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BUCKLING MODES OF PLATE-GIRDERS CURVED IN PLAN

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The behavior of instability of thin-walled, I-shaped plate girders curved in plan subjected to equal end moments is investigated by a nonlinear inelastic analysis. Their inelastic buckling modes are particularly focused. The investigation results show that the curved girders buckle in the three fundamental buckling mode shapes and the possibilities of these instabilities depend on not only the plate-slenderness but also the initial radius of curvature.

Key Words : *thin-walled plate girders, curved girders, I-shaped girders, local buckling, instability*

1. INTRODUCTION

Typical out of plane buckling of compression flange of straight girder with *I*-shaped cross section is well known as lateral and torsional buckling of the *T*-shaped flange[1]. Recently, as typical results of many important studies, web bucklings and the flange plate bucklings in horizontally curved plate-girders have been examined[2),3)]. The design aid for the lateral buckling of the girders is also presented[4]. However, the buckling modes in inelastic finite deformation of the curved *I*-girders composed by thin-walled plates are not sufficiently clear yet. The fundamental question is how much the initial radius of curvature influences the bifurcation of the inelastic buckling mode shapes of the curved plate girder. This question is not clarified satisfactorily.

The primary object of the present study is to examine the effect of the curvature on the inelastic buckling modes of the thin-walled, *I*-shaped plate girder curved in plan. An additional purpose is to classify its buckling modes. Their instability responses are investigated by an inelastic nonlinear finite element approach previously adopted in Ref.5). In the approach, von Mises yield criterion, Plandtl-Reuss flow rule and isotropic hardening rule were adopted in the development of elastic-plastic constitutive matrix for material inelasticity. Isoparametric shell elements are also used and their governing equations are based on general increment equations derived from Updated Lagrangian formulation. This paper focuses on the simply supported, steel-plated girder under equal end moments as shown in **Fig.1**.

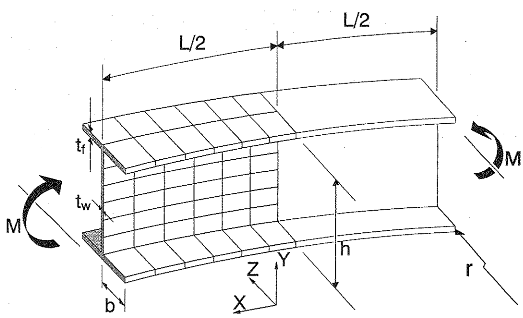

Fig.1 Analytical model.

2. NUMERICAL INVESTIGATIONS

(1) Numerical Model

In the following numerical model for the girder, mild steel is used with Young's modulus $E=2.1\times10^5$ N/mm^2, shear modulus $G=8.1\times10^4$ N/mm^2, Poisson's ratio $v=0.3$, initial yield stress $\sigma_Y=240$ N/mm^2 and initial yield strain $\varepsilon_Y=0.00114$. For the inelastic analyses, the bi-linear elastic-perfectly plastic stress-strain relationship is used. The width-

Table 1 Parameters and their ranges selected in the study.

Item (1)	Symbol (2)	Range of values (3)
Curvature parameter	L/r	0.0, 0.01, 0.03, 0.05
Sectional area parameter	A_f/A_w	0.2, 0.4, 0.8
Modified slenderness ratio	$\bar{\lambda}_g$	0.369, 0.615, 0.861, 1.107, 1.353, 1.599
Width-to-thickness ratio of flange	b_f/t_f	16
Width-to-thickness ratio of web	h_w/t_w	152
Initial imperfection	$\bar{w}_0$	$L/1000$
Yield stress parameter	E/σ_Y	875

to-thickness ratio of the web plate h/t_w of the girder model is kept constant as the limit value for bridge girders specified by JSSHB[6] at 152. Namely, the width-thickness parameter of web plate is :

$$\bar{\lambda}_w=\frac{h}{t_w}\sqrt{\frac{12(1-v^2)\sigma_Y}{23.9\pi^2 E}}=1.106 \qquad (1)$$

and the cross section of straight girder with this width-thickness parameter is in the design range limited by the local buckling[1]. The load is applied by incrementing the enforced displacement at the ends of the girder corresponding to the equal end moments. The enforced displacement approach for instability analysis is explained in Ref.7) in detail. The initial imperfection w_0 adopted herein is a sinusoidal in the normal direction of the curvilinear axial coordinate of the girder as follows :

$$w_0=\bar{w}_0\cos\frac{\pi x}{L} \qquad (2)$$

and is considered for each girder model. Varied parameters of the girder model in the study are the width-to-thickness ratio of the flange plate b/t_f, the span-to-initial radius of curvature ratio (herein termed as *curvature parameter*) L/r, the ratio of a flange sectional area and web sectional area (herein termed as *sectional area parameter*) A_f/A_w, and the modified slenderness ratio of *I*-girder defined as follows :

$$\bar{\lambda}_g=\frac{L}{\pi}\sqrt{\frac{\sigma_Y}{E}}\sqrt{\frac{A_f+A_w/6}{I_f}} \qquad (3)$$

where I_f is the moment of inertia of a flange. The ranges of these parameters are selected in the study as given in **Table 1** and generally within found in steel plate girder bridges. Particularly, the cross-section with the flange proportioned by the limit value of width-thickness ratio for bridge girders specified by JSSHB[6] at 16 is termed as "thin-walled" section herein. By referring the results of preexamination[5], a half of the girder model is meshed, as shown in **Fig.1**, with twelve elements for the flange plate (six longitudinal and two transverse) and thirty six elements for the web plate (six longitudinal and six for depth).

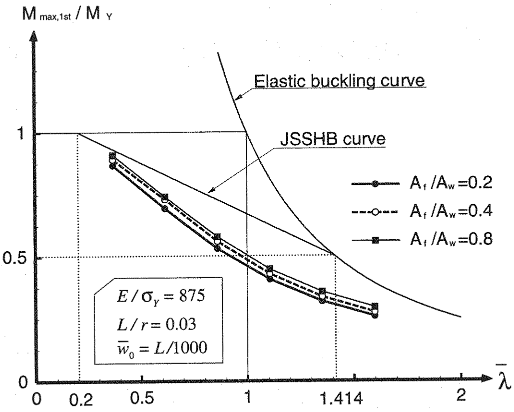

Fig.2 Effect of sectional area parameter.

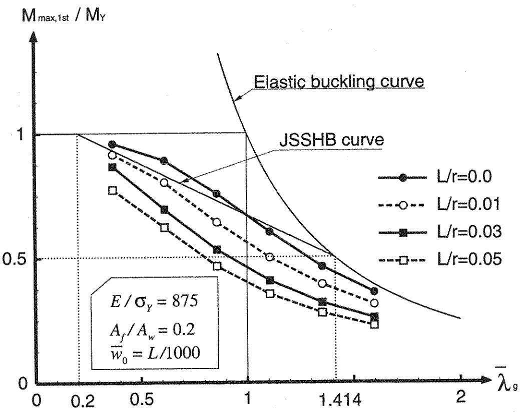

Fig.3 Effect of curvature parameter.

(2) Ultimate Strength Characteristics

a) Effect of sectional area parameter

Some selected results of the effects of sectional area parameter on dimensionless presentations of the girder strength and the modified slenderness ratios, i.e., the ultimate strength curves, for $L/r=0.03$ and $b/t_f=16$ are shown in **Fig.2**. In the figure, the strength design curve specified by JSSHB[6] and the elastic buckling curve are also shown for comparison purposes. A general conclusion which may be drawn from the parametric study for the effect of sectional area parameter is that the ultimate strength of the girder model increases slightly as increase of A_f/A_w within the structural dimensions adopted herein.

b) Effect of curvature parameter

Fig.3 shows typical ultimate strength curves for $A_f/A_w=0.2$, and four different curvatures of the curved girder, $L/r=0, 0.01, 0.03$, and 0.05. The

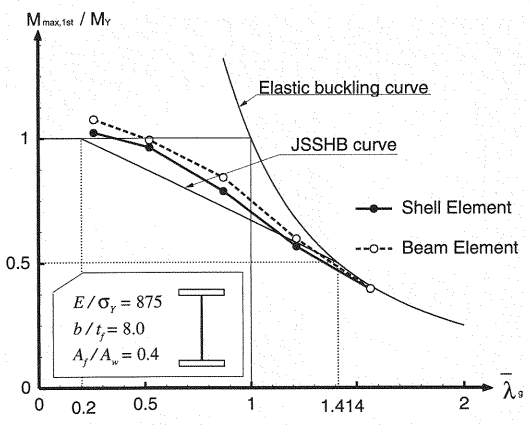

Fig.4 Effect of local instability for stocky flange.

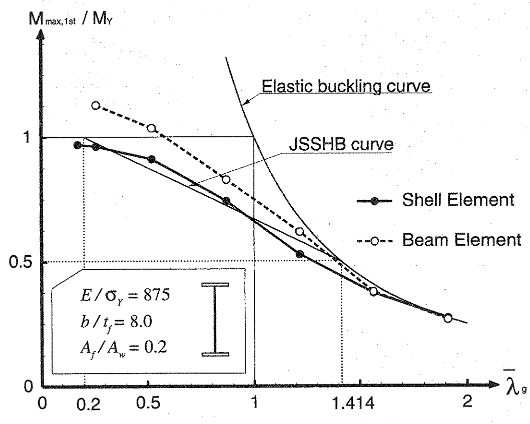

Fig.6 Effect of local instability for slender flange.

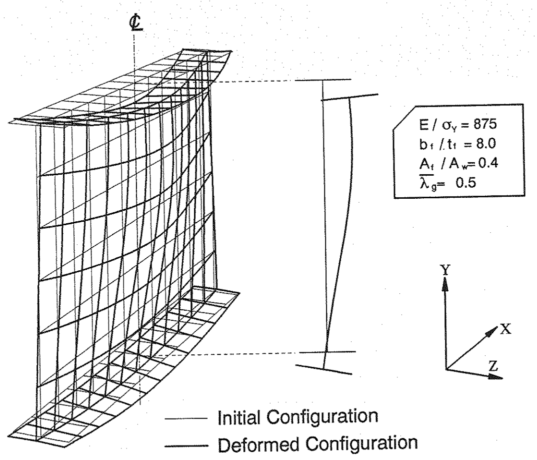

Fig.5 Inelastic buckling mode for stocky flange.

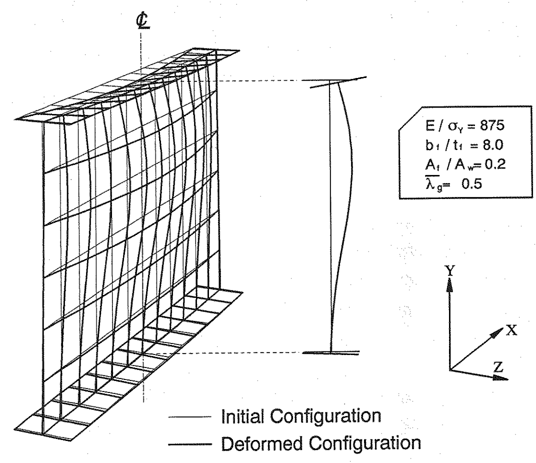

Fig.7 Inelastic buckling mode for slender flange.

ultimate strength curve for $L/r=0$, i.e., straight girder, shows good agreement with the design curve specified by JSSHB[6], approximately. On the other hand, the results for the curved girders show smaller strengths than the abovementioned design curve and the ultimate strength of the curved girder decreases as increase of the curvature parameter, L/r.

c) Effect of local instability

For a plate girder subjected to bending, the stabilities of the compression flange and the web must be considered. The effect of local instabilities on the ultimate strength of the girder is checked by varying the flange-sectional area and partially demonstrated in **Figs.4** and **5** for a stocky flange and **Figs.6** and **7** for a slender one. **Figs.4** and **6** show the ultimate strength curves for $b/t_f=8.0$ and their typical buckling modes are illustrated in **Figs.5** and **7** for a lower value of the girder-slenderness ratio, i.e., $\bar{\lambda}_g=0.5$. In order to focus on fundamental local instabilities, the straight girders are adopted. In these figures, the results obtained by the beam element approach in which the local instabilities are not considered[8),9)] are also shown for comparison purpose. The girder models shown in **Figs.4, 5, 6** and **7** have the same web size as each other. A compression flange with a stocky section receives the most of compressive force in a plate girder under equal end moments. Namely, since some of the compressive force to the web is redistributed to the compression flange, it is assumed that the contribution of the compressed portion of the web can be disregarded except for an effective strip along the compression flange. If material of the girder is elastic, its effective strip is evaluated as $A_w/6$ given in **Eq.(3)** by Basler[10)]. The strength of the girder with a stocky flange subjected to equal end moments is then characterized more strongly by its overall lateral instability than by the local one. It can be also seen from the fact that the result obtained by the beam element approach agrees with the result by the shell-element

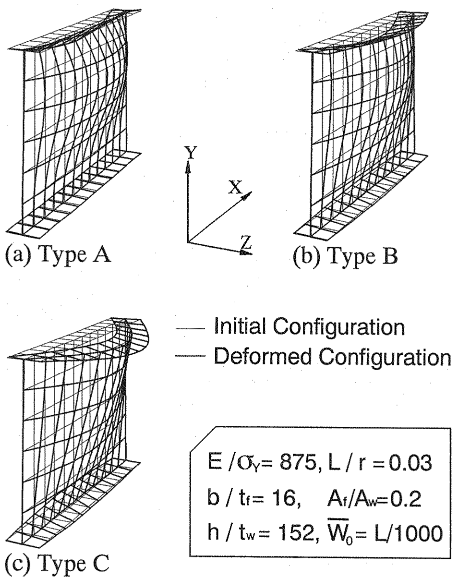

Fig.8 Fundamental buckling modes.

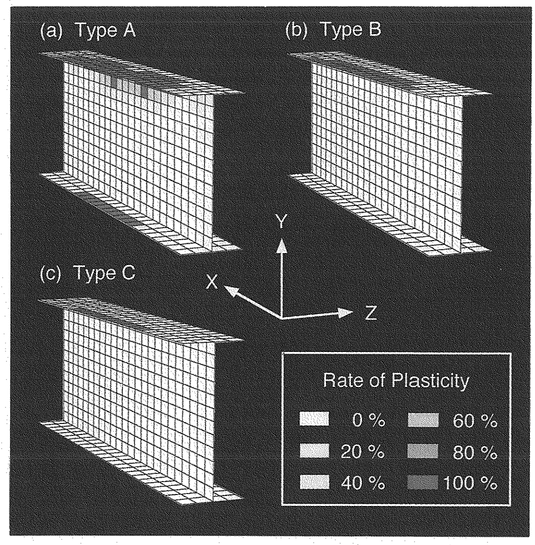

Fig.9 Spreads of plastic zones.

Table 2 Classification of inelastic buckling modes.

L/r	$\bar{\lambda}_g$					
	0.369	0.615	0.861	1.107	1.353	1.599
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0	Type-A	Type-A	Type-A	Type-B	Type-C	Type-C
0.01	Type-A	Type-A	Type-B	Type-B	Type-C	Type-C
0.03	Type-A	Type-A	Type-B	Type-B	Type-C	Type-C
0.05	Type-A	Type-A	Type-B	Type-C	Type-C	Type-C

approach, in the case of stocky flange as shown in **Fig.4**. In fact, a typical buckling mode of the girder obtained by the shell-element approach **(Fig.5)** shows the overall lateral buckling behavior rather than the local one. On the other hand, from **Fig.6**, the result for a slender flange obtained by the shell-element approach decreases, comparing with the result by the beam approach, as decrease of $\bar{\lambda}_g$. It shows following aspect. Namely, if the girder with slender compression flange is made as feasible to provide good lateral buckling resistance, i.e., higher strength for the overall lateral instability (lower value of $\bar{\lambda}_g$), it still may be possible that torsional buckling of the flange plate and adjacent portion of the web will occur. In fact, a typical buckling mode of the girder obtained by the shell-element approach **(Fig.7)** shows the local buckling behavior rather than the overall lateral one.

(3) Inelastic Buckling Modes

a) Fundamental buckling-mode shapes

The numerical studies performed herein may make the inelastic buckling-mode shapes for the curved girder with *I*-shaped thin-walled cross-section clear. As the results, three fundamental buckling modes are confirmed. These typical modes are shown in **Fig.8**. The inset (a) of **Fig.8** presents a local buckling mode in which lateral deflection of the web from the initial configuration becomes large and the compression flange deformes torsionally and laterally (herein expressed as Type-A). An overall lateral buckling mode expressed as Type-C in the inset (c) shows overall lateral-torsional deformation of the girder without any plate instabilities. An interactive instability of the local and the overall lateral instabilities, herein expressed as Type-B, is shown in the inset (b). The interactive instability causes the overall lateral buckling-mode shape accompanying with the abovemensioned local buckling-mode shape. **Fig.9** presents spreads of plastic zones in the cross sections and along the longitudinal axis of the girders corresponding to the fundamental buckling-mode shapes as shown in **Fig.8**. The plastic zone in the direction of plate-thickness is expressed as a percentage in the inset of **Fig.9**. It can be observed that the spread of plastic zones shown in the inset (a) corresponds to the deformation mode for the local instability, i.e., the remarkable lateral deflection of the compressed portion in the web and the torsional and lateral deflection of the compression flange. From the inset (c), the plastic zone for the overall lateral instability spreads with nearly uniform distribution along the compression flange length. It can be seen from this result that the compression flange in this case behaves as an isolated column. The spread of plastic zones in the inset (b) shows so-called combined distributions of the spreads of plastic zones for the local and the overall instabilities.

b) Classification of buckling modes

Based on the parametric study for the buckling modes, the spreads of plastic zones and the ultimate strength curves, the inelastic buckling modes of the curved girder with the thin-walled cross section is classified. The results are given in **Table 2**. The noticeable feature of these results is that the buckling mode of the thin-walled plate girder curved in plan is influenced by not only the girder slenderness but also the curvature of the curved girder. The local instability becomes considerable as decrease of L/r.

3. CONCLUSIONS

The buckling modes of *I*-shaped plate girder curved in plan has been investigated by the nonlinear finite element approach developed for isoparametric shell elements in the inelastic and the finite deformation range. The deformation of individual plate composing the cross section of the girder was examined in detail. Based on this study, the following conclusions can be drawn mainly :

1. The ultimate strength of the girder model increases slightly as increase of A_f/A_w within the structural dimensions adopted herein.
2. The curved girder shows smaller strength than the JSSHB specification and the ultimate strength of the curved girder decreases as increase of the curvature parameter, L/r.
3. Since the compressive flange with stocky section receives the most of compression force in the plate girder under equal end moments, its strength is characterized more strongly by the overall lateral instability than by the local one. If the girder with slender compression flange is made as feasible to provide good lateral buckling resistance, i.e., higher strength for the overall lateral instability (lower value of $\bar{\lambda}_g$), it still may be possible that torsional buckling of the flange plate and adjacent portion of the web will occur.
4. As a result on classificating the inelastic buckling mode shapes, three fundamental buckling modes are confirmed, i.e., a local buckling mode in which the lateral deflection of the web is remarkable and the compression flange deflects torsionally and laterally, an overall buckling mode in which the overall lateral torsional deflection occurs to the girder without any plate instabilities, and an interactive buckling mode of the loal and the overall lateral instabilities.
5. The buckling mode of the thin-walled plate girder curved in plan is influenced by not only the girder slenderness but also the curvature of the curved girder. Namely, the local instability becomes considerable as decrease of L/r.

From these results it can be seen that there are many aspects of stability of curved girders needing further attention. Some of which are summarized as studies of the other initial imperfections and loadings, the limitations on cross-sectional shape to prevent local buckling, bracing requirements, systematic experiment on the ultimate strength, and a rational ultimate load method for the design against the instability.

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