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SPATIAL VARIATION OF SEISMIC GROUND MOTION MODELED BY FK SPECTRUM

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Using a Fourier transform and coordinate transform, the space-time cross-spectral density function, which is often used to represent spatially varying seismic motion, is derived from a general frequency-wavenumber power spectrum model. Although the existence of a dominant wave component is not assumed, the resultant cross-spectrum model becomes the same as the one proposed by Loh. Hence the Loh's model is found to be applicable to express general stochastic wave fields.
Key Words : *seismic ground motion, spatial variation, FK spectrum, cross-spectral density function, coordinate transform.*

1. INTRODUCTION

Ground motion observed during strong earthquakes varies both in time and space. Spatial variation of seismic ground motion is one of the important issues to consider in the seismic design of spatially extended structures. To express such spatially varying seismic motion, the space-time cross-spectral density function is often utilized. Loh[1] and Harichandran and Vanmarcke[2] proposed models of the cross-spectral density function based on the recorded accelerograms from the SMART-1 array in Taiwan. Both models assume the existence of a dominant wave having a propagation direction and velocity.

The frequency-wavenumber (FK) power spectrum is an alternative way to represent spatial variation of seismic ground motion. Because of the Wiener-Khintchine relationship between the cross-spectral density function and the FK spectrum, it is actually equivalent to use either expression. An advantage of using the FK spectrum lies in the fact that it can be directly employed in the simulation of stochastic waves[3),4)] Hence, analytical modeling of the FK spectrum[5),6)] is also very useful.

This paper aims to demonstrate the meaning of the cross-spectrum model proposed by Loh from the viewpoint of the FK spectrum. In this stage, the model is shown to be applicable even to cases where no dominant wave exists.

2. MODELING OF FREQUENCY-WAVENUMBER POWER SPECTRUM

The frequency-wavenumber (FK) power spectrum can fully describe the second-order characteristics of stochastic seismic waves. Consider that the FK power spectrum model is given by some theoretical considerations or by dense array observations. Note that, however, the direct modeling of the FK spectrum from the observed one is not so common because the observed FK spectrum is sensitive to the weighting scheme and the layout of the array.

Assuming a space-time process is stationary in time, homogeneous in space, and ergodic in both, three-dimensional (3D) FK spectral density function $S(\boldsymbol{k}, f)$ is considered for wavenumber vector $\boldsymbol{k} = [k_x, k_y]^T$:

$$S(\boldsymbol{k}, f) = S_0(f) A(\boldsymbol{k}|f) \quad (1)$$

where $S_0(f)$ is the point power spectral density function and $A(\boldsymbol{k}|f)$ is the normalized conditional wavenumber spectrum at frequency f. From the definitions of the FK spectrum and the power spectrum, the variance σ^2 of the 3D process is given by the integration of $S(\boldsymbol{k}, f)$ over $\boldsymbol{k}$ and f and by the integration of $S_0(f)$ over f. Thus, the integration of $A(\boldsymbol{k}|f)$ over $\boldsymbol{k}$ becomes unity regardless of frequency f. The plots of this wavenumber spectrum A on $k_x - k_y$ plane are often shown for array records and referred to as the FK spectrum.

To model this wavenumber spectrum in the two-dimensional wavenumber space, two more coordinate systems shown in **Fig.1** are introduced for convenience. The first transformed coordinate system $\boldsymbol{k}^* = [k_x^*, k_y^*]^T$ can be obtained by the rotation of the original coordinate $\boldsymbol{k}$ by angle φ_0 :

$$\boldsymbol{k}^* = \boldsymbol{T}_\varphi \boldsymbol{k} \quad (2)$$

where $\boldsymbol{T}_\varphi$ is the two-dimensional coordinate rotation matrix. The second coordinate transform is the shift of the origin from point O to point P in **Fig.1**. The new coordinate system $\boldsymbol{k}' = [k_x', k_y']^T$ is represented by

$$\boldsymbol{k}' = \boldsymbol{k}^* - \boldsymbol{k}_0^* = \boldsymbol{T}_\varphi (\boldsymbol{k} - \boldsymbol{k}_0) \quad (3)$$

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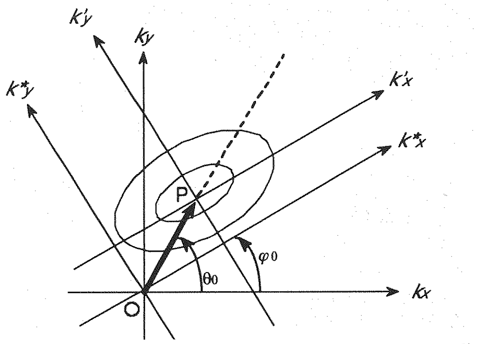


Fig.1 Wavenumber Spectrum Model and Three Coordinate Systems

where $\boldsymbol{k}_0$ and $\boldsymbol{k}_0^*$ denote the position of the origin P of $\boldsymbol{k}'$ coordinate in terms of the respective coordinates.

The centroid of the 2D wavenumber spectrum $A(\boldsymbol{k}|f)$ usually locates in some distance from the origin O. Hence, the model can be expressed more simply in the $\boldsymbol{k}'$ coordinate than in $\boldsymbol{k}$ coordinate as $A(\boldsymbol{k}'|f)$. The centroid of the wavenumber spectrum model is not necessarily a dominant peak. A rather flat shape of the spectrum is acceptable. This point simply represents a wave component having apparent velocity c_0 and azimuthal angle θ_0 as follows[7]:

$$|\boldsymbol{k}_0|=|\boldsymbol{k}_0^*|=\frac{f}{c_0}\ ;\ \theta_0=\tan^{-1}\left(\frac{k_{y0}}{k_{x0}}\right) \cdots\cdots\cdots\cdots\cdots(4)$$

Hence, this model can involve various wave components having different apparent velocities and azimuthal angles at frequency f. Note also that both c_0 and θ_0 can be functions of frequency.

As depicted in Fig.1, the two angles, φ_0 and θ_0, may be different. In such cases, however, the direction of wave propagation and the principal axis of wave coherency become different. Since it is difficult to explain such a state for a plane wave in homogeneous soil media, $\varphi_0=\theta_0$ may be assumed in modeling.

3. SPACE-TIME CROSS-SPECTRAL DENSITY FUNCTION DERIVED FROM FK SPECTRUM

The frequency-wavenumber power spectrum can be directly applicable to the simulation of stochastic waves. However, equivalent but different representations of the second-order statistics are also often utilized[8]. Because the three-dimensional FK spectrum, $S(\boldsymbol{k},f)$, is a function of two wavenumbers and frequency, several versions of its Fourier transform exist. If a two-fold Fourier transform with respect to the wavenumbers is taken, the space-time cross-spectral density function, $C(\xi,f)$, can be obtained:

$$C(\xi,f)=\int_k S(\boldsymbol{k},f)\exp[2\pi i\boldsymbol{\xi}\cdot\boldsymbol{k}]d\boldsymbol{k} \cdots\cdots(5)$$

where $\xi=[\xi_x,\ \xi_y]^T$ is the separation vector. The cross-spectral density function $C(\xi,f)$ is generally complex. Note that when the FK spectrum $S(\boldsymbol{k},f)$ is quadrant symmetric with respect to wavenumber axes, k_x and k_y, $C(\xi,f)$ becomes real. But that is an impractical assumption when propagating seismic waves are considered.

For the FK spectrum model defined by Eq.1, the analytical Fourier transform can be carried out. To do this, $\boldsymbol{k}^*$ coordinate in Fig.1 is used instead of $\boldsymbol{k}$ coordinate. The corresponding space lag vector is also replaced by $\xi^*=[\xi_x^*,\ \xi_y^*]^T$, which can be obtained by the same coordinate rotation in Eq.2. It is rather obvious that Eq.5 is also valid for other Cartesian coordinates. Thus, the following relationship exists for the set of wavenumber vector $\boldsymbol{k}^*$ and space lag vector ξ^*:

$$C(\xi^*,f)=\int_{k'} S(\boldsymbol{k}^*,f)\exp[2\pi i\xi^*\cdot\boldsymbol{k}^*]d\boldsymbol{k}^* \cdots\cdots\cdots\cdots(6)$$

The coordinate translation given by Eq.3 is then introduced in Eq.6. Replacing $\boldsymbol{k}^*$ by $\boldsymbol{k}'+\boldsymbol{k}_0^*$ and considering Eq.1, $\xi'=\xi^*$ and $d\boldsymbol{k}'=d\boldsymbol{k}^*$ give:

$$\begin{aligned}C(\xi^*,f)&=\int_{k'} S(\boldsymbol{k}^*,f)\exp[2\pi i\xi^*\cdot(\boldsymbol{k}'+\boldsymbol{k}_0^*)]d\boldsymbol{k}^*\\&=S_0(f)\int_{k'} A(\boldsymbol{k}'|f)\exp[2\pi i\xi'\cdot\boldsymbol{k}']d\boldsymbol{k}'\\&\quad\times\exp[2\pi i\xi'\cdot\boldsymbol{k}_0^*] \cdots\cdots\cdots\cdots(7)\end{aligned}$$

The integral on the left side of Eq.7 is the two-fold Fourier transform of $A(\boldsymbol{k}'|f)$. Hence, the following frequency-dependent spatial correlation function, γ, can be obtained:

$$\gamma(\xi'|f)=\int_{k'} A(\boldsymbol{k}'|f)\exp[2\pi i\xi'\cdot\boldsymbol{k}']d\boldsymbol{k}' \cdots(8)$$

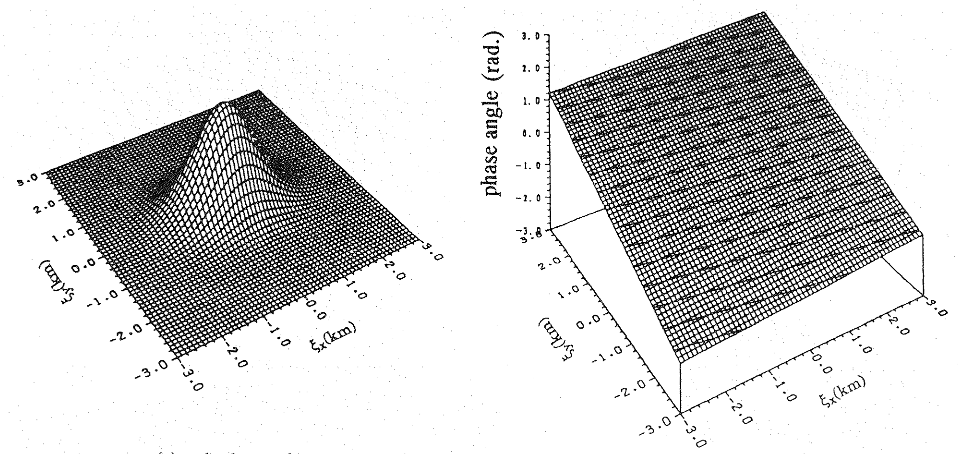
This integration becomes a real value if A is quadrant symmetric with respect to k_x' and k_y' axes. The last term on the left side of Eq.7 is a complex number which reflects the phase difference between two points with the separation ξ'. Considering the relationship given by Eq.4, this term can be rewritten as an ordinary plane wave equation having speed c_0:

$$\exp[2\pi i\xi'\cdot\boldsymbol{k}_0^*]=\exp\left[2\pi fi\left(\frac{\xi'\cdot\boldsymbol{a}_0}{c_0}\right)\right] \cdots\cdots(9)$$

where $\boldsymbol{a}_0$ is the directional cosine vector of $\boldsymbol{k}_0^*$ and indicates the wave propagation direction at the centroid of the wavenumber spectrum. Note again that both $\boldsymbol{a}_0$ and c_0 can be frequency-dependent.

Substituting Eqs.8 and 9 in Eq.7 gives:

$$C(\xi',f)=S_0(f)\gamma(\xi'|f)\exp\left[2\pi fi\left(\frac{\xi'\cdot\boldsymbol{a}_0}{c_0}\right)\right] \cdots\cdots\cdots(10)$$



(a) absolute value (b) phase angle

Fig.2 Absolute Value and Phase Angle of the Space-time Cross-spectral Density Function Model at a Fixed Frequency

This equation is actually the same as the one proposed by Loh[1]. But in this study, the existence of dominant waves was not assumed. It was simply derived through the coordinate transform and Fourier transform of the FK spectrum. Hence, Equation 10 can be valid in general wave fields having various wave components in one frequency ; $\gamma(\xi'|f)$ indicates the spatial wave coherency and the exponential term the phase delay due to wave propagation. It is interesting to observe that the wave coherency is still separable from the wave propagation direction and velocity for general wave fields.

4. EXAMPLE OF CROSS-SPECTRAL DENSITY FUNCTION FROM FK SPECTRUM

Consider that the wavenumber spectrum is modeled by the following anisotropic exponential function using the $\boldsymbol{k}'$ coordinate :

$$A(\boldsymbol{k}'|f)=\frac{\alpha_x\alpha_y}{\pi}\exp\left[-(\alpha_x k_x')^2-(\alpha_y k_y')^2\right] \qquad (11)$$

where α_x and α_y are parameters with the dimension of length and are expressed as functions of frequency. For a fixed frequency, this model has an ellipsoidal correlation structure. The two parameters control the shape of the wavenumber spectrum at each frequency. The larger they become, the flatter the wavenumber spectrum becomes, which means less spatial coherency.

The corresponding cross-spectral density function is obtained by the analytical Fourier transform as follows :

$$\gamma(\xi'|f)=\exp\left[-\left(\frac{2\xi_x'}{\alpha_x(f)}\right)^2-\left(\frac{2\xi_y'}{\alpha_y(f)}\right)^2\right] \qquad (12)$$

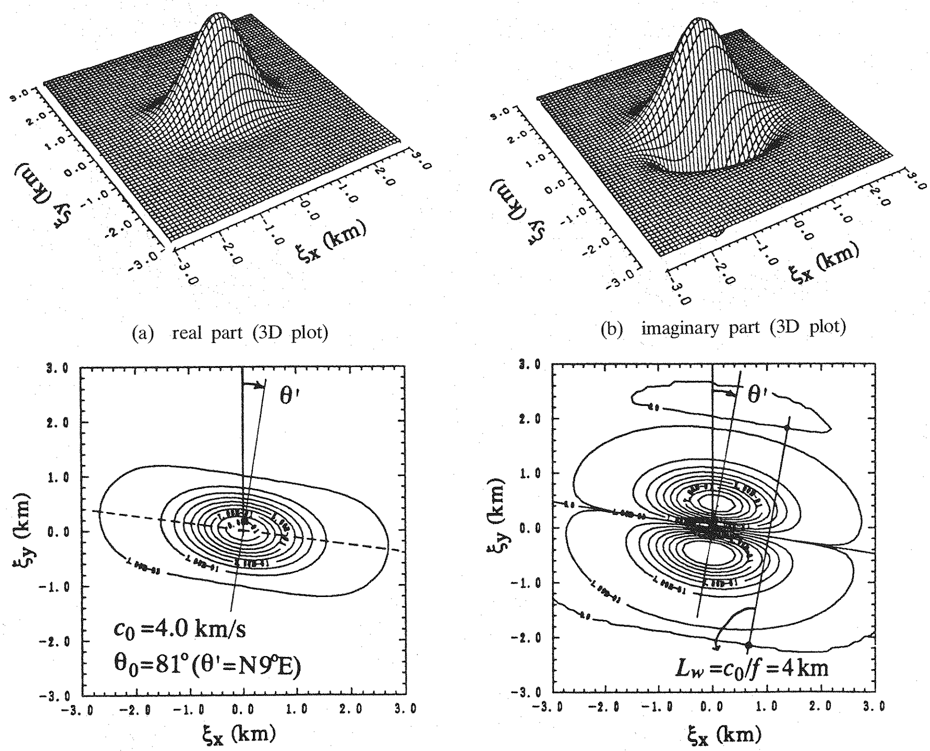
This function is real and symmetric with respect to the two separation distances.

At a fixed frequency, the cross-spectral density function, $C(\xi,f)$, is calculated for unit $S_0(f)$ value and assumed apparent velocity c_0 and azimuthal angle θ_0. **Fig.2** shows the absolute value and phase angle of $C(\xi,f)$ while **Fig.3** plots the real and imaginary parts of the same value. In both figures, the target azimuthal direction can be observed. The apparent wavelength ($L_w=1/|\boldsymbol{k}_0|=c_0/f$) can also be observed as the distance corresponding to the phase difference of 2π in **Fig.2**(b) or as the separation between the zero-crossing lines in **Fig.3**(d).

It is very interesting that although the original wavenumber spectrum contains seismic waves of various propagation directions and velocities at one frequency, the phase of the cross-spectral density function is that of the plane wave with constant velocity for the frequency. This fact supports the assumption of the existence of a dominant wave component for each frequency so long as an appropriate spatial coherency is considered.

5. CONCLUSION

This paper highlighted an analytical derivation of the space-time cross-spectral density function from the frequency-wavenumber (FK) power spectrum. Applying a two-step coordinate transform and a two-dimensional Fourier transform to an assumed general FK spectrum model, the corresponding cross-spectral density function was analytically derived. Although the existence of a dominant wave component for each frequency was not assumed, the resultant cross-spectral density function became the same expression as the one proposed by Loh. This fact validates the applicability of the Loh's model to general wave fields. Since the FK spectrum and the cross-spectral density function constitute the Wiener-Khintchine



(a) real part (3D plot) (b) imaginary part (3D plot)

(c) real part (2D plot) (d) imaginary part (2D plot)

Fig.3 Real and Imaginary Parts of the Space-time Cross-spectral Density Function Model at a Fixed Frequency

pair, such a cross-spectral density function model can be easily applied for the simulation of stochastic waves. A numerical example visually explained the meaning of the complex cross-spectral density function corresponding a wavenumber spectrum model.

REFERENCES

1) Loh, C.H. : Analysis of the Spatial Variation of Seismic Waves and Ground Movements from SMART-1 Array Data, *Earthquake Eng. and Struct. Dyn.*, Vol.13, pp.561～581, 1985.

2) Harichandran, R.S. and Vanmarcke, E.H. : Stochastic Variation of Earthquake Ground Motion in Space and Time, *J. Engrg. Mech.*, ASCE, Vol.112, No.2, pp.154～174, 198 .

3) Deodatis, G., Shinozuka, M. and Papageorgiou, A. : Stochastic Wave Representation of Seismic Ground Motion. II : Simulation, *J. Engrg. Mech.*, ASCE, Vol.116, No.11, pp.2381～2399, 1990.

4) Tüker, T., Yamazaki, F. and Katayama, T. Simulation of Earthquake Ground Motion Based on Frequency-Wave-number Spectrum, *Transaction of the 11th SMiRT,* K1, pp.21～26, 1991.

5) Deodatis, G., Shinozuka, M. and Papageorgiou, A. : Stochastic Wave Representation of Seismic Ground Motion. I : F-K Spectra, *J. Engrg. Mech.*, ASCE, Vol.116, No.11, pp.2363～2379, 1990.

6) Harada, T. : Seismic Response of Stochastic Ground, *Computational Stochastic Mechanics,* P.D. Spanos and C.A. Brebbia Eds., Computational Mechanics Publications & Elsevier Applied Science, pp.649～660, 1991.

7) Aki, K. and Richards, P.G. : Quantitative Seismology : Theory and Methods, Volume II, W.H. Freeman and Company, San Francisco, CA, 1980.

8) Vanmarcke, E. : *Random Fields : Analysis and synthesis,* MIT Press, Cambridge, MA, 1983.

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