FURTHER STUDY ON TWO-SURFACE MODEL FOR STRUCTURAL STEELS UNDER UNIAXIAL CYCLIC LOADING

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In this paper, a general definition of the accumulated effective plastic strain concept is introduced so that the two-surface model proposed by the authors in Ref. 1) can be extended easily to multiaxial stress state by combining with a proper hardening rule. In the treatment of the bounding line, the ultimate tensile stress is used as the limiting value of the bounding surface radius. In addition, the reduction of the bounding line slope is considered. Finally, the model parameters are calibrated according to the experimental data and presented. The predictions for various kinds of steels with the present model show a good agreement with the experimental results.

Keywords: *plasticity, two-surface model, cyclic loading, plastic work.*

1. INTRODUCTION

Following experimental observations, the authors proposed a two-surface model[1] for the uniaxial cyclic behavior of steels with yield plateau, such as SS400, SM490 and SM570. In order to extend this uniaxial model to the multiaxial stress state, further modification and refinement have been made in the present paper: firstly, the concept of the *accumulated effective plastic strain*[2] (as abbreviated to A.E.P.S.) is generalized to the multiaxial stress state by introducing a surface defined in the plastic strain space; secondly, the bounding surface is assumed to change in size with the plastic deformation but has some limiting value; thirdly, the bounding line slope is assumed to decrease with the plastic work.

2. EXTENSION OF A.E.P.S. IN MULTIAXIAL STRESS STATE

The concept of *accumulated effective plastic strain* (A.E.P.S.) under uniaxial condition in Refs. 1) and 2) is defined as the maximum amplitude of plastic strain that the material has ever experienced before and can be written as:

$$\bar{\varepsilon}^p = \varepsilon^p_{max} - \varepsilon^p_{min} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where ε^p_{max} and ε^p_{min} represent the arithmetic maximum and minimum values of the axial plastic strain respectively. Extending the concept of A.E.P.S. under the uniaxial case to the multiaxial stress state, a surface defined in the plastic strain space[3], called A.E.P.S. *surface*, is introduced and expressed as follows.

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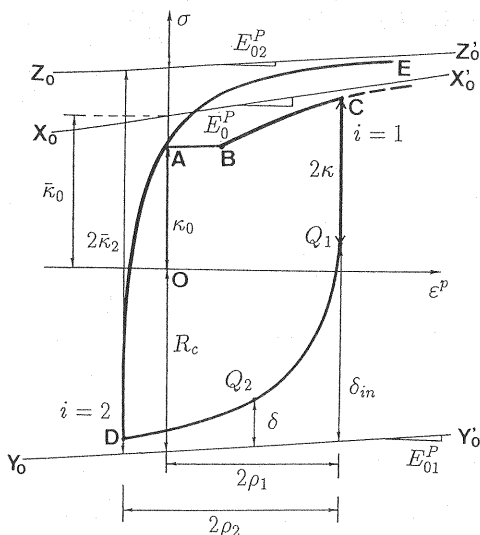***Member of JSCE, Dr. Eng., D.Sc., Professor, Dept. of Civil Engineering, Nagoya University, Nagoya

$$\phi = \phi(\varepsilon^p_{ij}) = \frac{2}{3}(\varepsilon^p_{ij} - \eta_{ij})(\varepsilon^p_{ij} - \eta_{ij}) - \rho^2 = 0 \quad \cdots(2)$$

where ε^p_{ij} is the component of plastic strain; η_{ij} and ρ represent the center and radius of the surface respectively. During the plastic deformation, A.E.P.S. surface moves and changes in size conditionally. According to the definition of A.E.P.S. in the uniaxial case, the motion of A.E.P.S. surface may be defined as follows.

$$d\eta_{ij} = \begin{cases} \frac{1}{2}d\varepsilon^p_{ij} & \text{if} \quad \phi(\varepsilon^p_{ij} + d\varepsilon^p_{ij}) > 0 \\ \\ 0 & \text{otherwise} \end{cases}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

From the condition $d\phi = 0$, there exists:

$$\frac{2}{3}(\varepsilon^p_{ij} - \eta_{ij})(d\varepsilon^p_{ij} - d\eta_{ij}) - \rho d\rho = 0 \quad \cdots\cdots\cdots (4)$$

Then substituting Eq.(3) into Eq.(4) yields:

$$d\rho = \begin{cases} \frac{1}{3}(\varepsilon^p_{ij} - \eta_{ij})d\varepsilon^p_{ij}/\rho & \text{if} \quad \phi(\varepsilon^p_{ij} + d\varepsilon^p_{ij}) > 0 \\ \\ 0 & \text{otherwise} \end{cases}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

It can be seen that, in the uniaxial case, ρ is one half of A.E.P.S. $\bar{\varepsilon}^p$. Therefore, the equations in Ref. 1) related to A.E.P.S., such as the equations to calculate the elastic range and the reduction of the yield plateau, can also be used in the multiaxial stress state by replacing $\bar{\varepsilon}^p$ with 2ρ.

3. RADIUS OF BOUNDING SURFACE

In Ref. 1), the heights of two bounding lines in tension and compression sides were assumed to change with A.E.P.S. linearly(i.e., Eq.(10) in Ref. 1)). However, since the bounding lines will become the bounding surface in the multiaxial stress state, it is important to know the radius of the bounding surface. Moreover, it is observed that the stress-strain

Fig.1 Axial Stress σ and Plastic Strain ε^p Curve
  in One Cycle Loading Experiment

curve will reach a saturated state with the increase
of plastic deformation under the cyclic loading. In
this paper, the radius $\bar{\kappa}_i$ of the bounding surface for
the loading path between the ith and the $(i + 1)$th
reversed loading points is assumed to follow

$$\bar{\kappa}_i = \bar{\kappa}_\infty + (\bar{\kappa}_0 - \bar{\kappa}_\infty)\exp\left(-\zeta\rho_i^2\right) \quad \cdots\cdots\cdots\cdots (6)$$

where $\bar{\kappa}_0$ is the radius of the initial bounding sur-
face(denoted by R in Ref. 1)); $\bar{\kappa}_\infty$ is the limiting
value of the bounding surface radius and defined to
be equal to the ultimate tensile stress σ_u; ρ_i repre-
sents one half of A.E.P.S. at the ith reversed loading
point(in Fig. 1, $i = 1$ for point C and $i = 2$ for
point D) and ζ is a constant. The parameters $\bar{\kappa}_0$
and ζ are determined from one cycle loading curve,
which spans both the tensile and compressive plastic
strain, as schematically shown in Fig. 1(i.e., loading
type (3) in Ref. 1)), in which about 5% A.E.P.S. is
required to get a stable state. As shown in Fig. 1,
the bounding lines for loading paths CD and DE are
supposed to be Y_0Y_0' and Z_0Z_0' respectively. Then,
the radii of the bounding surfaces for these two load-
ing paths are obtained to be $\bar{\kappa}_1$ and $\bar{\kappa}_2$ respectively,
where $\bar{\kappa}_1 = \frac{1}{2}(\bar{\kappa}_0 + R_c)$; R_c is the height of the bound-
ing line Y_0Y_0' from the origin point O(see Fig. 1) and
$\bar{\kappa}_2$ is the half distance between Z_0Z_0' and Y_0Y_0' along
the stress axis. Substituting these conditions into
Eq.(6) and solving the following nonlinear equations,

$$\begin{cases} \frac{1}{2}(\bar{\kappa}_0 + R_c) &= \bar{\kappa}_\infty + (\bar{\kappa}_0 - \bar{\kappa}_\infty)\exp\left(-\zeta\rho_1^2\right) \\[2mm] \bar{\kappa}_2 &= \bar{\kappa}_\infty + (\bar{\kappa}_0 - \bar{\kappa}_\infty)\exp\left(-\zeta\rho_2^2\right) \end{cases}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

the parameters $\bar{\kappa}_0$ and ζ for various steels are ob-

tained and presented in Table 1.
    It should be noted that the size of the bounding
surface is assumed to be constant during one load-
ing path and changes only at the reversed loading
points(i.e., C and D in **Fig.1**).

4. SLOPE OF BOUNDING LINE

    The slope of the bounding line is usually assumed
to be constant throughout the whole loading history
in the existing two-surface models[4],[5]. However, it
has been observed in the cyclic loading experiments
that the slope of the bounding line decreases with
the increase in loading cycles and approaches a lim-
iting value of zero. Under the previous assumption
that the slope of the bounding line dose not change
during the whole loading history in the model predic-
tion, an accurate prediction has not been obtained,
especially for the large plastic strain(say, larger than
5%). In the present paper, therefore, the slope of the
bounding line is supposed to decrease with the plas-
tic work and expressed as follows.

$$E_{0i}^P = \frac{E_0^P}{1 + \omega W_i^P} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

where E_{0i}^P represents the slope of bounding line for
the loading path between the ith and the $(i + 1)$th
reversed loading points; W_i^P is the plastic work ac-
cumulated from the beginning(i.e., point O in Fig.1)
until the ith reversed loading point; ω is a constant;
E_0^P indicates the slope of initial bounding line, which
is determined by using a monotonic loading curve up
to approximately 5% plastic strain and the size of the
initial bounding surface, $\bar{\kappa}_0$. On the other hand, the
slope E_{02}^P of the bounding line Z_0Z_0' and the plastic
work W_2^P accumulated till point D can be obtained
from the stress and plastic strain curve as shown in
Fig. 1. By substituting E_0^P, E_{02}^P and W_2^P into Eq.(8),
the parameter ω is thus determined.
    Similar to the case of bounding surface radius
$\bar{\kappa}_i$, E_{0i}^P is also assumed to be constant for one load-
ing path. Therefore, the relationship between stress
σ, plastic strain ε^p and distance δ from the current
loading point to the bounding line can be obtained
by replacing E_0^P with E_{0i}^P in Eqs. (2), (3), (4) and
(5) in Ref. 1).
    In the calibration of the model parameters, espe-
cially the bounding line slope, it is generally difficult
to obtain the suitable values at one time because
of the different personal evaluation. It has been
found that the bounding line slope affects the pre-
diction very much when the loading point lies near
the bounding line. Therefore, several sets of the pa-
rameter values were tried to predict the one cycle
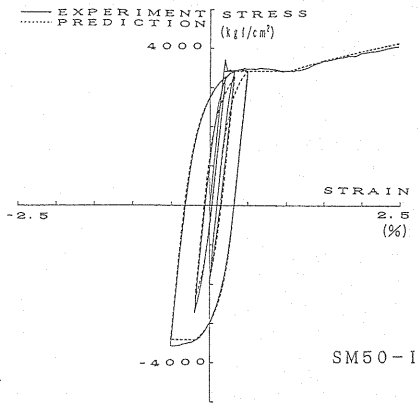loading curve and the best set was picked up.
    In order to determine the model parameters, the
slope of bounding line Z_0Z_0', E_{02}^P, should be first eval-
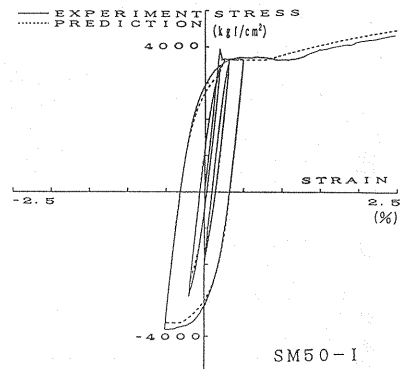uated from one cycle loading curve, that is, a stable
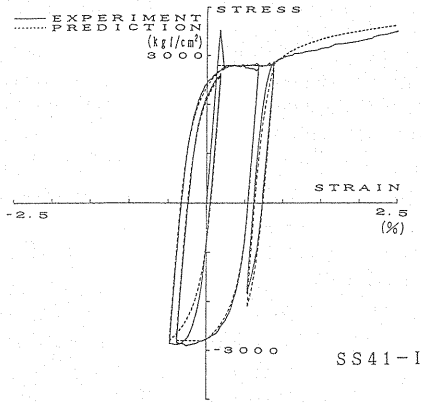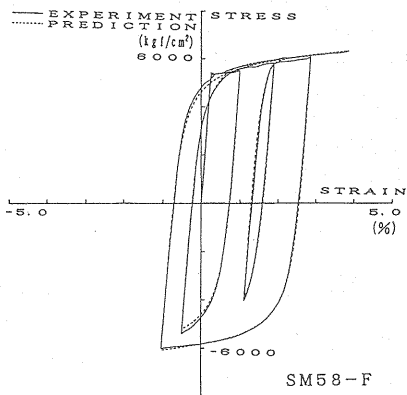
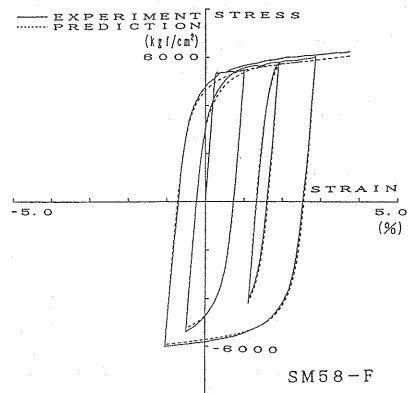(a) The example of steel SS400
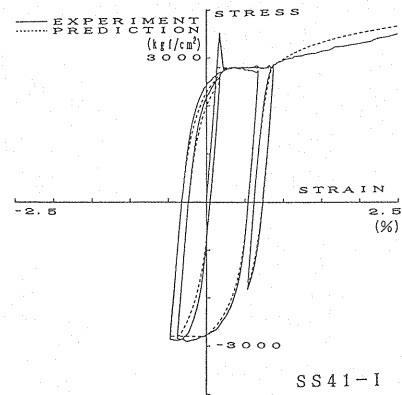
(b) The example of steel SM490

(c) The example of steel SM570

Fig. 2 The Comparison between Experiment
and Prediction by the Present Model

(a) The example of steel SS400

(b) The example of steel SM490

(c) The example of steel SM570

Fig. 3 The Comparison between Experiment
and Prediction by the Previous Model

Table 1 The parameters of steels SS400, SM490 and SM570 in the present model

material	SS400	SM490	SM570
$E(kgf/cm^2)$	2.11×10^6	2.10×10^6	2.20×10^6
$\sigma_y(kgf/cm^2)$	2.80×10^3	3.64×10^3	5.35×10^3
E_{st}^p/E	2.49×10^{-2}	3.40×10^{-2}	1.02×10^{-2}
E_0^p/E	8.96×10^{-3}	1.01×10^{-2}	7.85×10^{-3}
$\omega \cdot \sigma_y$	3.08	4.0	2.67
$\bar{\kappa}_0/\sigma_y$	1.15	1.13	1.06
σ_u/σ_y	1.81	1.61	1.22
$\zeta \cdot \varepsilon_y^2$	9.89×10^{-4}	1.52×10^{-3}	8.04×10^{-3}

Note: (1) $\bar{\kappa}_\infty = \sigma_u$ is the limiting value of the bounding surface radius; (2) ε_y is the strain at the initial yield point, i.e., $\varepsilon_y = \sigma_y/E$; (3) see Table 2 in Ref. 1) for the other parameters.

stress-plastic strain loading curve up to 5% in tension side(DE in Fig. 1). As for the compression part, i.e., CD in Fig. 1, it is difficult to evaluate the bounding line Y_0Y_0' because of a possible occurrence of buckling and therefore the same slope of the bounding line Z_0Z_0' as in tension part is assumed in the determination of the model parameters.

5. EXAMPLES

In the uniaxial case, the present model is the same as the previous one except the calculation of the bounding surface radius and the slope of the bounding line. The present model is applied to predict the uniaxial cyclic experiment for steels SS400, SM490 and SM570. The comparison between the experiment and the prediction by the present model is shown in Fig. 2. At the same time, the prediction by the previously proposed model is shown in Fig. 3 for the same experiments. It can be seen that the prediction by the present model is accurate enough even for the yield plateau.

6. SUMMARY AND CONCLUSIONS

From the present study, it can be concluded that:
(1) With the introduction of A.E.P.S. surface, the present uniaxial two-surface model can be easily extended to the multiaxial stress state.
(2) By considering reduction of the bounding line slope, the present model is more accurate, especially when the plastic strain is relatively large.
(3) The assumption that the radius of bounding line changes with plastic deformation and limits to the ultimate stress σ_u is in accord with the experimental results. Moreover, Eq.(6) is convenient in the case of multiaxial stress state.

The present model can be extended to the multiaxial stress state easily and will be presented in a different paper. The parameters newly introduced in the present model, such as ω, ζ, $\bar{\kappa}_0$ and $\bar{\kappa}_\infty$, are simply and clearly determined without increasing the experiment work.

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