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BLOCK-DIAGONALIZATION METHOD FOR SYMMETRIC STRUCTURES WITH ROTATIONAL DISPLACEMENTS

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The group-representation theory guarantees that the (tangent) stiffness matrix of symmetric structures can be put into a block-diagonal form by means of a suitable (local) geometric transformation. This transformation decomposes the linear equilibrium equation of symmetric structures into a number of independent equations, and hence is advantageous for parallel analysis. The block-diagonalization method, which so far has mainly been applied for translational displacements, is extended here to rotational ones. The interrelationship between the symmetries of rotational and translational displacements is investigated by means of group theory to arrive at the transformation matrix of rotational ones.

Key Words : *block diagonalization, dihedral group, group-representation theory, orbit, parallel analysis, symmetry, rotational displacement*

1. INTRODUCTION

Geometric symmetry has extensively been exploited in structural analysis of symmetric structures. For example, only a part of a symmetric structure is cut out and analyzed to reduce the degrees of freedom involved. It is customary to impose the axisymmetry in shell analysis. The description and use of symmetry, however, are currently done in a semi-empirical manner.

A systematic strategy for exploiting symmetry has already been established in many other fields of physical science and engineering. For the description of symmetry (e.g., the structure of the crystal lattice etc.[2)]), it is standard to use groups[1)] that consist of rotational and reflectional transformation. The block-diagonalization method is used to decompose the governing equation of a symmetric system into a number of independent equations by means of a suitable transformation[3)]. This method is systematically and completely described by means of the group-representation theory in the field of applied mathematics[4),5),6)].

The method has come to be put to use also in the field of structural engineering [7) ~18)]. Since the (tangent) stiffness matrix of symmetric structures can be transformed into a block-diagonal form by means of a suitable local geometric transformation, their equilibrium equations can be decomposed into a number of independent equations. This method, therefore, is suitable for parallel computation, and hence can improve the computational efficiency and decrease the array capacity. The method, however, has a problem regarding the compatibility with the finite element method.

With the use of the concept of the orbit, a systematic method to compute the transformation matrix compatibly with the finite element method[14),17),18)] has been presented for translational displacements. The extension to the rotational displacements, however, is a pressing need to make the block-diagonalization method applicable to general symmetric structures.

This paper offers a block-diagonalization method for generalized displacements. We investigate the interrelationships of rotational and translational displacements by means of the group-representation theory to arrive at the transformation matrix for rotational ones. The method has thus been made accessible for general symmetric structures.

2. BLOCK DIAGONALIZATION

This chapter introduces a method for describing the geometrical symmetry of structures as a summary of previous papers[14),17),18)].

(1) Equivariance of equilibrium equation

Denote by $U(\boldsymbol{f}, \boldsymbol{u})$ the total potential energy function [1] of a discretized system, where $\boldsymbol{f}$ stands for the load pattern vector and $\boldsymbol{u}$ for the displacement vector, respectively. The (N-dimensional) equilibrium equation of this system becomes:

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[1] Although the following discussion is applicable for non-potential systems, the total potential is used here to make the discussion more comprehensive.

$$\left(\frac{\partial U}{\partial \boldsymbol{u}}\right)^{\mathrm{T}} \equiv \boldsymbol{F}(\boldsymbol{f},\boldsymbol{u}) = \boldsymbol{0} \quad \cdots\cdots (1)$$

In describing the symmetry of the equilibrium equation we consider a group G composed of geometric transformation g (such as reflections and rotations). It is assumed that when an element g of G acts on an N-dimensional vector $\boldsymbol{u}$ (respectively, $\boldsymbol{F}$), $\boldsymbol{u}$ is transformed into $g(\boldsymbol{u})$ (respectively, $\boldsymbol{F}$ into $g(\boldsymbol{F})$). An N-dimensional ($N \times N$) representation matrix $T(g)$ that describes the action of g on the corresponding vector space is defined by

$$T(g)\boldsymbol{u} = g(\boldsymbol{u}), \quad T(g)\boldsymbol{F} = g(\boldsymbol{F}), \quad g \in G \quad \cdots\cdots (2)$$

The representation matrix of the load vector [2] $\boldsymbol{f}$ is defined by

$$\tilde{T}(g)\boldsymbol{f} = g(\boldsymbol{f}), \quad g \in G \quad \cdots\cdots (3)$$

The representation matrices $T(g)$ and $\tilde{T}(g)$ are assumed to be orthogonal.

The symmetry of this system is expressed in terms of the invariance of the total potential energy U with respect to the transformation by all elements g of group G. U is called invariant with respect to G, when

$$U(\tilde{T}(g)\boldsymbol{f}, T(g)\boldsymbol{u}) = U(\boldsymbol{f},\boldsymbol{u}), \quad g \in G \quad \cdots\cdots (4)$$

is satisfied. Such invariance is inherited to $\boldsymbol{F}$ such that

$$T(g)\boldsymbol{F}(\boldsymbol{f},\boldsymbol{u}) = \boldsymbol{F}(\tilde{T}(g)\boldsymbol{f}, T(g)\boldsymbol{u}), \quad g \in G \quad \cdots (5)$$

Eq. (5) is a general symmetry condition applicable for non-potential systems, and is called the equivariance of $\boldsymbol{F}$ to G. **Eq.** (5) means that the transforming of independent variables $\boldsymbol{f}$ and $\boldsymbol{u}$ respectively by $\tilde{T}(g)$ and $T(g)$ is the same as the transforming of the whole equation $\boldsymbol{F}$ by $T(g)$.

We consider a linear equilibrium equation [3]

$$\boldsymbol{F} \equiv K\boldsymbol{u} - \boldsymbol{f} = \boldsymbol{0} \quad \cdots\cdots (6)$$

that satisfies the symmetry condition (5). By virtue of this condition, the linear stiffness matrix K in (6) satisfies the symmetry condition

$$T(g)K = KT(g), \quad g \in G \quad \cdots\cdots (7)$$

and hence can be block-diagonalized by means of a suitable geometric transformation.

The linear equilibrium equation equivariant to some group is known to be transformed into a set of independent equations corresponding to the irreducible representations of the group. The forms of the transformation matrix and block-diagonal matrix vary with individual groups.

Define by

$$T^{\mu}(g) = T_i^{\mu}(g), \ i = 1,\cdots,a^{\mu}, \ g \in G, \ \mu \in R(G) \quad (8)$$

the irreducible representation matrices of the group G, which do not depend on the structure but only on G. Here μ indicates the irreducible representation of G, $R(G)$ denotes the whole set of irreducible representations, and a^{μ} is the multiplicity of the irreducible representation μ in the representation matrix $T(g)$, being given by

$$a^{\mu} = \frac{1}{|G|}\sum_{g\in G}\chi(g)\overline{\chi^{\mu}(g)}, \quad \mu \in R(G) \quad \cdots\cdots (9)$$

Here $(\overline{\cdots})$ denotes the complex conjugate and $\chi(g)$ stands for the character (the sum of the diagonal components) of $T(g)$and $\chi^{\mu}(g)$ for that of $T(g)$, respectively. It is a basic concept of the "group-representation theory" to formulate general principles for the irreducible representation matrices $T^{\mu}(g)$, and in turn to obtain representation matrix $T(g)$ and transformation matrix H of each structure. This corresponds to obtaining H such that

$$H^{\mathrm{T}}T(g)H = \bigoplus_{\mu\in R(G)}\bigoplus_{i=1}^{a^{\mu}} T_i^{\mu}(g), \quad g \in G \quad \cdots\cdots (10)$$

(2) Block diagonalization for dihedral group

In this paper we focus on the dihedral group $G = D_n$, which represents the symmetry of a regular-polygon (n-gon), though the block-diagonalization is a general principle approved to hold for an arbitrary group. The dihedral group of degree n is defined by

$$D_n \equiv \{1, r, \cdots, r^{n-1}, s, sr, \cdots, sr^{n-1}\} \quad \cdots\cdots (11)$$

with $r^n = s^2 = (sr)^2 = 1$. Here 1 is the identity element that leaves everything unchanged, the element s stands for the reflection with respect to the XZ-plane, and r^j for the counter-clockwise rotation around the Z-axis at an angle of $2j\pi/n$ ($j = 1,\cdots,n-1$). Subgroups of D_n are defined by

$$D_m^j \equiv \{r^{kn/m}, sr^{kn/m+j-1} \mid k = 0,\cdots,m-1\} \quad (12)$$

$$C_m \equiv \{r^{kn/m} \mid k = 0,\cdots,m-1\} \quad \cdots\cdots (13)$$

Here $D_m = D_m^1$ and $C_1 = \{1\}$; $m = \gcd(j,n)$ shows the greatest common divisor of j and n. The dihedral groups D_m^j of order m denote regular m-gonal symmetries and the cyclic groups C_m indicate the rotational symmetry with respect to an angle of $2\pi/m$. Deformation patterns of D_n-invariant structures are expressed by these subgroups.

Denote by

$$R(D_n) = \{\mu \equiv (d,j) \mid j = 1,\cdots,m_d; d = 1,2\} \quad (14)$$

the whole set of irreducible representations of D_n (refer to Murota · Ikeda[14]) for details of notations). Here d denotes the degree of the irreducible representation $\mu = (d,j)$, j means the jth irreducible representation of degree d, and m_d is the number of non-equivalent d-dimensional irreducible representations, being given by

[2] In general, the representation matrix of $\boldsymbol{f}$ is different from that of $\boldsymbol{u}$ and $\boldsymbol{F}$ because the dimension of $\boldsymbol{f}$ in general is different from that of $\boldsymbol{u}$ and $\boldsymbol{F}$.

[3] Since the symmetry condition (7) holds also for the tangent stiffness matrix of nonlinear problems, the results of the present paper are applicable also for these problems.

$$\begin{cases} m_1 = 4, & m_2 = n/2 - 1, \quad \text{when } n = \text{even} \\ m_1 = 2, & m_2 = (n-1)/2, \quad \text{when } n = \text{odd} \end{cases} \tag{15}$$

The one-dimensional irreducible representation matrices of D_n are uniquely determined as

$$\begin{array}{ll} T^{(1,1)}(r) = 1, & T^{(1,1)}(s) = 1 \\ T^{(1,2)}(r) = 1, & T^{(1,2)}(s) = -1 \\ T^{(1,3)}(r) = -1, & T^{(1,3)}(s) = 1 \\ T^{(1,4)}(r) = -1, & T^{(1,4)}(s) = -1 \end{array} \tag{16}$$

The two-dimensional ones, which are not unique, are chosen to be

$$T^{(2,j)}(r) = \begin{pmatrix} \cos(2\pi j/n) & -\sin(2\pi j/n) \\ \sin(2\pi j/n) & \cos(2\pi j/n) \end{pmatrix}, \tag{17}$$

$$T^{(2,j)}(s) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{18}$$

according to Murota・Ikeda[14].

The character is given by

$$\chi^{(2,j)}(r) = 2\cos(2\pi j/n), \quad \chi^{(2,j)}(s) = 0 \tag{19}$$

The geometric transformation matrix which decomposes equilibrium equation (1) into the components of irreducible representations is defined by

$$\begin{aligned} H &\equiv [\cdots, H^{\mu}, \cdots] \\ &= [\, H^{(1,1)}, \cdots, H^{(1,m_1)}, \\ &\quad H^{(2,1)}, \cdots, H^{(2,m_2)}, \\ &\quad H^{(2,1)-}, \cdots, H^{(2,m_2)-} \,] \end{aligned} \tag{20}$$

Here $H^{(1,j)}$ indicates blocks for the one-dimensional irreducible representations $(1,j)$, $H^{(2,j)}$ and $H^{(2,j)-}$ express those for the two-dimensional ones $(2,j)$, respectively. By means of this transformation matrix H, the stiffness matrix can be transformed into a block-diagonal form:

$$\begin{aligned} \widetilde{K} &= H^{\mathrm{T}} K H = \mathrm{diag}[\cdots, \widetilde{K}^{\mu}, \cdots] \\ &= \mathrm{diag}[\widetilde{K}^{(1,1)}, \cdots, \widetilde{K}^{(1,m_1)}, \\ &\quad \widetilde{K}^{(2,1)}, \cdots, \widetilde{K}^{(2,m_2)}, \\ &\quad \widetilde{K}^{(2,1)}, \cdots, \widetilde{K}^{(2,m_2)} \,] \end{aligned} \tag{21}$$

where diag[$\cdots$] denotes a block-diagonal matrix with the diagonal blocks in the parentheses. It is to be noted that two identical diagonal blocks correspond to a two-dimensional irreducible representation. It is computationally efficient to compute each diagonal block by the formula

$$\widetilde{K}^{\mu} = (H^{\mu})^{\mathrm{T}} K H^{\mu}, \quad \mu \in R(D_n) \tag{22}$$

which exploits the orthogonality of H^{μ} among irreducible representations. For the translational displacements[17),18)], H^{μ} has the following symmetry

$$\begin{array}{ll} \Sigma(H^{(1,1)}) = D_n, & \Sigma(H^{(1,2)}) = C_n \\ \Sigma(H^{(1,3)}) = D_{n/2}, & \Sigma(H^{(1,4)}) = D^2_{n/2} \\ \Sigma(H^{(2,j)}) = D^k_{\mathrm{gcd}(j,n)} & \end{array} \tag{23}$$

$$\Sigma(H^{(2,j)-}) = \begin{cases} D^{k+n'/2}_{\mathrm{gcd}(j,n)}, & \text{when } n' = \text{even} \\ C_{\mathrm{gcd}(j,n)}, & \text{when } n' = \text{odd} \end{cases} \tag{24}$$

$$1 \le k \le n', \quad j = 1, \cdots, m_d, \quad n' = n/\mathrm{gcd}(j,n)$$

where $\Sigma(\cdot)$ means the group that labels the symmetry of the deformation patterns expressed by the column vectors of the matrix in the parentheses (see **Fig.6** and **Fig.7**).

The coordinate system associated with the irreducible representation is defined by

$$\boldsymbol{u} = H\boldsymbol{w} = \sum_{\mu \in R(G)} H^{\mu} \boldsymbol{w}^{\mu} \tag{25}$$

$$\begin{aligned} &= \sum_{j=1}^{m_1} H^{(1,j)} \boldsymbol{w}^{(1,j)} \\ &\quad + \sum_{j=1}^{m_2} (H^{(2,j)} \boldsymbol{w}^{(2,j)} + H^{(2,j)-} \boldsymbol{w}^{(2,j)-}) \end{aligned} \tag{26}$$

where the new independent variable

$$\begin{aligned} \boldsymbol{w} = [\ & (\boldsymbol{w}^{(1,1)})^{\mathrm{T}}, \cdots, (\boldsymbol{w}^{(1,m_1)})^{\mathrm{T}}, \\ & (\boldsymbol{w}^{(2,1)})^{\mathrm{T}}, \cdots, (\boldsymbol{w}^{(2,m_2)})^{\mathrm{T}}, \\ & (\boldsymbol{w}^{(2,1)-})^{\mathrm{T}}, \cdots, (\boldsymbol{w}^{(2,m_2)-})^{\mathrm{T}} \ ]^{\mathrm{T}} \end{aligned} \tag{27}$$

is expressed as the assemblage of the components for the irreducible representation. The linear equilibrium equation (6) can be decomposed into a set of independent equations

$$\begin{aligned} & (H^{(d,j)})^{\mathrm{T}} \boldsymbol{f} = \widetilde{K}^{(d,j)} \boldsymbol{w}^{(d,j)} \\ & (H^{(2,j)-})^{\mathrm{T}} \boldsymbol{f} = \widetilde{K}^{(2,j)} \boldsymbol{w}^{(2,j)-} \\ & j = 1, \cdots, m_d\,; \quad d = 1, 2 \end{aligned} \tag{28}$$

compatibly with the irreducible representation through the transfomation by **Eq.** (26). The solution $\boldsymbol{u}$ is obtained by substituting the solutions of **Eqs.** (28) into **Eq.** (26).

3. EXTENSION TO ROTATIONAL DISPLACEMENTS

(1) Representation matrix of nodal displacements

Consider the generalized displacement vector

$$\boldsymbol{u} = \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\theta} \end{pmatrix} = \begin{pmatrix} [\cdots, (\boldsymbol{v}^i)^{\mathrm{T}}, \cdots]^{\mathrm{T}} \\ [\cdots, (\boldsymbol{\theta}^i)^{\mathrm{T}}, \cdots]^{\mathrm{T}} \end{pmatrix} \tag{29}$$

which consists of a translational displacement vector $\boldsymbol{v}^i$ and rotational one $\boldsymbol{\theta}^i$ of node i ($i = 1, 2, \cdots$) shown in **Fig.1**. Here

$$\boldsymbol{v}^i = (v^i_X, v^i_Y, v^i_Z)^{\mathrm{T}}, \quad \boldsymbol{\theta}^i = (\theta^i_X, \theta^i_Y, \theta^i_Z)^{\mathrm{T}} \tag{30}$$

and the arrow → in **Fig.1** shows the translational displacements, and ↠ shows the rotational displacements around the axis. Since a D_n-invariant structure has rotational symmetry around the Z-axis, the X- and Y-directional displacements, and the Z-directional ones have different properties in the framework of the present theory. In order to exploit such a difference, which will turn out to realize the sparsity of H in **Eqs.** (47) and (48), we decompose $\boldsymbol{v}^i$ and $\boldsymbol{\theta}^i$ as follows:

$$\boldsymbol{v}^i = \begin{pmatrix} \boldsymbol{v}^i_{XY} \\ v^i_Z \end{pmatrix}, \quad \boldsymbol{\theta}^i = \begin{pmatrix} \boldsymbol{\theta}^i_{XY} \\ \theta^i_Z \end{pmatrix} \tag{31}$$

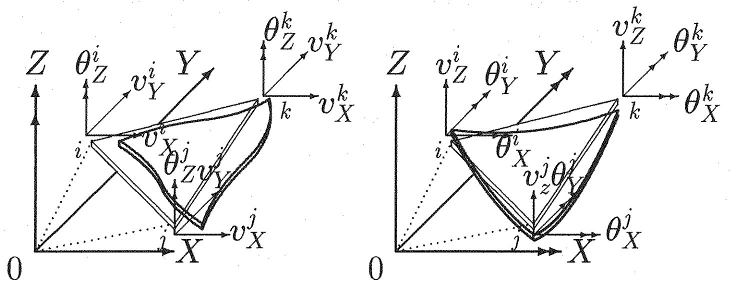
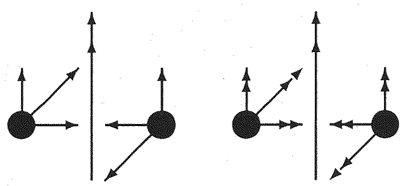
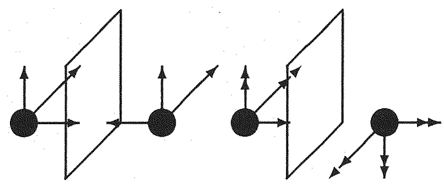


Fig.1 Definition of displacements



(a) translational displacements (b) rotational displacements

Fig.2 Action of rotation r



(a) translational displacements (b) rotational displacements

Fig.3 Action of reflection s

where the subscript XY of each vector (respectively, Z) shows that the relevant vector is associated with the XY-direction (respectively, Z-direction).

Because translational displacements $\boldsymbol{v}^i$ and rotational ones $\boldsymbol{\theta}^i$ are independent, representation matrix $\widehat{T}(g)$ of each node with 6 degrees of freedom is expressed as:

$$\widehat{T}(g) = \begin{pmatrix} \widehat{T}_v(g) & \mathrm{O} \\ \mathrm{O} & \widehat{T}_\theta(g) \end{pmatrix}, \quad g \in G \quad \cdots\cdots\cdots (32)$$

which is the direct sum [4] of representation matrices $\widehat{T}_\theta(g)$ and $\widehat{T}_v(g)$ associated with the translational and the rotational ones, respectively. Because rotation causes the same action on the translational and the rotational displacements as shown in **Fig.2**, we have

$$\widehat{T}_\theta(r) = \widehat{T}_v(r) \quad \cdots\cdots\cdots (33)$$

The reflection s transforms the translational and the rotational displacement vectors in opposite directions, as shown in **Fig.3**. This leads to

$$\widehat{T}_\theta(s) = -\widehat{T}_v(s) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \cdots\cdots\cdots (34)$$

The representation matrices $\widehat{T}_\theta(s)$ and $\widehat{T}_v(s)$ have opposite signs to show the difference of the action of the reflection s on the translational and the rotational displacements. Such difference arises from the fact that the translational ones are of an axial vector, while the rotational ones of a polar vector.

[4] The representation matrix for one orbit is a tensor product of $\widehat{T}(g)$ and the permutation representation, and $T(g)$ is given as a direct sum of this matrix over all orbits. Refer to Murota · Ikeda[14] for details.

(2) Transformation matrices

Murota · Ikeda[14] has obtained a concrete form of the geometric transformation matrix $H = H_v$ associated with the representation matrix

$$T(g) = T_v(g) \quad \cdots\cdots\cdots (35)$$

for the translational displacements. Note here that the representation matrix

$$T(g) = \begin{pmatrix} T_v(g) & \mathrm{O} \\ \mathrm{O} & T_\theta(g) \end{pmatrix}, \quad g \in G \quad \cdots\cdots\cdots (36)$$

and the geometric transformation matrix for each irreducible representation

$$H^\mu = \begin{pmatrix} H_v^\mu & \mathrm{O} \\ \mathrm{O} & H_\theta^\mu \end{pmatrix}, \quad \mu \in R(G) \quad \cdots\cdots\cdots (37)$$

both have block-diagonal forms.

Eq. (10) for $\boldsymbol{v}$ and $\boldsymbol{\theta}$ becomes

$$H_\Phi^{\mathrm{T}} T_\Phi(g) H_\Phi = \bigoplus_{\mu \in R(G)} \bigoplus_{i=1}^{a_\Phi^\mu} T_i^\mu(g)$$
$$g \in G, \quad \Phi = v \text{ or } \theta \quad \cdots\cdots (38)$$

where a_Φ^μ ($\Phi = v$ or θ) denotes the multiplicity of the irreducible representation μ for the representation matrix $T_\Phi(g)$. We obtain below the interrelationships between a_v^μ and a_θ^μ, and H_v^μ and H_θ^μ, and in turn to obtain the concrete form of a_θ^μ and H_θ^μ with reference to that of a_v^μ and H_v^μ obtained in Murota · Ikeda[14].

Since **Eqs.** (33) and (34) for the vector $\boldsymbol{u}^i$ of the ith node hold also for the vector $\boldsymbol{u}$ of all nodes, we have

$$T_\theta(g) = \sigma(g) T_v(g), \quad g \in G \quad \cdots\cdots\cdots (39)$$

$$\sigma(g) = \begin{cases} 1, & g = r \\ -1, & g = s \end{cases} \quad \cdots\cdots\cdots (40)$$

Fig.4 Four types of orbits

For a pair of irreducible representations μ and $\bar{\mu}$ defined as listed in **Table-1**, the relationship

$$\sigma(g)T^{\mu}(g)=T^{\bar{\mu}}(g),\quad g\in G \quad\cdots\cdots\cdots\cdots\cdots (41)$$

is satisfied.

The following relationships concerning the multiplicity and the coordinate transformation matrix are obtained from **Eqs.** (38), (39), and (41). Translational displacements $\boldsymbol{v}$ and rotational ones $\boldsymbol{\theta}$ have the same multiplicity

$$a^{\mu}_{\theta}=a^{\mu}_{v} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (42)$$

The irreducible representations μ and $\bar{\mu}$, which satisfy the relationship in **Table-1**, are associated with the translational displacements $\boldsymbol{v}$ and with the rotational ones $\boldsymbol{\theta}$. The combination of these relationships with **Eqs.** (23) and (40) leads to

$$H^{(1,1)}_{\theta}=H^{(1,2)}_{v},\qquad H^{(1,2)}_{\theta}=H^{(1,1)}_{v} \quad\cdots\cdots\cdots (43)$$

$$H^{(1,3)}_{\theta}=H^{(1,4)}_{v},\qquad H^{(1,4)}_{\theta}=H^{(1,3)}_{v} \quad\cdots\cdots\cdots (44)$$

$$H^{(2,j)}_{\theta}=H^{(2,j)-}_{v},\quad H^{(2,j)-}_{\theta}=H^{(2,j)}_{v} \quad\cdots\cdots\cdots (45)$$

$(j=1,\cdots,m_2)$. The blocks H^{μ}_{θ} for the rotational displacements can be calculated by **Eqs.** (43), (44), and (45) based on H^{μ}_{v} of the translational ones. Then the transformation matrix H can be calculated from **Eqs.** (20) and (37) with the use of H^{μ}_{v} and H^{μ}_{θ} obtained in this manner.

Table-1 The relationship between irreducible representations μ and $\bar{\mu}$

One-dimensional irreducible rep.		Two-dimensional irreducible rep.	
μ	$\bar{\mu}$	μ	$\bar{\mu}$
$(1,1)$	$(1,2)$	$(2,1)$	$(2,1)$
$(1,2)$	$(1,1)$	$(2,2)$	$(2,2)$
$(1,3)$	$(1,4)$	$\vdots$	$\vdots$
$(1,4)$	$(1,3)$	$(2,m_2)$	$(2,m_2)$

4. ORBITS AND SPARSITY OF H

A set of nodes of D_n−invariant structure can be decomposed into a series of D_n-invariant minimum subsets, which are called the "orbits." An orbit is defined by a set of points

$$\{r^k(\boldsymbol{x}),\ sr^k(\boldsymbol{x})\mid k=0,1,\cdots,n-1\} \quad\cdots\cdots (46)$$

where $r^k(\boldsymbol{x})$ (respectively, $sr^k(\boldsymbol{x})$) denotes the point transformed from a point $\boldsymbol{x}$ by means of the transformation r^k (respectively, sr^k) $(k=0,1,\cdots,n-1)$ of D_n. Though an individual node may be moved by the transformation caused by D_n, the orbit as a whole remains unchanged. A set of D_n−invariant nodes can be classified into the following four types of D_n−invariant orbits[14),18)] as shown in **Fig.4**.

type of orbits
- Center type (0)
- n-gon type (1V)
- n-gon type (1M)
- $2n$-gon type (2)

The column vectors of H can be defined orbit by orbit to assemble them systematically and to make it sparse, and in turn to enhance numerical efficiency.

The substituting of the formulas[14),15)] for the transformation matrix H_v of the translational displacements into the right hand sides of **Eqs.** (43) − (45) leads to H_{θ} of the rotational ones.

The blocks H^{μ} of H with $N_{\rm O}$ orbits are expressed as

$$H^{\mu}=\begin{pmatrix} H^{\mu}_1 & 0 & 0 & 0 & 0\\ 0 & \cdot & 0 & 0 & 0\\ 0 & 0 & \cdot & 0 & 0\\ 0 & 0 & 0 & \cdot & 0\\ 0 & 0 & 0 & 0 & H^{\mu}_{N_{\rm O}}\end{pmatrix},\mu\in R(G) \quad (47)$$

and hence are very sparse due to the independence of orbits. The representation matrices of the translational displacements $\boldsymbol{v}$ and rotational ones $\boldsymbol{\theta}$ are independent. Furthermore, the representation matrices for the X- and Y-directions and those for the Z−direction are independent as shown in **Eq.** (37). Owing to such independence, each block H^{μ}_q of **Eq.** (47) has a further block-diagonal structure

$$H^{\mu}_q=\begin{pmatrix} H^{\mu}_{q,vXY} & 0 & 0 & 0\\ 0 & H^{\mu}_{q,vZ} & 0 & 0\\ 0 & 0 & H^{\mu}_{q,\theta XY} & 0\\ 0 & 0 & 0 & H^{\mu}_{q,\theta Z}\end{pmatrix} \cdots (48)$$

When we calculate the blocks $\widetilde{K}^{\mu}$ of the stiffness matrix by **Eq.** (22), the sparsity of the H matrix due to **Eqs.** (47) and (48) is very advantageous. Refer to Ikeda et al.[18)] for the efficient calculation of $\widetilde{K}^{\mu}$.

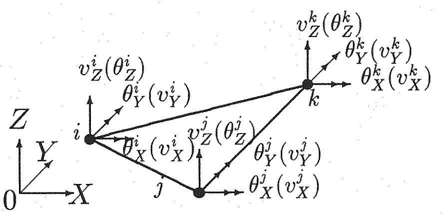


Fig.5 Regular-triangular plate element

5. ANALYTICAL RESULTS

The block-diagonalization method is applied in this section to regular polygonal plates.

(1) Regular-triangular plate element

We consider the regular-triangular plate element shown in **Fig.5**, the three nodes of which form the orbit of type 1M. Each node of this element has six degrees of freedom (three translational and three rotational displacements). The stiffness matrix of this element can be block-diagonalized by the present method. The transformation matrix (20) for this element is

$$H = [H^{(1,1)}, H^{(1,2)}, H^{(2,1)}, H^{(2,1)-}] \quad \cdots\cdots\cdots (49)$$

The deformation patterns represented by the column vectors of H are shown in **Fig.6**, while their components are listed in **Fig.7**. As can be seen, H is very sparse due to the double block structure in **Eqs.** (37) and (48) In numerical analysis, one needs not to compute H but to refer to the information on the orbit shown in **Fig.6** to decrease the requisite amount of storage and the computational cost.

The substituting of the element stiffness matrix K_e of this regular-triangular plate shown in **Fig.8** into the transformation

$$\begin{aligned}\widetilde{K}_e &= H^{\mathrm{T}} K_e H \\ &= \mathrm{diag}[\widetilde{K}_e^{(1,1)}, \widetilde{K}_e^{(1,2)}, \widetilde{K}_e^{(2,1)}, \widetilde{K}_e^{(2,1)}]\end{aligned} \quad \cdots (50)$$

yields a block-diagonalized form shown in **Fig.9**.

(2) Regular polygonal plates

The D_n-invariant plates in **Fig.10** with triangular meshes are considered. We employ ACM (Adini, Clough and Melosh) [19] linear element, which has three degrees of freedom for each node, consisting of rotational displacement θ_X^i around the X-axis and rotational one θ_Y^i around the Y-axis and the vertical translational displacement v_Z^i. The shape functions of the bending and the bending angle are assumed to be C^1-continuous [5].

The stiffness matrix of D_n-invariant plates (with D_n-invariant meshes and with uniform material and stiffness distribution) shown in **Fig.10** can be transformed into a block-diagonal form in **Fig.11**, regardless of the numbering of nodes. Here $(\cdot)$ shows zero values and $(+)$ shows positive ones and $(-)$ negative ones, respectively. For example, for $n = 3$, the whole set of nodal points consists of four orbits, including: one type 0 (original point), two type 1M, and one type 1V. In association with the increase in the degree n, the number $(m_1 + 2m_2)$ of the block increases, but the sizes of block matrices remain almost constant. Though further block structures are observed for some blocks, these structures do not arise from the group-theoretic nature but accidental one. **Fig.12** compares the stiffness matrix K and the block-diagonalized matrix $\widetilde{K}$. When n increases the band width of K increases significantly, while $\widetilde{K}$ has a very narrow band and independent blocks.

Fig.13 compares the variation of the array capacity in association with the increase of n for various methods. Here the ordinate denotes n and the abscissa in the left shows the array capacity of $\widetilde{K}$ by the present method (•), the capacity by the skyline method (◇), and that for the whole array of K (○). All these capacities are normalized with respect to the whole array of K for $n = 20$. The present method requires far less array than the skyline method, which is usually noted to save array. The ratio of the capacity of $\widetilde{K}$ relative to that of K (shown by the line graph without symbols and associated with the abscissa in the right) converges to 0 with a slight oscillation [6], and hence demonstrates the advantage of the present method.

Fig.14 shows the ratio of the computational time consumed by the NEC workstation EWS4800/350 with non-parallel CPU for the present method and the Cholesky method, though such time may be machine dependent. This computational time is normalized with respect to the time of K for $n = 20$. The present method, which demands less than half computing time of the Cholesky method does (for $n = 20$), is far more computationally efficient. The use of parallel machines will further enhance the efficiency of the present method.

6. CONCLUSION

In this paper, the block-diagonalization method has been extended to the rotational displacements, and its usefulness and validity are assessed. Although the numerical examples employed are limited to plates, the present formulation is general and is applicable to other structures.

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[5] Though the bending angle between elements is not continuous, it does not affect the present formulation, because this formulation is applicable both for compatible and non-compatible elements so far as **Eq.** (7) is satisfied.

[6] This oscillation arises from the difference in the composition of irreducible representations for n even and odd.

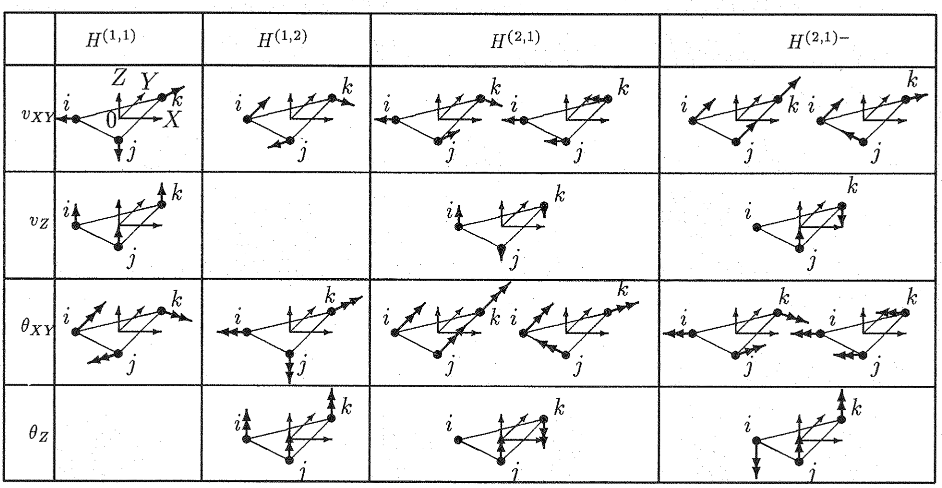

Fig.6 Deformation patterns represented by column vectors of H (1M type)

	$H^{(1,1)}_{v_{XY}}$	$H^{(1,1)}_{v_Z}$	$H^{(1,1)}_{\theta_{XY}}$	$H^{(1,2)}_{v_{XY}}$	$H^{(1,2)}_{\theta_{XY}}$	$H^{(1,2)}_{\theta_Z}$	$H^{(2,1)}_{v_{XY}}$	$H^{(2,1)}_{v_{XY}}$	$H^{(2,1)}_{v_Z}$	$H^{(2,1)}_{\theta_{XY}}$	$H^{(2,1)}_{\theta_{XY}}$	$H^{(2,1)}_{\theta_Z}$	$H^{(2,1)-}_{v_{XY}}$	$H^{(2,1)-}_{v_{XY}}$	$H^{(2,1)-}_{v_Z}$	$H^{(2,1)-}_{\theta_{XY}}$	$H^{(2,1)-}_{\theta_{XY}}$	$H^{(2,1)-}_{\theta_Z}$
v^i_X	$\frac{-1}{\sqrt{3}}$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	0	0	0	0	0	0	0	0	0
v^i_Y	0	0	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	0	0	0
θ^i_Z	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	0	0	0	0	0	0	$\frac{2}{\sqrt{6}}$
v^i_Z	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	0	$\frac{2}{\sqrt{6}}$	0	0	0	0	0	0	0	0	0
θ^i_X	0	0	0	0	$\frac{-1}{\sqrt{3}}$	0	0	0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0
θ^i_Y	0	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	0	0	0	0	0	0
v^j_X	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{-1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	0	0	0	$\frac{-1}{2}$	0	0	0	0
v^j_Y	$\frac{-1}{2}$	0	0	$\frac{-1}{2\sqrt{3}}$	0	0	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	0	0
θ^j_Z	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	$\frac{-1}{\sqrt{2}}$	0	0	0	0	0	$\frac{-1}{\sqrt{6}}$
v^j_Z	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	0	$\frac{-1}{\sqrt{6}}$	0	0	0	0	0	$\frac{-1}{\sqrt{2}}$	0	0	0
θ^j_X	0	0	$\frac{-1}{2}$	0	$\frac{1}{2\sqrt{3}}$	0	0	0	0	0	$\frac{-1}{2}$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0
θ^j_Y	0	0	$\frac{-1}{2\sqrt{3}}$	0	$\frac{-1}{2}$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	0	0	0	$\frac{1}{2}$	0
v^k_X	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0
v^k_Y	$\frac{1}{2}$	0	0	$\frac{-1}{2\sqrt{3}}$	0	0	0	$\frac{-1}{2}$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	0	0
θ^k_Z	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	$\frac{1}{\sqrt{2}}$	0	0	0	0	0	$\frac{-1}{\sqrt{6}}$
v^k_Z	0	$\frac{1}{\sqrt{3}}$	0	0	0	0	0	0	$\frac{-1}{\sqrt{6}}$	0	0	0	0	0	$\frac{1}{\sqrt{2}}$	0	0	0
θ^k_X	0	0	$\frac{1}{2}$	0	$\frac{1}{2\sqrt{3}}$	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0
θ^k_Y	0	0	$\frac{-1}{2\sqrt{3}}$	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	0	0	0	$\frac{-1}{2}$	0

Fig.7 Transformation matrix H (18×18)

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	v^i_X	v^i_Y	θ^i_Z	v^i_Z	θ^i_X	θ^i_Y	v^j_X	v^j_Y	θ^j_Z	v^j_Z	θ^j_X	θ^j_Y	v^k_X	v^k_Y	θ^k_Z	v^k_Z	θ^k_X	θ^k_Y
v^i_X	$\frac{64D_P}{3}$	0	0	0	0	0	$\frac{-32D_P}{3}$	$\frac{48D_P}{5\sqrt{3}}$	0	0	0	0	$\frac{-32D_P}{3}$	$\frac{-48D_P}{5\sqrt{3}}$	0	0	0	0
v^i_Y	0	$\frac{112D_P}{15}$	0	0	0	0	$\frac{56D_P}{5\sqrt{3}}$	$\frac{-56D_P}{15}$	0	0	0	0	$\frac{-56D_P}{5\sqrt{3}}$	$\frac{-56D_P}{15}$	0	0	0	0
θ^i_Z	0	0	D_T	0	0	0	0	0	$\frac{-D_T}{2}$	0	0	0	0	0	$\frac{-D_T}{2}$	0	0	0
v^i_Z	0	0	0	$\frac{128D_B}{9L^2}$	0	$\frac{-44D_B}{5\sqrt{3}L}$	0	0	0	$\frac{-64D_B}{9L^2}$	$\frac{-32D_B}{9L}$	$\frac{28\sqrt{3}D_B}{45L}$	0	0	0	$\frac{-64D_B}{9L^2}$	$\frac{32D_B}{9L}$	$\frac{28\sqrt{3}D_B}{45L}$
θ^i_X	0	0	0	0	$\frac{208D_B}{135}$	0	0	0	0	$\frac{-122D_B}{45L}$	$\frac{-139D_B}{270}$	$\frac{-59\sqrt{3}D_B}{270}$	0	0	0	$\frac{122D_B}{45L}$	$\frac{-139D_B}{270}$	$\frac{59\sqrt{3}D_B}{270}$
θ^i_Y	0	0	0	$\frac{-44D_B}{5\sqrt{3}L}$	0	$\frac{46D_B}{15}$	0	0	0	$\frac{22D_B}{5\sqrt{3}L}$	$\frac{9D_B}{10\sqrt{3}}$	$\frac{-13D_B}{30}$	0	0	0	$\frac{22D_B}{5\sqrt{3}L}$	$\frac{-9D_B}{10\sqrt{3}}$	$\frac{-13D_B}{30}$
v^j_X	$\frac{-32D_P}{3}$	$\frac{56D_P}{5\sqrt{3}}$	0	0	0	0	$\frac{164D_P}{15}$	$\frac{-52D_P}{5\sqrt{3}}$	0	0	0	0	$\frac{-4D_P}{15}$	$\frac{-4D_P}{5\sqrt{3}}$	0	0	0	0
v^j_Y	$\frac{48D_P}{5\sqrt{3}}$	$\frac{-56D_P}{15}$	0	0	0	0	$\frac{-52D_P}{5\sqrt{3}}$	$\frac{268D_P}{15}$	0	0	0	0	$\frac{4D_P}{5\sqrt{3}}$	$\frac{-212D_P}{15}$	0	0	0	0
θ^j_Z	0	0	$\frac{-D_T}{2}$	0	0	0	0	0	D_T	0	0	0	0	0	$\frac{-D_T}{2}$	0	0	0
v^j_Z	0	0	0	$\frac{-64D_B}{9L^2}$	$\frac{-122D_B}{45L}$	$\frac{22D_B}{5\sqrt{3}L}$	0	0	0	$\frac{128D_B}{9L^2}$	$\frac{22D_B}{5L}$	$\frac{22\sqrt{3}D_B}{15L}$	0	0	0	$\frac{-64D_B}{9L^2}$	$\frac{38D_B}{45L}$	$\frac{-94\sqrt{3}D_B}{45L}$
θ^j_X	0	0	0	$\frac{-32D_B}{9L}$	$\frac{-139D_B}{270}$	$\frac{9D_B}{10\sqrt{3}}$	0	0	0	$\frac{22D_B}{5L}$	$\frac{145D_B}{54}$	$\frac{103\sqrt{3}D_B}{270}$	0	0	0	$\frac{-38D_B}{45L}$	$\frac{-53D_B}{135}$	$\frac{-7\sqrt{3}D_B}{27}$
θ^j_Y	0	0	0	$\frac{28\sqrt{3}D_B}{45L}$	$\frac{-59\sqrt{3}D_B}{270}$	$\frac{-13D_B}{30}$	0	0	0	$\frac{22\sqrt{3}D_B}{15L}$	$\frac{103\sqrt{3}D_B}{270}$	$\frac{173D_B}{90}$	0	0	0	$\frac{-94\sqrt{3}D_B}{45L}$	$\frac{7\sqrt{3}D_B}{27}$	$\frac{-5D_B}{9}$
v^k_X	$\frac{-32D_P}{3}$	$\frac{-56D_P}{5\sqrt{3}}$	0	0	0	0	$\frac{-4D_P}{15}$	$\frac{4D_P}{5\sqrt{3}}$	0	0	0	0	$\frac{164D_P}{15}$	$\frac{52D_P}{5\sqrt{3}}$	0	0	0	0
v^k_Y	$\frac{-48D_P}{5\sqrt{3}}$	$\frac{-56D_P}{15}$	0	0	0	0	$\frac{-4D_P}{5\sqrt{3}}$	$\frac{-212D_P}{15}$	0	0	0	0	$\frac{52D_P}{5\sqrt{3}}$	$\frac{268D_P}{15}$	0	0	0	0
θ^k_Z	0	0	$\frac{-D_T}{2}$	0	0	0	0	0	$\frac{-D_T}{2}$	0	0	0	0	0	D_T	0	0	0
v^k_Z	0	0	0	$\frac{-64D_B}{9L^2}$	$\frac{122D_B}{45L}$	$\frac{22D_B}{5\sqrt{3}L}$	0	0	0	$\frac{-64D_B}{9L^2}$	$\frac{-38D_B}{45L}$	$\frac{-94\sqrt{3}D_B}{45L}$	0	0	0	$\frac{128D_B}{9L^2}$	$\frac{-22D_B}{5L}$	$\frac{22\sqrt{3}D_B}{15L}$
θ^k_X	0	0	0	$\frac{32D_B}{9L}$	$\frac{-139D_B}{270}$	$\frac{-9D_B}{10\sqrt{3}}$	0	0	0	$\frac{38D_B}{45L}$	$\frac{-53D_B}{135}$	$\frac{7\sqrt{3}D_B}{27}$	0	0	0	$\frac{-22D_B}{5L}$	$\frac{145D_B}{54}$	$\frac{-103\sqrt{3}D_B}{270}$
θ^k_Y	0	0	0	$\frac{28\sqrt{3}D_B}{45L}$	$\frac{59\sqrt{3}D_B}{270}$	$\frac{-13D_B}{30}$	0	0	0	$\frac{-94\sqrt{3}D_B}{45L}$	$\frac{-7\sqrt{3}D_B}{27}$	$\frac{-5D_B}{9}$	0	0	0	$\frac{22\sqrt{3}D_B}{15L}$	$\frac{-103\sqrt{3}D_B}{270}$	$\frac{173D_B}{90}$

where $D_P = \frac{E\,t}{1-\nu^2}\,\frac{A}{L^2}$, $D_T = \alpha\,E\,t\,A$, $D_B = \frac{E\,t^3}{12(1-\nu^2)}\,\frac{A}{L^2}$, and $\nu = 0.3$.

Fig.8 Element stiffness matrix of a regular-trianglar plate K_e (18×18)

	$H^{(1,1)}_{vXY}$	$H^{(1,1)}_{vZ}$	$H^{(1,1)}_{\theta XY}$	$H^{(1,2)}_{vXY}$	$H^{(1,2)}_{\theta XY}$	$H^{(1,2)}_{\theta Z}$	$H^{(2,1)}_{vXY}$	$H^{(2,1)}_{vXY}$	$H^{(2,1)}_{vZ}$	$H^{(2,1)}_{\theta XY}$	$H^{(2,1)}_{\theta XY}$	$H^{(2,1)}_{\theta Z}$	$H^{(2,1)-}_{vXY}$	$H^{(2,1)-}_{vXY}$	$H^{(2,1)-}_{vZ}$	$H^{(2,1)-}_{\theta XY}$	$H^{(2,1)-}_{\theta XY}$	$H^{(2,1)-}_{\theta Z}$
$H^{(1,1)}_{vXY}$	$\frac{208D_P}{5}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(1,1)}_{vZ}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(1,1)}_{\theta XY}$	0	0	$\frac{13D_B}{5}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(1,2)}_{vXY}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(1,2)}_{\theta XY}$	0	0	0	0	$\frac{7D_B}{5}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(1,2)}_{\theta Z}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(2,1)}_{vXY}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(2,1)}_{vXY}$	0	0	0	0	0	0	0	$\frac{112D_P}{5}$	0	0	0	0	0	0	0	0	0	0
$H^{(2,1)}_{vZ}$	0	0	0	0	0	0	0	0	$\frac{64D_B}{3L^2}$	$\frac{-76D_B}{15\sqrt{6}L}$	$\frac{64D_B}{3\sqrt{6}L}$	0	0	0	0	0	0	0
$H^{(2,1)}_{\theta XY}$	0	0	0	0	0	0	0	0	$\frac{-76D_B}{15\sqrt{6}L}$	$\frac{61D_B}{45}$	$\frac{-38D_B}{45}$	0	0	0	0	0	0	0
$H^{(2,1)}_{\theta XY}$	0	0	0	0	0	0	0	0	$\frac{64D_B}{3\sqrt{6}L}$	$\frac{-38D_B}{45}$	$\frac{32D_B}{9}$	0	0	0	0	0	0	0
$H^{(2,1)}_{\theta Z}$	0	0	0	0	0	0	0	0	0	0	0	$\frac{3D_T}{2}$	0	0	0	0	0	0
$H^{(2,1)-}_{vXY}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$H^{(2,1)-}_{vXY}$	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{112D_P}{5}$	0	0	0	0
$H^{(2,1)-}_{vZ}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{64D_B}{3L^2}$	$\frac{76D_B}{15\sqrt{6}L}$	$\frac{64D_B}{3\sqrt{6}L}$	0
$H^{(2,1)-}_{\theta XY}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{76D_B}{15\sqrt{6}L}$	$\frac{61D_B}{45}$	$\frac{38D_B}{45}$	0
$H^{(2,1)-}_{\theta XY}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{64D_B}{3\sqrt{6}L}$	$\frac{38D_B}{45}$	$\frac{32D_B}{9}$	0
$H^{(2,1)-}_{\theta Z}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{3D_T}{2}$

Fig.9 Block-diagonalized element stiffness matrix $\widetilde{K_e}$ (18×18)

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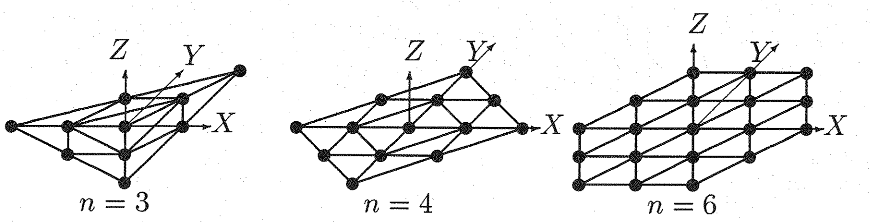

Fig.10 D_n-invariant plates ($n = 3, 4, 6$)

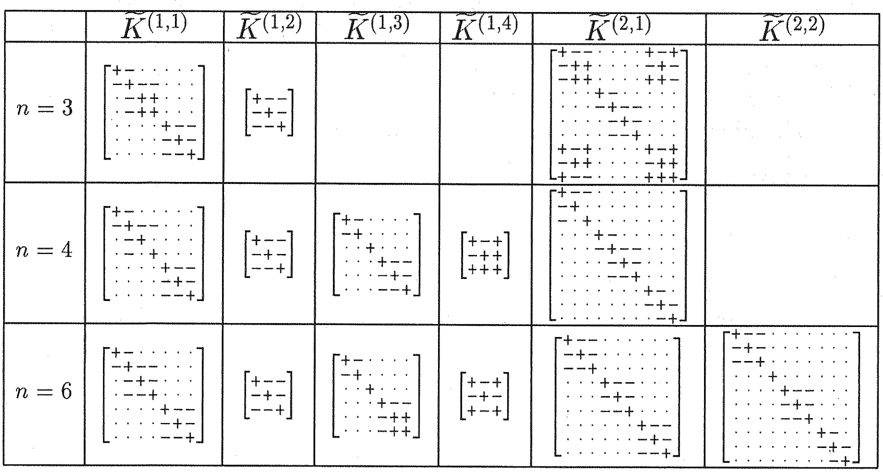

Fig.11 Block-diagonalized matrix $\widetilde{K}$

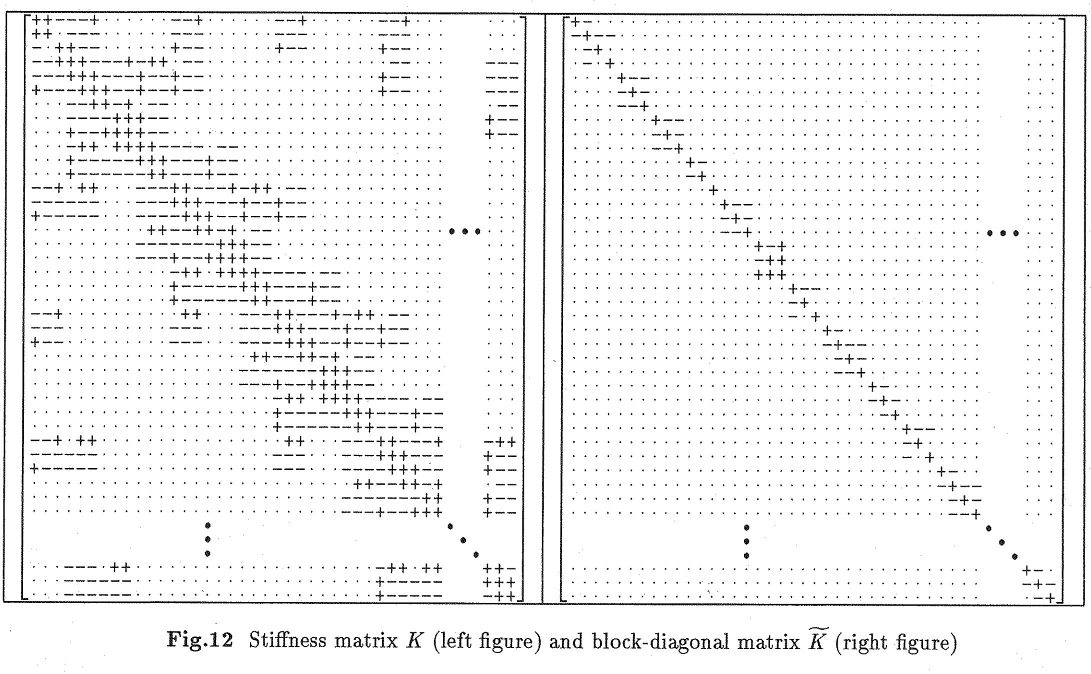

Fig.12 Stiffness matrix K (left figure) and block-diagonal matrix $\widetilde{K}$ (right figure)

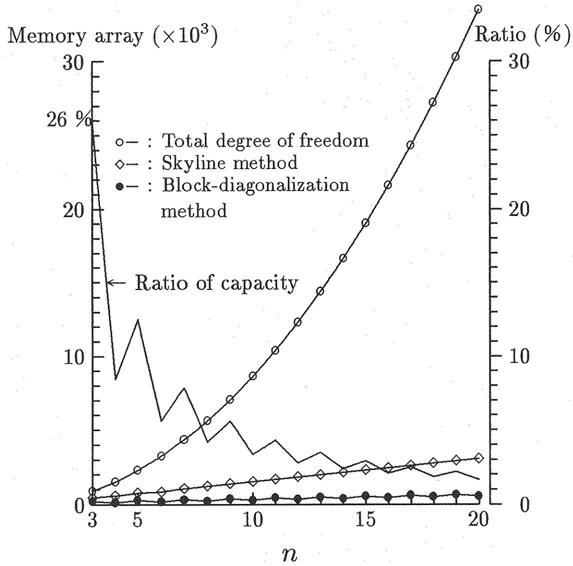

Fig.13 Computer memory

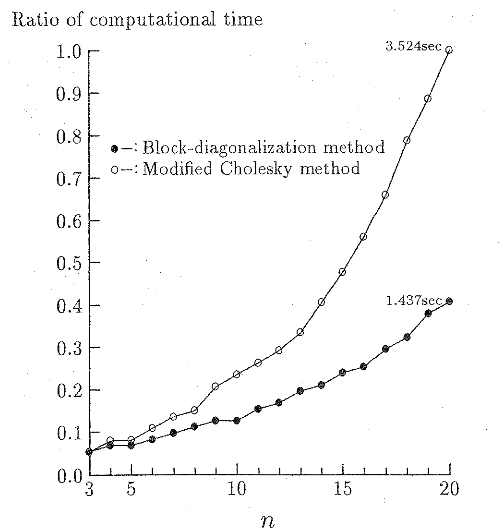

Fig.14 Computing time

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