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EVALUATION OF DYNAMIC COLLAPSE OF CANTILEVER COLUMN UNDER AXIAL COMPRESSION AND LATERAL DYNAMIC FORCE

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The objective of this paper is to construct a method of evaluating the dynamic collapse of a cantilever column under the axial compression and the lateral disturbing force simultaneously. First, a parametric study is carried out to investigate the dynamic collapse behavior of the column idealized by the flexural multi-degree spring-mass system, under the axial compression and the harmonic disturbing force. Secondly, through the results, a method of evaluating the dynamic collapse of the column is proposed by developing the method for this type of SDOF systems. Finally, the dynamic collapse of a uniform column and a stepped one is evaluated by the proposed method.
Key Words : *dynamic collapse, cantilever column, axial compression, power, energy absorption, input energy, multi-degree system*

1. INTRODUCTION

Recently, a number of cable supported bridges have been built or are under construction all over the world. In this type of bridges, the towers are subjected to the static axial compression, which may cause the structural instability, and the lateral dynamic force simultaneously. Hence, the tower must be designed based on the dynamic strength under this load condition as well as the static strength.

From this point of view, the authors et al. investigated the dynamic collapse behavior of the fundamental model with structural instability[1]. As a result, a method of evaluating the dynamic collapse of the system, by comparing the strength power corresponding to the energy absorption of the system with the disturbing force power corresponding to the input energy exerted by the disturbing force, was proposed.

However, in order to establish the dynamic limit state design method for the tower, the dynamic collapse behavior and energy quantities of the fundamental model should be related to those of the multi-degree system with practical structural properties. Furthermore, to investigate the dynamic collapse behavior of a tower with non-prismatic cross section, it is necessary to employ an adequate multi-degree system.

Kato and Akiyama[2)-5)] Yamada, Iemura et al.[6)], and Ohno and Nishioka[7),8)] investigated the dynamic elasto-plastic behavior of the multi-degree shear vibrating systems and they developed a kind of earthquake resistant design procedure based on the plastic deformation or the energy concept. However, it is not easy to incorporate a tower of the cable supported bridge into the shear vibrating system, because the flexural behavior is dominant in these structures and for a tower with non-prismatic cross section, a so-called plastic hinge is not always formed at the base.

The elasto-plastic behavior of such structures is possible to be analyzed by FEM taking into account the nonlinear behaviors. However, extensive parametric analysis is required to investigate the relation between the energy absorption of the system and the input energy by the disturbing force. Therefore, it is desirable to employ an adequate flexural vibrating system into which the generalized force (bending moment) and the generalized displacement (curvature) is incorporated.

In this paper, firstly, a uniform cantilever column and a stepped cantilever column are idealized by a flexural system with finite number of degrees of freedom. Secondly, a parametric study is carried out to investigate the dynamic ultimate strength and the energy quantities of the system subjected to the static axial compression and the lateral harmonic disturbing force. In this case, the dynamic collapse behavior of the system under this load condition is largely affected by the flexural vibration. Thus, the mass matrix of the system is formulated so that each natural frequency of the system with arbitrary number of degrees of freedom may agree with the corresponding flexural natural frequency of the column. Finally, through the results analyzed, a method of evaluating the dynamic collapse of a

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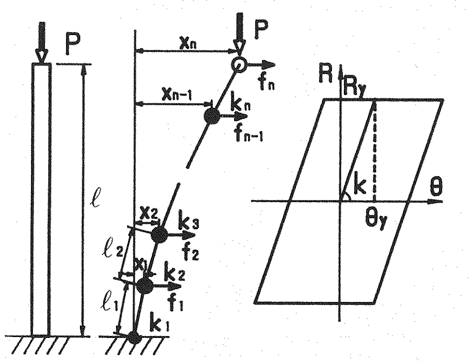


(a) Uniform column (b) Spring-mass system (c) Restoring moment characteristics of rotational spring

Fig. 1 Modelling of column

uniform cantilever column and a stepped one is proposed by developing the method for this type of an SDOF system.

Although the proposed method focuses on the critical state which is specified by the dynamic collapse of the system, the method may be applicable to the seismic design method based on different critical state such as the yielding, the maximum strength capacity or the ductility capacity.

2. MODELLING OF CANTILEVER COLUMN

A cantilever column subjected to both the static axial compression and the lateral disturbing force as shown in **Fig.1(a)** is considered in the analysis. The column is modelled as the flexural system with arbitrary number of degrees of freedom, which consists of rigid bars and rotational springs as shown in **Fig.1(b)**[9]. Considering the equilibrium of moment about each rotational spring, the equation of motion is

$$\boldsymbol{LM\ddot{x}} + \boldsymbol{R} - \boldsymbol{Px} = \boldsymbol{Lf} \quad \cdots\cdots (1)$$

in which

$$\boldsymbol{L} = \begin{pmatrix} \ell_1 & \ell_1+\ell_2 & \ell_1+\ell_2+\ell_3 & \cdot & \cdot \\ 0 & \ell_2 & \ell_2+\ell_3 & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \ell_{n-1} & \ell_{n-1}+\ell_n \\ \cdot & \cdot & \cdot & 0 & \ell_n \end{pmatrix}$$

$$\boldsymbol{M} = \begin{pmatrix} m_{11} & & & Sym. \\ m_{21} & m_{22} & & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot \\ m_{n1} & \cdot & \cdot & m_{nn} \end{pmatrix}$$

$$\boldsymbol{P} = \begin{pmatrix} 0 & 0 & 0 & \cdot & P \\ -P & 0 & 0 & \cdot & P \\ 0 & -P & 0 & \cdot & P \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & -P & P \end{pmatrix}$$

$$\boldsymbol{x} = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{Bmatrix} \qquad \boldsymbol{f} = \begin{Bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ f_n \end{Bmatrix} \quad \cdots\cdots (2)$$

These matrices and vectors are composed of the properties shown in **Fig.1(b)** except for mass matrix $\boldsymbol{M}$. It is assumed that there is no viscous damping in the system and that each rotational angle is small enough to neglect the geometrical nonlinearity. But the effect of the static vertical load on the potential energy is taken into account. The moment vector $\boldsymbol{R}$ is expressed as the product of the rotational spring constant matrix $\boldsymbol{K}$ and the angle of rotation vector $\boldsymbol{\theta}$ within the elastic range of the spring. Then

$$\boldsymbol{R} = \boldsymbol{K\theta} \quad \cdots\cdots (3)$$

in which

$$\boldsymbol{K} = \begin{pmatrix} k_1 & & & & Sym. \\ 0 & k_2 & & & \\ 0 & 0 & k_3 & & \\ \cdot & \cdot & \cdot & \cdot & \\ 0 & 0 & 0 & \cdot & k_n \end{pmatrix}$$

$$\boldsymbol{R} = \begin{Bmatrix} R_1 \\ R_2 \\ \cdot \\ \cdot \\ R_n \end{Bmatrix} \qquad \boldsymbol{\theta} = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \theta_n \end{Bmatrix} \quad \cdots\cdots (4)$$

The geometrical relationship between $\boldsymbol{x}$ and $\boldsymbol{\theta}$ yields

$$\boldsymbol{x} = \boldsymbol{L}^T\boldsymbol{\theta} \quad \cdots\cdots (5)$$

The restoring moment characteristics of the rotational spring, which are the analytical model for the relation between the bending moment and the curvature of the structural member, are modelled as the ideal elasto-plastic ones as shown in **Fig.1(c)**. The spring constant of the rotational spring in the elastic range is determined so that the strain energy U_R stored in one rotational spring may be equal to the strain energy U_C stored in the corresponding segment of the column. If it is assumed that the bending moment applied to the segment ℓ_i shown in **Fig.1(b)** is constant and that its flexural stiffness is denoted as EI_i, thus,

$$U_C = \frac{1}{2}EI_i(1/\rho_i)^2\ell_i \quad \cdots\cdots (6\text{-a})$$

$$U_R = \frac{1}{2}k_i{\theta_i}^2 \quad \cdots\cdots (6\text{-b})$$

The spring constant therefore becomes

$$k_i = EI_i/\ell_i \quad \cdots\cdots (7)$$

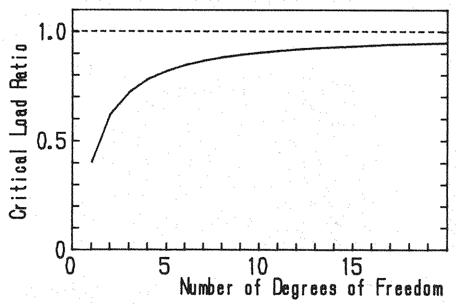


Fig. 2 Relation between critical load and number of degrees of freedom

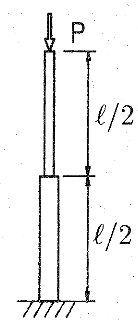


Fig. 3 Stepped column

in which ρ_i is the radius of curvature and $1/\rho_i$ is assumed to be approximately equal to θ_i/ℓ_i. The yield rotational angle θ_{Yi} is set equal to θ_Y/n (θ_Y : reference yield rotational angle, n : number of degrees of freedom) so that the rotational spring may yield in the same restoring moment, irrespective of the number of degrees of freedom.

Fig.2 shows the critical load P_{cr} against the number of degrees of freedom, when the uniform column is idealized by the spring system as shown in **Fig.1(b)**. The ordinate shows the critical load P_{cr} normalized by Euler load of the cantilever column. It is noted from this figure that the critical load of the spring system approaches Euler load, as the number of degrees of freedom increases. This implies that the stiffness property of the cantilever column with structural instability is roughly evaluated by the spring system with an adequate number of degrees of freedom employed here.

However, the stiffness property of the spring system with a small number of degrees of freedom does not represent accurately the one of the actual column as shown in **Fig.2**. Moreover, the natural frequencies of the model with lumped-mass idealization differ from those of an actual structure generally. Therefore, it is undesirable that the actual structure is idealized by a model not only with a smaller number of degrees of freedom, but also with lumped-mass idealization. Thus, the mass matrix is evaluated so that each natural frequency of the model with an employed number of degrees of freedom may agree with the corresponding flexural natural frequency of the actual column.

According to Eqs.(1), (3) and (5), for a freely vibrating undamped multi-degree elastic system, the equation of motion becomes

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \overline{\boldsymbol{K}}\boldsymbol{x} = \boldsymbol{0} \quad \cdots\cdots (8)$$

in which $\boldsymbol{M}$ is the mass matrix to be evaluated and $\overline{\boldsymbol{K}} = \boldsymbol{L}^{-1}\,[\boldsymbol{K}(\boldsymbol{L}^T)^{-1} - \boldsymbol{P}]$. In calculating the mass matrix $\boldsymbol{M}$ for a uniform column, $\boldsymbol{M}$ is assumed to be a diagonal matrix having equal value in each diagonal element first. For a stepped column, the value proportional to the mass of each segment is arranged in the corresponding diagonal element of the matrix. The mode-shape matrix $\boldsymbol{\phi}$ is obtained by solving the eigenvalue problem of the frequency equation corresponding to Eq.(8). Both $\boldsymbol{M}'$ ($= \boldsymbol{\phi}^T\ \boldsymbol{M}\ \boldsymbol{\phi}$) and $\boldsymbol{K}'$ ($= \boldsymbol{\phi}^T\ \overline{\boldsymbol{K}}\ \boldsymbol{\phi}$) become diagonal matrices because of the orthogonality property of mode-shape matrix.

If each flexural natural circular frequency ω_i ($i = 1, 2...$) of the column under the static axial compression is computed by using FEM for a beam element, the diagonal element in $\boldsymbol{M}'$ is alternatively obtained from

$$M'_{ii} = K'_{ii}/\omega_i{}^2 \quad \cdots\cdots (9)$$

Thus, by pre- and postmultiplying $\boldsymbol{M}'$ obtained from Eq.(9) by the inverse of the mode-shape matrix or its transpose, the mass matrix $\boldsymbol{M}$ of the model is obtained from

$$\boldsymbol{M} = (\boldsymbol{\phi}^T)^{-1}\boldsymbol{M}'\boldsymbol{\phi}^{-1} \quad \cdots\cdots (10)$$

This is not a diagonal matrix generally.

3. EXAMPLE COLUMN AND PARAMETER

On the basis of the above procedure, a cantilever column ($\ell = 10m$) under the static axial compression is idealized by the spring-mass system and the dynamic collapse behavior of the column is investigated for various sets of parameters.

The flexural stiffness EI, the reference yield rotational angle θ_Y and α, the ratio of the static vertical load to the critical load are selected as the structural parameters for a uniform column as shown in **Fig.1(a)**. On the other hand, η, the ratio of the flexural stiffness of the upper section to that of the lower section is added to the parameters for a stepped column shown in **Fig.3**, which consists of two different sections of equal lengths. These parameters employed here are summarized in **Table 1** and Euler load of each column is also shown in the table. α is defined as the ratio of the static vertical load to

Table 1 Analyzed parameters

	EI (MNm^2)	α	θ_Y	η	f_1 (Hz)	*Euler Load(kN)*
Uniform Column	13.38	0.5	0.002	–	1.48	330.1
	14.41	0.3	0.002	–	2.02	355.5
		0.5	0.001	–	1.72	
			0.002			
			0.003			
		0.7	0.002	–	1.34	
	15.44	0.5	0.002	–	2.07	380.8
Stepped Column	14.41	0.5	0.002	0.3	2.22	240.4
				0.5	2.0	297.8
				0.7	1.86	329.1

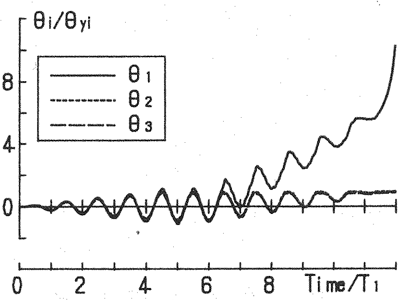
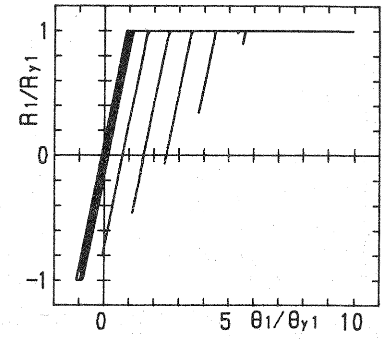


Fig. 4 Time hystories of rotational angles (uniform column)

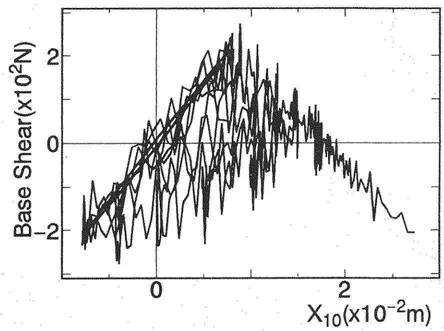
the corresponding critical load, because the critical load varies with the number of degrees of freedom of the system as shown in **Fig.2**. For a stepped column, the value of EI shown in **Table 1** is that of the lower section of the column.

The fundamental dynamic collapse behavior of the system subjected to the harmonic ground motion $Z\sin\omega_1 t$ is examined in the analysis. In this case, ω_1 is the first natural circular frequency of the column shown in **Table 1** and five different acceleration amplitudes ranging from 0.0469 to $0.2346 m/s^2$ are used. The dynamic responses are calculated by using a step-by-step integration procedure where the modified Newton-Raphson method and the Newmark-β method with β equal to 1/4 are combined.

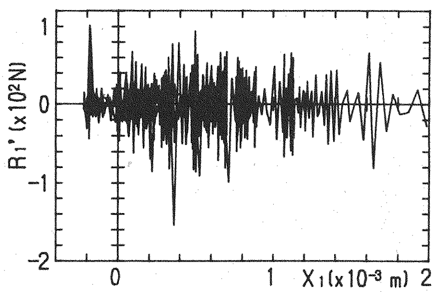
Examples of the time histories of the rotational angles of three springs at and near the base are shown in **Fig.4**, when the uniform column is idealized by a spring-mass system with ten degrees of freedom. The ordinate shows the rotational angle normalized by each yield rotational angle and the abscissa shows the elapsed time normalized by T_1, the first natural period of the system. When the system is subjected to a harmonic disturbing force whose frequency is equal to its first natural frequency, the system collapses as a result of the yielding of the rotational spring at the base. This tendency is perceived irrespective of the number of degrees of freedom considered in modelling the column.



(a) Restoring moment versus rotational angle at base



(b) Base shear versus horizontal displacement at top



(c) Horizontal restoring force versus horizontal displacement of first story

Fig. 5 Restoring force characteristics of ten-degree system

Fig.5(a) shows the relation between the restoring moment and the rotational angle of the spring at the base, **Fig.5(b)** shows the relation between the base shear and the horizontal displacement at the top, and **Fig.5(c)** shows the relation between the horizontal restoring force and the horizontal displacement of the first story. The relation between the restoring moment and the rotational angle of the spring results in the assumed ideal elasto-plastic one. On the other hand, the envelope of the relation between the base shear and the horizontal displace-

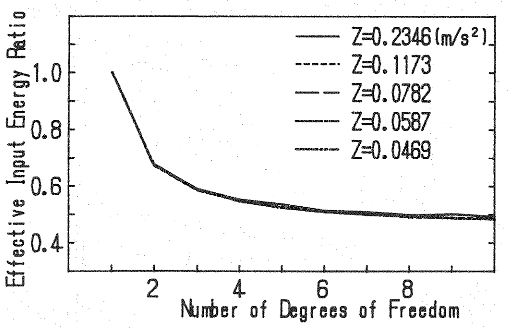


Fig. 6 Relation between E_{EF} and number of degrees of freedom (uniform column, $EI = 14.41MNm^2$, $\alpha = 0.5$, $\theta_Y = 0.002$)

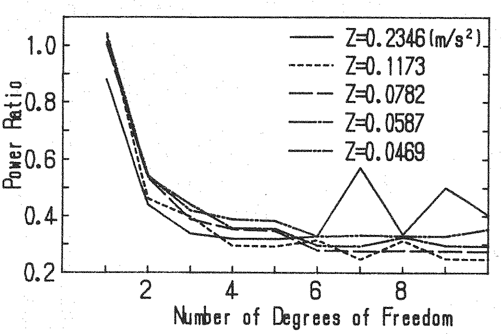
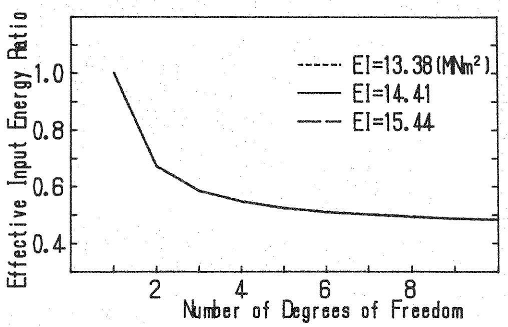


Fig. 7 Relation between power and number of degrees of freedom (uniform column, $EI = 14.41MNm^2$, $\alpha = 0.5$, $\theta_Y = 0.002$)

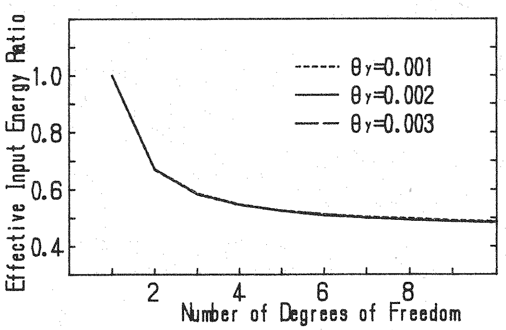
ment at the top becomes the degrading restoring force characteristics. However, the relation between the horizontal restoring force and the horizontal displacement of the first story becomes a complicated loop, because the relation is affected by the rotational angle of the other springs, its yielding and the vertical static load, which causes the structural instability. This tendency is observed for this type of two-degree system in Ref.(12). This implies that this type of structure is not easily idealized by the multi-degree shear vibrating system.

4. EFFECTIVE INPUT ENERGY AND POWER OF UNIFORM COLUMN

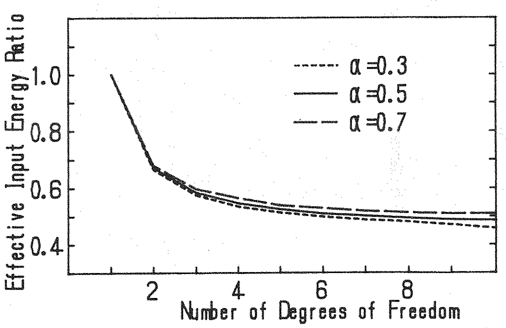
The authors et al. showed that the effective input energy governed the dynamic ultimate state for SDOF systems with structural instability[10] and that the dynamic collapse of the SDOF systems was evaluated without employing any dynamic analysis by using the power corresponding to the input energy exerted by the harmonic and random disturbing force[1),11)]. Therefore, the parametric study is carried out focusing on the effective input energy and the disturbing force power. The effective input en-



(a) Effect of flexural stiffness



(b) Effect of yield rotational angle

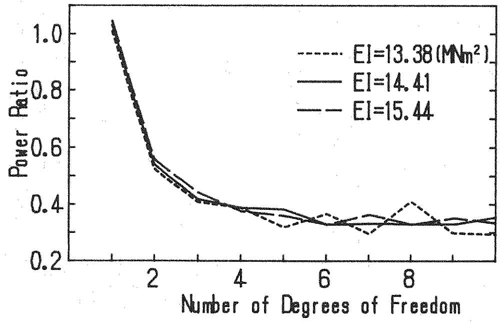


(c) Effect of static load ratio

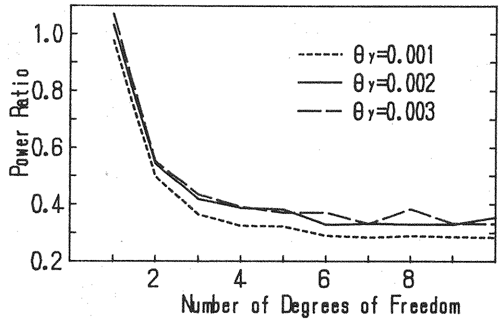
Fig. 8 Relation between E_{EF} and number of degrees of freedom (uniform column)

ergy E_{EF} is defined as the amount of energy which is obtained by subtracting the energy dissipated by the hysteretic damping from the total input energy. On the other hand, the power L is defined as Eq.(11) in chapter 6 and is employed to compare the relative magnitudes of the disturbing force with the structural strength.

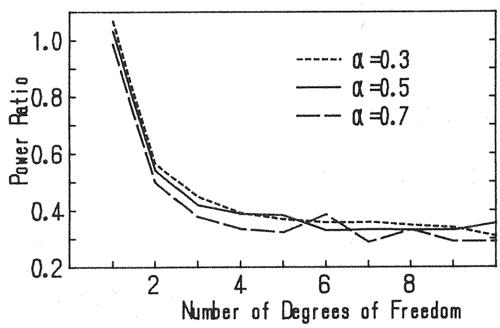
Fig.6 shows the relation between E_{EF} up to the dynamic ultimate state, Z and the number of degrees of freedom of the system. The ordinate shows E_{EF} and the value of E_{EF} for the corresponding single-degree system is set equal to unity. From the figure, it is revealed that E_{EF} decreases and approaches a certain value, as the number of degrees of freedom increases, irrespective of Z. This is because the yield-



(a) Effect of flexural stiffness



(b) Effect of yield rotational angle



(c) Effect of static load ratio

Fig. 9 Relation between power and number of degrees of freedom (uniform column)

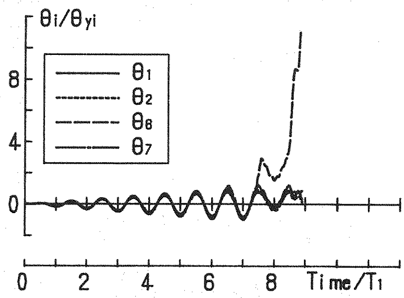


Fig. 10 Time hystories of rotational angles (stepped column)

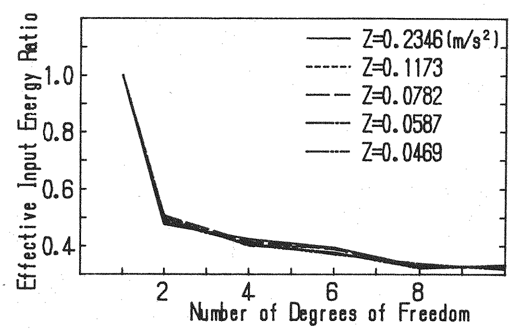


Fig. 11 Relation between E_{EF} and number of degrees of freedom (stepped column, $\alpha = 0.5$, $\theta_Y = 0.002$, $\eta = 0.5$)

ing is concentrated on the spring at the base and the total energy absorption of the system is largely influenced by the energy absorption of the spring.

Fig.7 also shows the relation between L obtained by Eq.(11), Z and the number of degrees of freedom. The ordinate shows L normalized by S_1 for the corresponding single-degree system, which agrees well with L for the single-degree system[1] and is given by Eq.(12) in chapter 6. As the increase of the energy dissipated by the hysteretic damping, the total input energy increases and then L increases. Therefore, the results shown in **Fig.7** are scattered, but in the same manner as E_{EF}, L decreases and approaches a certain value according to the increase of the number of degrees of freedom. It is also clear from this figure that the ratio of the power L for the ten-degree system to the power S_1 for the single-degree system is almost constant (about 0.3) irrespective of Z.

Figs.8 and **9** show the relation between E_{EF}, L and the number of degrees of freedom, for different values of EI, the yield rotational angle θ_Y and the static load ratio α. The results are plotted against only one amplitude of the harmonic ground motion ($Z = 0.0469 m/s^2$) in these figures. It is also noted that both E_{EF} and L approach respective certain values with the increase of the number of degrees of freedom and that the ratio of the power L for the ten-degree system to the power S_1 for the single-degree system becomes about 0.3.

5. EFFECTIVE INPUT ENERGY AND POWER OF STEPPED COLUMN

Examples of time histories of the rotational angles of two springs at and near the base and two springs, where the cross section changes, are shown in **Fig.10**, when the column with a stepped cross section is idealized by the ten-degree system. In this system, η, which is the ratio of the flexural stiffness of the upper section to that of the lower section, is set equal to 0.5 and each spring has the same yield

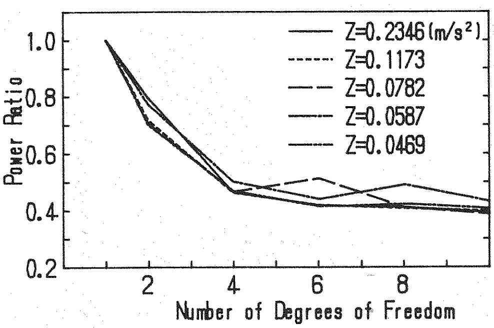


Fig. 12 Relation between power and number of degrees of freedom (stepped column, α = 0.5, θ_Y = 0.002, η = 0.5)

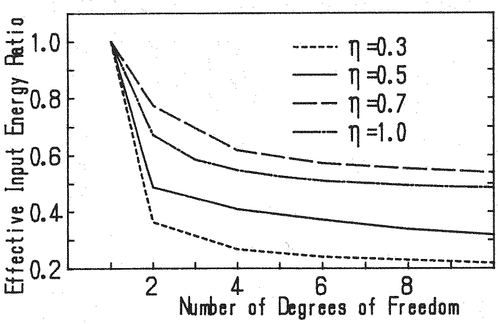


Fig. 13 Relation between E_{EF} and number of degrees of freedom (stepped column)

rotational angle. It is clear from this figure that θ_6, the rotational angle of the spring at the changing point of the cross section becomes larger and that the yielding is concentrated on this spring.

The relation between E_{EF} up to the dynamic ultimate state, Z and the number of degrees of freedom is shown in **Fig.11**, and the relation between L, Z and the number of degrees of freedom is shown in **Fig.12**. In these cases, the column is idealized by the system with even degrees of freedom up to ten. The value of E_{EF} and L for the single-degree system in **Figs.11** and **12** are calculated from E_{SU} and S_1 where only the upper portion of the column is idealized by the single-degree system. The ordinates in **Figs.11** and **12** are normalized by the value of E_{SU} and S_1, respectively. In these cases, yielding is concentrated on the spring at the changing point of the cross section and thereafter the system collapses. Moreover, the relation between E_{EF} and the number of degrees of freedom also resembles that for the uniform column and the ratio of L for the ten-degree system to L for the single-degree system agrees well with the one for the uniform column.

Figs.13 and **14** show E_{EF} and L against the number of degrees of freedom, for a stepped column with different η. While the yielding is concentrated on the spring at the changing point of the section for η=0.3 and 0.5, the yielding is concentrated on the spring at the base for η=0.7. Therefore, each E_{EF} and L for the single-degree system in **Figs.13** and **14** is calculated from E_{SU} and S_1, where only the upper portion or the entire of the column is idealized by the single-degree system. E_{EF} and L for the case of the uniform column are also shown in these figures. It is clear from these figures that the tendencies of E_{EF} and L against the number of degrees of freedom for the stepped column resemble those for the uniform column.

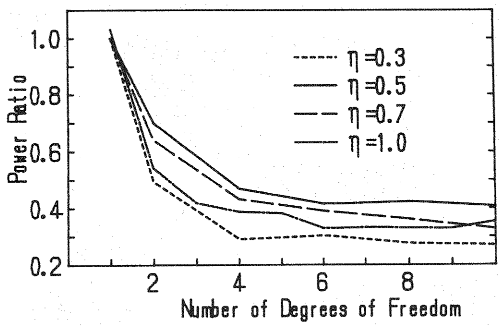


Fig. 14 Relation between power and number of degrees of freedom (stepped column)

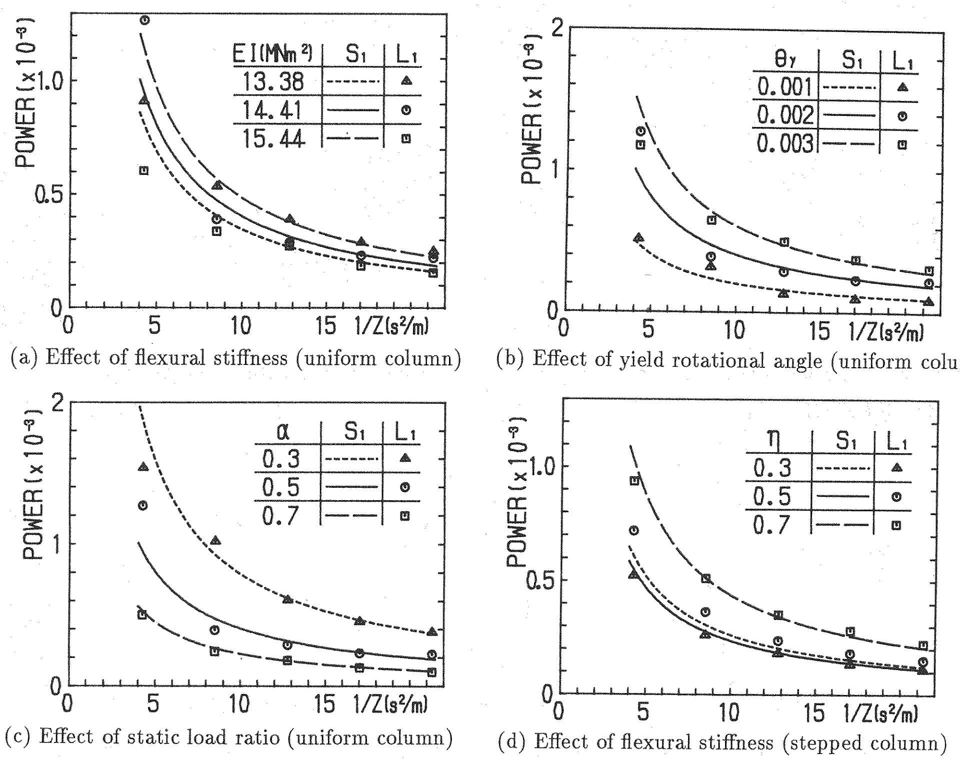
6. EVALUATION METHOD OF DYNAMIC COLLAPSE FOR MULTI-DEGREE SYSTEM

As described above, it is revealed that E_{EF} and L for the uniform column and the stepped column idealized by the spring-mass system decrease and approach respective certain values, as the number of degrees of freedom increases. This implies that L for the actual column can be approximately evaluated by L for the ten-degree system. Furthermore, the ratio of L for the ten-degree system to S_1 for the single-degree system becomes almost the same value, irrespective of the parameters analyzed. Consequently, L for the single-degree system, which is almost equal to S_1, is considered to be evaluated by L for the ten-degree system.

Then, when the ten-degree system, which is almost equivalent to the actual column, is subjected to the static axial compression and the harmonic ground motion whose frequency is equal to the first natural frequency of the system, the dynamic collapse of the system can be evaluated by the following procedure:

(1) Calculate the disturbing force power L for the ten-degree system as follows:

$$L = \int_0^{T_d} \{Z \sin \omega_1 t\}^2 dt \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (11)$$



(a) Effect of flexural stiffness (uniform column) (b) Effect of yield rotational angle (uniform column)

(c) Effect of static load ratio (uniform column) (d) Effect of flexural stiffness (stepped column)

Fig. 15 Relation between strength power S_1 and disturbing force power L_1

where T_d is the duration of the harmonic disturbing force applied to the system.

(2) Evaluate L_1 for the corresponding single-degree system by multiplying L by the factor (1/0.3).

(3) Modelling the actual column as a single-degree system according to the predicted collapse mode shape, the strength power S_1 is calculated by

$$S_1 = \frac{Z}{m x_m \omega_1} E_{SU} \quad \cdots\cdots (12)$$

in which m is the mass of the single-degree system $= k(1-\alpha)/(\omega_1 \ell)^2$, k : rotational spring constant for the single-degree system and x_m=0.537x_Y, x_Y : horizontal yield displacement for the single-degree system, $=\ell\theta_Y$. E_{SU} is also the energy absorption for the single-degree system and is represented by

$$E_{SU} = E_Y(1-\alpha)/\alpha \quad \cdots\cdots (13)$$

in which E_Y is the energy absorption up to the yielding of the spring for the single-degree system $(= 0.5k\theta_Y{}^2)$. If the system has the possibilities of the multiple collapse modes, S_1 for the collapse mode with the smaller value of E_{SU} is required to be employed.

(4) Evaluate the dynamic collapse of the column, by comparing L_1 obtained by step(2) with S_1 obtained by step(3).

Fig.15 shows L_1 evaluated by L for the ten-degree system and S_1 calculated by Eq.(12) against $1/Z$. The ordinate shows the power and the abscissa shows the reciprocal of the amplitude Z of the harmonic ground motion. It is clear from these figures that S_1 agrees well with L_1 irrespective of parameters analyzed. This implies that the dynamic collapse of the actual column is roughly evaluated by the procedure described above.

7. CONCLUSIONS

The objective of this paper is to construct a method of evaluating the dynamic collapse of a cantilever column which is subjected to a static axial compression and a lateral dynamic loading. Therefore, a parametric study was carried out to investigate the effective input energy and the disturbing force power for a uniform column and for a stepped column.

In the analysis, the column is modelled as a flexural vibrating system with finite number of degree of freedom and the system is assumed to be subjected to the harmonic ground motion whose frequency is equal to the first natural frequency of the system. Furthermore, the mass matrix of the system is formulated so that each natural frequency of the system with the employed number of degrees of freedom may agree with the corresponding flexural natural frequency of the actual structure.

Consequently, the proposed evaluation procedure of the dynamic collapse of a cantilever column is summarized as follows:

(1) When a column is modelled as a flexural vibrating system with finite number of degree of freedom, the effective input energy E_{EF} and the disturbing force power L up to the dynamic ultimate state decrease and approach respective certain values, as the number of degrees of freedom increases. This tendency is perceived, irrespective of parameters analyzed here.

(2) Therefore, L for the actual column is approximately evaluated by L for the ten-degree system. Furthermore, since the ratio of L for the ten-degree system to the strength power S_1 for the corresponding single-degree system becomes almost constant value, L_1, which almost coincides with S_1, for the single-degree system can be evaluated by L for the ten-degree system.

(3) Hence by comparing the translated L_1 for the single-degree system with the strength power S_1 for the single-degree system, the dynamic collapse of the actual column can be roughly evaluated.

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