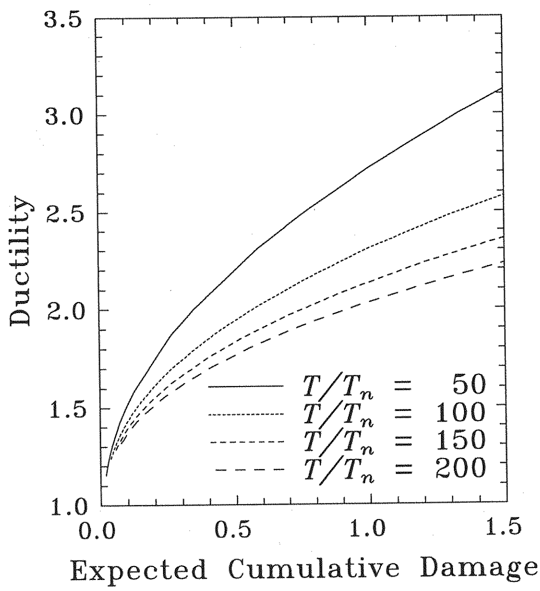

Fig.4 Variation in Ductility, μ with Expected Cumulative Damage, $E(D)$ for Steel.

$E(D)$ with the parameter, T/T_n is shown in **Figs.1** and **2** for different yield levels and for the cases of concrete and steel respectively. For concrete, the yield levels are taken as l=1.7, 2.0, 2.3 and 2.6 while for the case of steel, l has been taken as 1.5, 1.8, 2.1 and 2.4. It is seen that damage increases with the increase in T/T_n. This indicates that for a given yield level, there is more damage for longer duration response or stiff structures. This increase in damage is however insignificant at the high yield levels. This may be due to lesser non-linear excursions associated with the higher yield levels even when the duration of response is very long or the structure is very stiff.

Figs.3 and **4** show the variation in ductility with the expected cumulative damage and the parameter, T/T_n, for concrete and steel respectively. There are four curves in each figure for T/T_n=50, 100, 150 and 200. All curves are seen to converge to a ductility of 1.0 for the "no damage" situation. It is also seen that the increase in damage with ductility is steeper at higher ductility levels. Further, for the same ductility ratio, greater damages result from the higher values of T/T_n i. e., long duration response or stiff structures cases involve more damage. In other words, for a given damage, the allowable ductility may be less for longer duration response, especially in case of the stiff structures. This is because of more zero crossings or peaks in the long duration response for stiff structures which in turn are associated with greater possibility of non-linear excursions and thus of the damage. Therefore, the duration of response and the stiffness of structures both should play an important role in fixing the allowable ductility level, particularly when the ductility levels are kept relatively high. In fact, based on the structural damage to be allowed in a structure, there exists an upper bound on the ductility which is governed by the expected response duration and the structural stiffness.

5. CONCLUSIONS

This study has clearly highlighted the insufficiency of the traditional concept of ductility to assess the damage in the structures. In fact, a proper damage index should also account for the damage accumulation from the inelastic excursions. The model considered in this study can assess the damage and ductility for a narrow-banded response process directly in terms of the parameters like the response duration and the natural time period of the structural system. It has been seen that there exists an upper limit on the ductility which is based on the allowable damage, for a given response duration and system stiffness.

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(Received October 20, 1994)