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[DISCUSSION/CLOSURE]

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STIFFNESS EVALUATION OF SUBMERGED FLOATING FOUNDATION MOORED BY FOUR CABLES

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▶Discussion

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The idea of a mooring foundation seems very hopeful in the future. The writer is especially interested in the authers' treatment of the buoyancy acting on a long cable. At the same time, the writer wishes some questions on that point be clarified.

For a simplified discussion, let the followings be assumed : a completely flexible cable is deformed in a vertical plane $\{x, z\}$; and its extension and volume change by the tensile force and the hydraulic pressure are negligible.

In case a cable element ds is isolated in the water, the Archimedes' principle gives us the buoyant force

$$\boldsymbol{b}=A\gamma\{0, 1\} \cdots\cdots (12)$$

where $\boldsymbol{b}$ is defined per unit length; and brace { } means components into $\{x, z\}$. As well known, this buoyancy is a resultant of the hydraulic pressure acting on the two cross sections and the side surface. If element ds has no curvature as shown in **Fig.15**, the cross-sectional pressures are given by

$$\boldsymbol{P}_A=A\gamma h\{\cos\phi, \sin\phi\},$$

$$\boldsymbol{P}_B=-A\gamma(h+dh)\{\cos\phi, \sin\phi\} \cdots\cdots (13)$$

where h is the depth of water; and ϕ is angle of line ds from the x-direction. Then, the pressure on the side surface per unit length is derived as follows :

$$\boldsymbol{p}=\frac{\boldsymbol{b}ds-(\boldsymbol{P}_A+\boldsymbol{P}_B)}{ds}$$

$$=A\gamma\{-\cos\phi\sin\phi, \cos^2\phi\} \cdots\cdots (14)$$

Under the former assumptions, this result leads to the authors' Eqs.(1.a-c).

Instead of the usual $\boldsymbol{b}$, the authors employed side force $\boldsymbol{p}$ as an alternative buoyancy to describe the equilibrium state. But, is this expression itself correct for a general element ds? For instance, consider a (rigid) ring of radius R in the water (**Fig.16**). By the integration of (14), we have

$$\int \boldsymbol{p}ds=A\gamma\int_0^{2\pi}\{-\cos\phi\sin\phi, \cos^2\phi\}Rd\phi$$

$$=\cdots=\{0, \pi RA\gamma\} \cdots\cdots (15)$$

This result does not agree with the usual buoyancy: $2\pi RA\gamma$ into z.

In case of an element ds having curvature κ, let

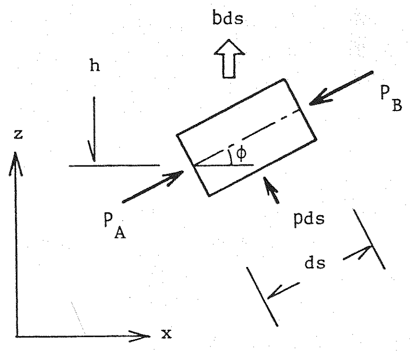

Fig.15 Straight cable element

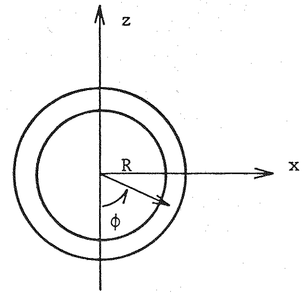

Fig.16 Circular ring

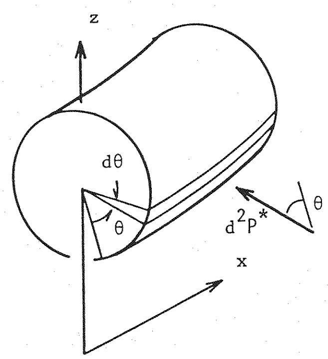

Fig.17 Curved cable element

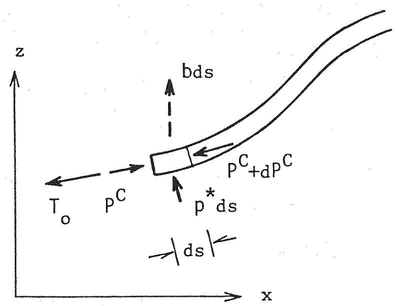

Fig.18 Forces at cable end

the writer develop the hydrostatic forces, more precisely. Since the cross-sectional pressure at s is written as

$$\boldsymbol{P}^C = A\gamma h(s)\{\cos\phi(s), \sin\phi(s)\} \cdots\cdots\cdots (16)$$

its change for distance ds is given by the differential

$$d\boldsymbol{P}^C = A\gamma\frac{dh}{ds}\begin{Bmatrix}\cos\phi\\ \sin\phi\end{Bmatrix}ds + A\gamma h\begin{Bmatrix}-\sin\phi\\ \cos\phi\end{Bmatrix}\frac{d\phi}{ds}ds$$

$$= A\gamma\begin{Bmatrix}-\cos\phi\sin\phi - \kappa h\sin\phi\\ -\sin^2\phi + \kappa h\cos\phi\end{Bmatrix}ds \cdots\cdots (17)$$

where relations $dh/ds = -\sin\phi$ and $d\phi/ds = \kappa$ have been adopted.

Next, we consider the hydrostatic pressure on the cylindrical side surface. Angular coordinate θ is introduced in the cross section from the intersection line with the $\{x, z\}$ plane, as shown in **Fig.17**. Since the fiber length at θ is $ds^* = (1 + \kappa r\cos\theta)ds$ with r being the radius of corss-section, the pressure on area element $rd\theta \times ds^*$ is given by

$$d^2P^* = rd\theta\cdot(1 + \kappa r\cos\theta)ds\cdot(h + r\cos\phi\cos\theta)\gamma \cdots\cdots (18)$$

Apparently, the pressure around the cylindrical surface is in a self balance into the y-direction. By the integration of its projection onto the $\{x, z\}$-plane, $d^2P^*\cos\theta$, we have the magnitude

$$dP_n^* = \int d^2P^*\cos\theta = \cdots$$

$$= A\gamma(h\kappa + \cos\phi)ds \cdots\cdots\cdots (19)$$

This force acts into normal to line ds in the $\{x, z\}$ plane. By the resolution into the spatial $\{x, z\}$, we have th pressure force per unit length :

$$\boldsymbol{p}^*\left(=\frac{d\boldsymbol{P}_n^*}{ds}\right) = A\gamma(h(s)\kappa(s) + \cos\phi)\begin{Bmatrix}-\sin\phi\\ \cos\phi\end{Bmatrix} \cdots\cdots\cdots (20)$$

The buoyancy of the Archimedes' principle is obtained by the sum of the former pressure forces:

$$\boldsymbol{P}_A + \boldsymbol{P}_B + \boldsymbol{p}^*ds = \boldsymbol{P}^C - (\boldsymbol{P}^C + d\boldsymbol{P}^C) + \boldsymbol{p}^*ds$$

$$= \cdots\cdots = A\gamma\, ds\begin{Bmatrix}0\\ 1\end{Bmatrix} \cdots\cdots\cdots (21)$$

In a usual mooring system, the radius of cable curvature is comparative to the depth of water. The hydrostatic pressure on the side surface is estimated by $\boldsymbol{p}^*$ of (20), instead of by $\boldsymbol{p}$ of (14).

We now consider element ds in a long cable anchored on the bottom of water. At the bottom end (see **Fig.18**), together with the anchor force $\boldsymbol{T}_0$, the hydrostatic forces act: $\boldsymbol{P}^C$ of (16) on the end cross-section, and $\boldsymbol{p}^*$ of (20) on the side surface. By the statics (21) on any cable configuration, the effect of $\boldsymbol{P}^C$ and $\boldsymbol{p}^*$ is equivalent to the change $d\boldsymbol{P}^C$ by (17) of cross-sectional pressure and the buoyancy $A\gamma ds$ into z, and so on along the cable length. Then, either under the side pressure $\boldsymbol{p}^*$ and the total end force $\boldsymbol{T}_0 + \boldsymbol{P}^C|_{s=0}$, or under the buoyancy $\boldsymbol{b}$ and the anchor force $\boldsymbol{T}_0$, a mooring cable can be analyzed. But, the writer suppose the latter method is more effective in our usual engineering. In this treatment, the hydrostatic $h(s)\gamma$ in the material is an additional force which has no effect on the behavior of cable. It is only the buoyancy given by (12) to be taken into account. The simple expression $A\gamma\{0, 1\}$ for the buoyancy is not affected by any other factors such as the configuration of cable. This is why the Archimedes' buoyancy exists as a principle. In the former treatment, the expression (20) for the side pressure $\boldsymbol{p}^*$ is relatively complicated, containing direction angle ϕ and curvature κ. The dealing with the higher derivative κ can cause a numerical error in the actual analysis. Even, in case there exist the elongation of cable and the change of cross-section area, the circumstances do not change in principle : it is enough for the usual buoyancy to be defined per unit volume after the deformation.

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The authors greatly appreciate the important comments and indication of limitations in our paper by Dr. Ai. Judging from the period of review of this discussion because the authors received it in the middle of January, we felt it was a very delicate problem on buoyancy of cables.

The authors accept the writer's indications and must admit that the side pressure must be replaced by Eq. (20) of the writer. Since the authors' original governing equations exactly hold for the straight cables and since the cables in this particular problems are almost straight and the total buoyancy of cables is much smaller than the total cable tension and buoyancy of the floating body in any configuration, the conclusions on the resisting behavior and usefulness of the submerged floating foundations do not change.

As has been pointed out by the writer, the authors have neglected the effect of curvature of cables in estimating buoyancy because it has been assumed to be in the order of strains just like the formulation of a finite displacement beam theory in small strains. But the writer has shown that the accumulation of such errors due to the curvature leads to a significant error of the results of long cables.

Furthermore the writer has proved that the precise extimate of side pressure leads to correctness of usage of under-water weight of cables. Therefore it has been proved that the equilibrium equations using under-water weight yield exact solutions provided the final cable tension is evaluated by subtracting the total hydrostatic pressure acting on that cross section.

Finally again the authors thank Dr. Ai for the discussion to our paper.

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