MODAL ANALYSIS METHOD FOR STIFFNESS DEGRADATION IDENTIFICATION OF NON-PROPORTIONALLY DAMPED STRUCTURES

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A modal analysis method is presented to identify both the location and the severity of stiffness degradation for nonproportionally damped structures by using measured lower modes. A ten-story shear building has been analyzed to demonstrate the availability of the method. Also the effects of measurement error of modes on identification accuracy has been examined.

Key Words : stiffness degradation, nonproportional damping, modal analysis method

1. INTRODUCTION

It is generally recognized that damage of structures caused by an earthquake etc. will result in degradation of stiffness and in change of damping. Many methods in time domain and frequency domain by the use of observed vibration record have been developed for detecting the damage of structures. However, someties the identification accuracy of those methods is greatly affected by noise[1]. Also in civil engineering, the instrument for earthquake record is installed only in a small number of important structures, but not most structures. Therefore, an evaluation of damage through modal analysis methods[2] which are based on the measured modal parameters from ordinary vibration testing is paid increasing attentions.

In our previous studies[3],[4], we dealt with the damage evaluation for a structure without damping or with proportional damping[5]. However, it is necessary to evaluate the damage of a structure with non-proportional damping[5] which yields complex mode shapes, because non-proportional damping exists in realistic structures[2], e.g. cable-stay-bridges[6] or RC bridges with some deterioration[7]. In this study, a modal analysis method is presented to identify both the location

and the severity of damage of a structure with nonproportional damping. The numerical result shows that this method has significant sensitivity for detecting the location of stiffness degradation. Also the effects of measurement error of modes on identification accuracy has been examined.

2. MODAL ANALYSIS INSPECTION FOR NONPROPORTIONAL DAMPING

A system with damping matrix $[C]$ which can not be considered as a proportional type is described by its free vibration equation of motion

$$[M]\{\ddot{Z}\}_p + [C]\{\dot{Z}\}_p + [K]\{Z\}_p = 0 ;$$
$$p = 1, 2, \cdots, N \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where $[M]$ = mass matrix; $[C]$ = damping matrix; $[K]$ = stiffness matrix; $\{Z\}_p$ = the pth vector of relative displacements and N = the number of system's degree of freedom. The solutions of Eq.(1) can be expressed[8] as

$$\{Z_p\} = \{X\}_p e^{\mu_p t} ; \quad p = 1, 2, \cdots, 2N \cdots\cdots (2)$$

which $\{\mu_p\}$ = the pth complex natural frequency and $\{X\}_p$ = the pth complex eigenvector. Eq.(1) is rewritten as

$$[M]\{X\}_p \mu_p^2 + [C]\{X\}_p \mu_p + [K]\{X\}_p = 0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

If Eq.(3) is premultiplied by the transpose of the complex conjugate of the complex mode shape $\{X\}_p$ i.e., $\{\bar{X}\}_p^T$, the system's free vibration equation of motion may be written alternatively as

$$m_p{}^* \mu_p{}^2 + c_p{}^* \mu_p + k_p{}^* = 0 \cdots\cdots\cdots\cdots\cdots (4)$$

where $m_p{}^*$, $c_p{}^*$ and $k_p{}^*$ = real-value generalized parameters defined as

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$$m_p^* = \{\bar{X}\}_p^T [M] \{X\}_p ;$$
$$c_p^* = \{\bar{X}\}_p^T [C] \{X\}_p ; \qquad \Bigg\} \cdots\cdots\cdots\cdots (5)$$
$$k_p^* = \{\bar{X}\}_p^T [K] \{X\}_p$$

Eq.(4) is thus an equation of μ_p with real coefficients whose solution yields[9]

$$\mu_p = -\xi_p \omega_p + i\omega'_p ; \quad i^2 = -1 \cdots\cdots\cdots (6)$$

where

$$\omega_p = \sqrt{k_p^*/m_p^*} ;$$
$$\omega'_p = \omega_p \sqrt{1 - \xi_p^2} ; \qquad \Bigg\} \cdots\cdots\cdots\cdots (7)$$
$$\xi_p = c_p^*/2\omega_p m_p^*$$

The ω_p, ω'_p and ξ_p represent the system's pth undamped natural frequency, pth damped natural frequency and pth damping ratio respectively. Moreover, if the $1 \sim L$th conjugate pairs of modes have been measured, the real-part and imaginary-part of natural frequency matrix $[\Omega]$ and mode shape matrix $[\Phi]$ can be expressed as follows

$$[\Omega]_{L \times L} = [\Omega]_a + i[\Omega]_b = \mathrm{diag}[\mu_1, \mu_2, \cdots, \mu_L]$$
$$[\Phi]_{N \times L} = [\Phi]_a + i[\Phi]_b = [\{X\}_1, \{X\}_2,$$
$$\cdots, \{X\}_L]$$
$$\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

A commonly used, nonproportional damping matrix with sufficient accuracy[5] is defined as

$$[C] = \beta_0 [I] + \beta_1 [M] + \beta_2 [K] \cdots\cdots\cdots (9)$$

where, $\beta_0 \ (\neq 0)$, β_1 and β_2 = the coefficents regarding to damping ; $[I]$ = identity matrix. It should be noted that the β_0, β_1 and β_2 may be determined experimentally[10].

Let $[K] = [K_0] + [\Delta K]$, in which $[K_0]$ is undamaged stiffness matrix and $[\Delta K]$ is the change of stiffness before and after damage. Substituting the $[\Omega]$, $[\Phi]$, $[M]$, $[C]$ and $[K]$ into Eq.(3), and then arranging the obtained equations, the separated real-part and imaginary-part of the equations can be obtained, respectively. The equations for the real-part is

$$[\Delta K][\Gamma_1] = [\Xi_1] \cdots\cdots\cdots\cdots\cdots\cdots (10)$$

where $[\Gamma_1]_{N \times L} = (\beta_2 [\Phi]_a [\Omega]_a - \beta_2 [\Phi]_b [\Omega]_b + [\Phi]_a)$ and $[\Xi_1]_{N \times L} = -\beta_0 ([\Phi]_a [\Omega]_a - [\Phi]_b [\Omega]_b) - [M]([\Phi]_a [\Omega^2]_a - [\Phi]_a [\Omega^2]_b - 2[\Phi]_b [\Omega]_a [\Omega]_b) - \beta_1 [M]([\Phi]_a [\Omega]_a - [\Phi]_b [\Omega]_b) - [K_0][\Gamma_1]$.

Also the equations for the imaginary-part is

$$[\Delta K][\Gamma_2] = [\Xi_2] \cdots\cdots\cdots\cdots\cdots\cdots (11)$$

where $[\Gamma_2]_{N \times L} = (\beta_2 [\Phi]_b [\Omega]_a + \beta_2 [\Phi]_a [\Omega]_b + [\Phi]_b)$ and $[\Xi_2]_{N \times L} = -\beta_0 ([\Phi]_a [\Omega]_b + [\Phi]_b [\Omega]_a) - [M]([\Phi]_b [\Omega^2]_a - [\Phi]_b [\Omega^2]_b + 2[\Phi]_a$

$[\Omega]_a [\Omega]_b) - \beta_1 [M]([\Phi]_b [\Omega]_a + [\Phi]_a [\Omega]_b) - [K_0][\Gamma_2]$.

Eq.(10) and Eq.(11) can be further expressed as

$$[\Delta K][[\Gamma_1], [\Gamma_2]] = [[\Xi_1], [\Xi_2]] \cdots\cdots\cdots (12)$$

The least-square estimate[11] of $[\Delta K]$ in the sense of trace $\{([\Delta K][[\Gamma_1], [\Gamma_2]] - [[\Xi_1], [\Xi_2]])^T ([\Delta K][[\Gamma_1], [\Gamma_2]] - [[\Xi_1], [\Xi_2]])\}$ to reach the minimum value is

$$[\Delta K] = [[\Xi_1], [\Xi_2]][[\Gamma_1], [\Gamma_2]]^+ \cdots\cdots\cdots (13)$$

where $[[\Gamma_1], [\Gamma_2]]^+$ = the pseudoinverse matrix[11] of $[[\Gamma_1], [\Gamma_2]]$.

The estimated $[\Delta K]$ can be used to detect the location of damage. Differing from the method for undamped and proportional damping problem which lead to a normal (real) mode shape[8], the method in this study for nonproportional damping problem has a better sensitivity to detect the degradation of stiffness. In fact, while there are $N(N+1)/2$ unknown coefficents of $[\Delta K]$ and $N \times L$ equations in Eq.(10) for undamped problem, there are $2 \times N \times L$ equations (see Eq.(12)) and $N(N+1)/2$ unknown coefficents of $[\Delta K]$ in Eq.(13).

3. SOLUTION OF UNKNOWN CO-EFFICIENTS

The percentage of change ratio $\Delta k_{pp}/k_{0pp}$ ($p=1$, 2, $\cdots$, N) for diagonal stiffness coefficients can be used to detect the location of damage. When a node with a remarkable change ratio is detected, each non-zero stiffness coefficient in the column (or row) corresponding to this node in stiffness matrix $[K]$ is multiplied by an unknown coefficient α_k respectively. The total number of α_k is written as NF, thus the $[K]$ is expressed as $[K(\alpha)]$, where $\{\alpha\} = \{\alpha_1, \alpha_2, \cdots, \alpha_{NF}\}^T$. In Eq.(3), there are N equations corresponding to each mode. If all those L measured modes are used, the $L \times N$ equations can be obtained. Substituting the measured $[\Omega^2]$, $[\Phi]$, together with $[M]$, $[C(\alpha)]$ and $[K(\alpha)]$ into Eq.(3), further arranging the $L \times N$ equations, the following new equations are obtained

$$[A]\{\alpha\} = \{B\} \cdots\cdots\cdots\cdots\cdots\cdots (14)$$

where, $[A]$ and $\{B\}$ are known $NF \times NF$ matrix and $NF \times 1$ vector, respectively. After all the unknown coefficients $\{\alpha\}$ are solved, the $[K(\alpha)]$ is the identified result of $[K]$. Therefore, the severity of damage can be identified.

4. NUMERICAL EXAMPLE

A numerical example[9] of a ten-story shear building is shown in **Fig.1**. The structure is described as a FEM model with 11 nodes, 10 elements and 10 degrees of freedoms. Also the
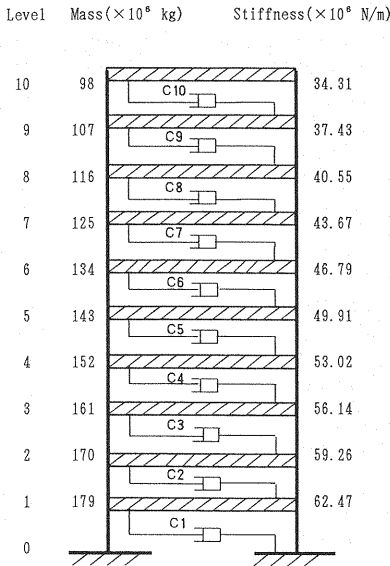
Fig.1 Ten-story shear building

Table 2 Identified coefficents of $[K]$; unit : $10^6 \times N/m$

BEFORE DAMAGE	AFTER DAMAGE		
NODE	EXACT	EXACT	IDENT.	E_{pp}
k_{22}	113.400	98.558	98.558	0.0
k_{23}	−56.140	−44.910	−44.910	0.0
k_{33}	109.160	92.318	92.318	0.0
k_{77}	84.220	72.055	72.055	0.0
k_{78}	−40.550	−32.440	−32.440	0.0
k_{88}	77.980	65.815	65.815	0.0

Fig.2 Damage location detection of this method for the structure with nonproportional damping.
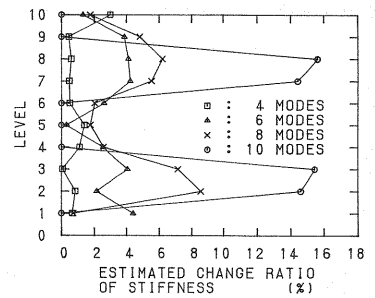
Fig.3 Damage location detection of previous study[3] for undamped structures.

Fig.4 The effect of measurement errors of modal parameters on E_m for the structure with nonproportional damping.

Table 1 Natural frequencies and damping ratios

	BEFORE DAMAGE		AFTER DAMAGE	
N	f_p(Hz)	ξ_p	f_p(Hz)	ξ_p
1	0.500	0.020	0.480	0.054
2	1.326	0.020	1.264	0.037
3	2.151	0.027	2.097	0.035
4	2.934	0.035	2.838	0.051

order of nodes agrees with the numbered levels in the model. The undamped natural frequencies and damping ratios of the structure are listed in **Table 1**. In this example, it is assumed that, a damage occurs in element 3 (level 2-3) and element 8 (level 7-8). Therefore, the coefficents of stiffness in nodes 2, 3, 7 and 8 corresponding to those elements will be degraded. An approximate 15 % minor degradation of stiffness is considered in those nodes. Moreover, referring to the study[5], the coefficents regarding to damping β_0, β_1 and β_2 are taken as the known values in this study, i.e. $\beta_0 = 1$, $\beta_1 = 1.31 \times 10^{-1}$ and $\beta_2 = 5.47 \times 10^{-3}$. For both before and after the damage, the calculated modal parameters are hereby taken as the measured ones. Furthermore, from observing **Fig.2** in which the nodes 2, 3, 7 and 8 are remarkable, the locations of damage are detected by using 1~4 th measured
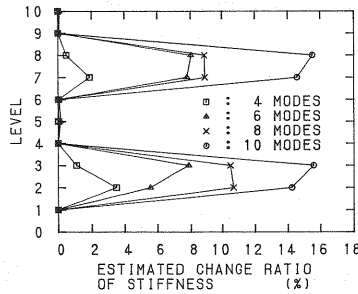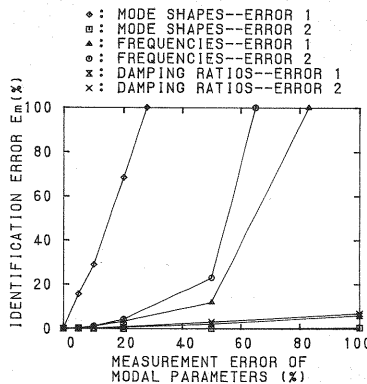
modes. **Fig.3** is the result of the detection of our previous method[3] by using the 1~10 th measured modes. In **Fig.3**, even if 8 modes are used, it seems there are stiffness changes in almost all the nodes. On the contrary, this method is significantly sensitive (see **Fig.2**). By the same way as stated in Section 3, the severity of damage can be identified. A percentage error E_{pp} and a mean error E_m of diagonal stiffness coefficients are defined as

$$E_{pp} = |\{(k_{pp})_{\text{EXACT}} - (k_{pp})_{\text{IDENT.}}\}/(k_{pp})_{\text{EXACT}}|$$
$$\times 100 ;$$
$$E_m = (\sum E_{pp})/ND$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

where the ND is the number of diagonal unknown stiffness coefficients among the NF (the total number of α_k). The calculated E_{pp} and identified result are shown in **Table 2**, in which the identified result is well satisfied. Moreover, the mean error E_m of identification is shown in **Fig.4**. Referring to the report[12], two kinds of measurement errors (Error 1 and Error 2) for all the measured modal parameters (natural frequencies, mode shapes and

damping ratios) are assumed in this study. The Error 1 is defined : Each measured value varies with a same percentage of the value. The Error 2 for mode shape is defined : If a percentage of the measured value is increased/or decreased in one node, the same percentage is decreased/or increased in the neighbour nodes. Also the Error 2 for natural frequency or damping ratio is defined : If a percentage of the measured value is increased/or decreased in a certain order, the same percentage is decreased/or increased in the neighbour order. The calculated results for the two kinds of errors show that the measurement error of mode shape has a strong effect on identification accuracy. On the other hand, damping ratio has a comparatively weak one. However, as for the measurement error of modal parameters on damage location detection, in this example, the detected locations remain unchanged within 20 % measurement error of modes.

5. CONCLUSIONS

(1) This study presents a modal analysis method which can be applied to nonproportional damping. By this method, both the location and severity of damage can be identified.

(2) The method for nonproportional damping problem may have better sensitivity than the undamped or proportional one.

(3) The measurement error of mode shape has a strong effect on identification accuracy. On the contrary, that of damping ratio has a comparatively weak one. As for the measurement error of modal parameters on damage location detection for nonproportional damping, the detected locations remain unchanged within a certain range of measurement error of modes.

Moreover, the calculated results show that a sensitivity for detecting the location of damage is not affected by β_0 ($\beta_0 \neq 0$, see Eq.(9)). Therefore, considering the conclusion (2), the sensitivity may be significantly improved by treating a damped structure (usually with Rayleigh damping) to be a nonproportionally damped one which is close to the structure with Rayleigh damping, i.e., $\beta_0 \doteq 0$.

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