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EVALUATION OF DYNAMIC COLLAPSE OF SDOF SYSTEM WITH DEGRADING RESTORING FORCE CHARACTERISTICS UNDER RANDOM EXCITATION

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The objective of this paper is to construct a method of evaluating the dynamic collapse of an SDOF system with the degrading restoring force characteristics under random excitation. Firstly, an extensive parametric study is carried out to investigate the duration up to the dynamic ultimate state of the system and the power, when the system is subjected to the combined harmonic disturbing force. Secondly, through the results, a method of evaluating the dynamic collapse of the system subjected to a random excitation is proposed, in which the random excitation is replaced by a harmonic disturbing force of the resonance frequency. Finally, numerical examples are developed to illustrate the application of the method.

Keywords : *dynamic collapse, SDOF system, degrading restoring force characteristics, power, maximum energy absorption, input energy, random excitation*

1. INTRODUCTION

Recently, it has been pointed out that the conventional earthquake resistant design procedure, in which the effect of dynamic load is replaced by the static one, has disadvantage in estimating the dynamic strength. Therefore, in order to establish the dynamic limit state design method based on the true dynamic ultimate strength of structures, a number of researches on the input energy imparted to a structure by a dynamic load and on the energy absorption up to the failure of the structure have been carried out. However, to accomplish this object, the relation between the dynamic ultimate strength of a structure subjected to a random dynamic loading and its applied dynamic loading characteristics has to be investigated. Namely, the relation between the load and the strength of structures should be clarified.

From this point of view, there are many researches dealing with the parametric studies on the energy quantities of the structures subjected to the random dynamic loading[1)~11)]. These researches also discussed qualitatively the relation between the input energy exerted by the dynamic loading and the characteristics of the dynamic loading. However, since so-called random dynamic loading is irregular in character by nature, further extensive parametric study is required to establish the dynamic strength design method based on the way of the above researches.

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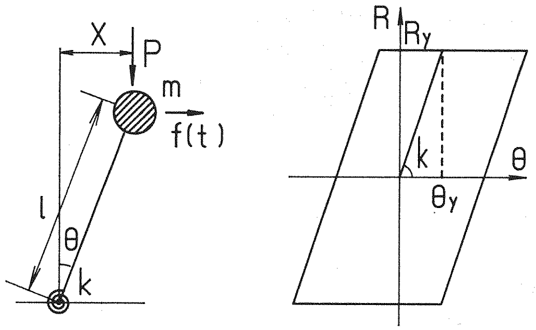
Then the authors et al. investigated the relation between a harmonic disturbing force and the strength of a single-degree-of-freedom (SDOF) system with the degrading restoring force characteristics[12)]. As a result, a method of evaluating the dynamic collapse of the system without employing any dynamic analysis was proposed using the power corresponding to the input energy exerted by the disturbing force. However, practical random dynamic loadings such as earthquake excitation differ from a harmonic disturbing force. Therefore, it is necessary to develop this procedure into the case where the system is subjected to the random excitation.

In this paper, for an SDOF system with the degrading restoring force characteristics, a parametric study is carried out to investigate the duration up to the dynamic ultimate state of the system and the power, when the system is subjected to the combined harmonic disturbing force. According to these results, a method of evaluating the dynamic collapse of the system, which is subjected to a harmonic disturbing force with the same frequency as the natural frequency of the system, is extended to the case where the system is subjected to the random excitation by replacing the random excitation with an equivalent harmonic disturbing force.

2. EVALUATION OF DYNAMIC COLLAPSE UNDER SINGLE HARMONIC EXCITATION[12)]

A method of evaluating the dynamic collapse of an SDOF system subjected to a harmonic disturb-



(a) SDOF system with structural instability (b) Ideal elasto-plastic restoring moment characteristics

Fig.1 Structural model

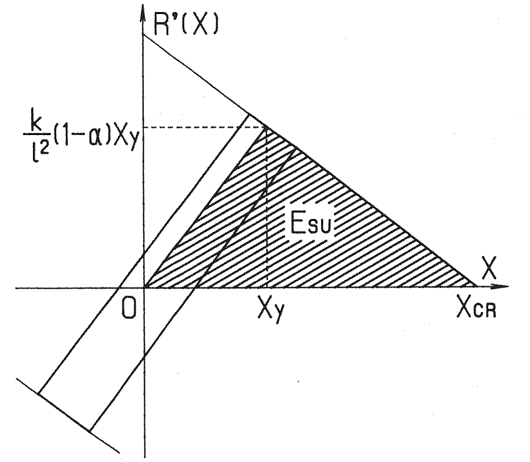


Fig.2 Degrading restoring force characteristics of system

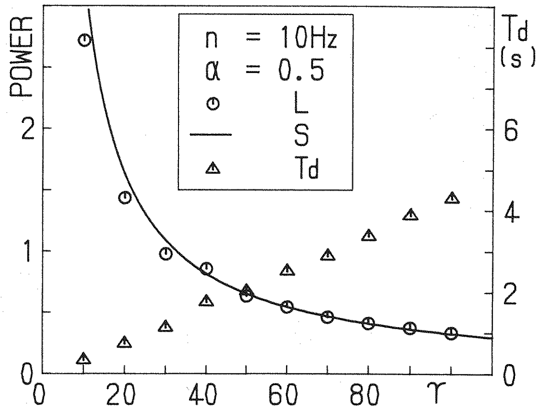


Fig.3 Relation between L, S, T_d and γ under harmonic disturbing force

ing force is summarized in this chapter. The details are described in Ref. 12).

If the restoring moment characteristics of the rotational spring of an SDOF system shown in **Fig.1**(a) are the ideal elasto-plastic characteristics shown in **Fig.1**(b), the horizontal restoring force R'(x)-displacement x relation becomes the degrading restoring force characteristics as shown in **Fig.2**[13]. Therefore, the system has a maximum elasto-plastic strain energy absorption E_{SU}, which is shown by the hatched area in **Fig.2** and is analytically obtained from

$$E_{SU}=E_Y(1-\alpha)/\alpha \cdots\cdots (1)$$

in which E_Y is the maximum elastic strain energy $(=k\theta_Y{}^2/2)$ and α is the ratio of the applied static vertical load P to the critical load $P_{CR}(=k/l)$. From the results in Ref.13), it is revealed that the state of the SDOF system shown in **Fig.1** reaches the dynamic ultimate state, when the input energy E_F exerted by a disturbing force $f(t)$ exceeds E_{SU}, if the system has no energy loss by a viscous and hysteretic damping. The relation is written as

$$E_F=\int_0^{T_d} f(t)\cdot \dot{x}(t)dt \geqq E_{SU} \cdots\cdots (2)$$

in which T_d is the duration of the applied disturbing force. According to Eq.(1), E_{SU} is equal to E_Y, if α $=0.5$. In this case, however, the rotational spring yields and the state is reached in the plastic region. The detailed discussion is left to Ref.13). If the system is subjected to the following harmonic disturbing force due to the ground motion

$$f(t)=-mZ\sin\omega t \cdots\cdots (3)$$

and the displacement response is assumed as

$$X(t)=X_m\cos\omega t \cdots\cdots (4)$$

then Eq.(2) becomes as follows：

$$E_F=\frac{mX_m\omega}{Z}\int_0^{T_d}(Z\sin\omega t)^2dt \geqq E_{SU} \cdots\cdots (5)$$

in which Z is the amplitude of the ground acceleration, and X_m is the steady-state amplitude of the displacement response and is assumed to be given as $X_m=0.537\,X_Y(X_Y$：horizontal yield displacement, $=l\theta_Y)$.

Thus the dynamic collapse of the system can be evaluated by comparing the strength power S given by Eq.(6) with the disturbing force power L given by Eq.(7).

$$S=\frac{Z}{mX_m\omega}E_{SU} \cdots\cdots (6)$$

$$L=\int_0^{T_d}(Z\sin\omega t)^2dt \cdots\cdots (7)$$

An example of the disturbing force power L, the strength power S and the duration T_d up to the dynamic ultimate state against the amplitude Z of the acceleration is shown in **Fig.3**, in the case where the system is subjected to a harmonic disturbing force at resonance and the dynamic ultimate state is just reached. The ordinates show the disturbing force power L by circles, the strength power S by a solid line and the duration T_d by triangles. The abscissa shows the acceleration amplitude Z in terms of the yield strength coefficient γ ($\gamma=R_Y/lmZ$, R_Y : yield restoring moment). Here, the results are shown for $n=10$ Hz, in which n is the natural frequency of the corresponding elastic system under no static vertical load. It will be noted from the figure that the strength power S agrees well with the disturbing force power L and that the duration T_d up to the dynamic ultimate state increases as γ increases, that is, Z decreases. Therefore, in the case where a system with an arbitrary property is subjected to a harmonic disturbing force with an arbitrary amplitude Z and duration T_d at resonance, whether the system collapses or not can be evaluated by comparing the power L with S.

3. EVALUATION OF DYNAMIC COLLAPSE UNDER COMBINED HARMONIC EXCITATION

(1) Duration up to dynamic ultimate state

The final objective of the paper is to establish a method of evaluating the dynamic collapse of the SDOF system with the degrading restoring force characteristics, when the system is subjected to the random excitation. As the first step, the parametric study on dynamic behavior is carried out in this section, when the system is subjected to a set of two single harmonic disturbing forces. It is revealed that the dynamic collapse of the SDOF system subjected to a harmonic disturbing force is hard to occur, as the exciting frequency recedes from the natural frequency of the system[12].

Therefore, in the case where either of the frequencies of the combined harmonic disturbing force is not near the resonance frequency, the system may not collapse under the combined one. Then it is assumed that one of the exciting frequencies is equal to the resonance frequency and that the other is not equal to the resonance one. The former is called the resonance disturbing force which has the circular frequency ω_A and the latter is called the non-resonance disturbing force which has the circular frequency ω_B.

When the non-resonance disturbing force and the resonance one are applied to the system, the former is considered to have some influence on the dynamic collapse of the system. Then it may be inferred that the duration up to the dynamic ultimate state of the system under the combined harmonic disturbing force differs from the one under the application of the resonance disturbing force only. Namely, by investigating the duration up to the dynamic ultimate state, the effect of the non-resonance disturbing force on the dynamic collapse of the system can be evaluated to some extent.

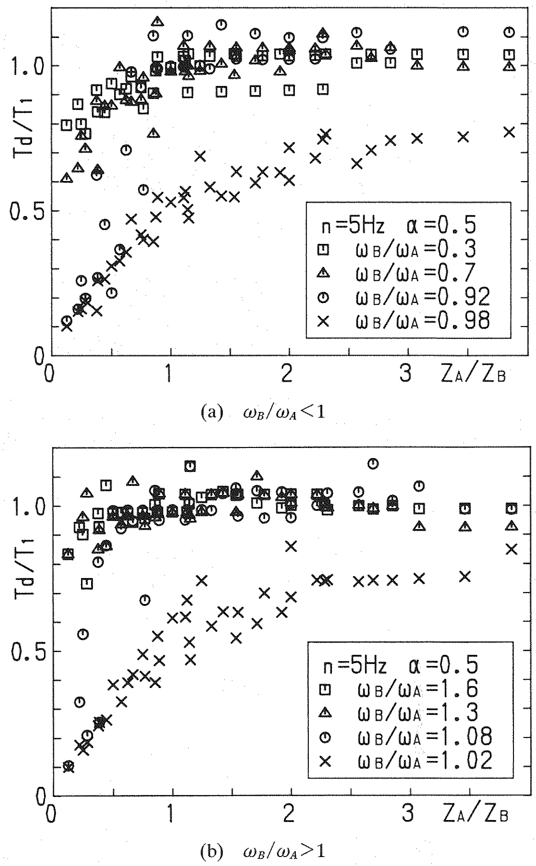
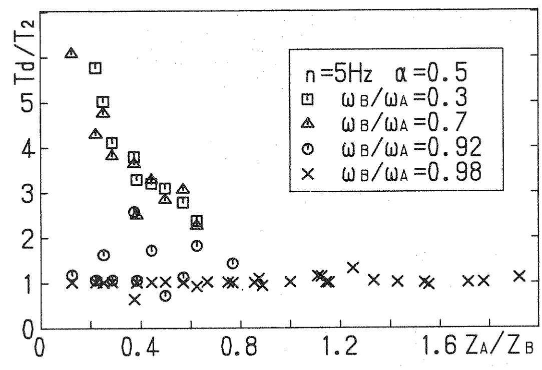


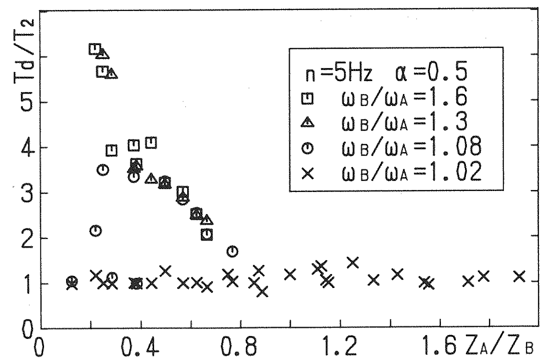
Fig.4 Relation between T_d/T_1, Z_A/Z_B and ω_B/ω_A

Fig.4 shows the relation between T_d, the duration up to the dynamic ultimate state, the ratio of the amplitude Z_A of the resonance disturbing force to the amplitude Z_B of the non-resonance one and its frequency ratio ω_B/ω_A, when the system is subjected to the combined disturbing force. The ordinate shows the duration T_d normalized by T_1, the duration up to the dynamic ultimate state under the resonance disturbing force only. The abscissa shows the amplitude ratio Z_A/Z_B. It will be noted that there are cases where the ratio T_d/T_1 is nearly equal to unity, or less than unity. The former corresponds to the case where the non-resonance disturbing force has little effect on the dynamic collapse and the latter corresponds to the case where the non-resonance one may have some effect.

Fig.5 shows the relation between Z_A/Z_B, ω_B/ω_A and the duration T_d normalized by the duration T_2. The duration T_2 is the one up to the dynamic ultimate state, when the system is subjected to the resonance disturbing force of amplitude Z_A+Z_B. It



(a) $\omega_B/\omega_A<1$



(b) $\omega_B/\omega_A>1$

Fig.5 Relation between T_d/T_2, Z_A/Z_B and ω_B/ω_A

is clear from **Fig.5** that T_d agrees well with T_2 as ω_B approaches ω_A. This implies that the non-resonance disturbing force can be replaced by the resonance disturbing force of the same amplitude Z_B.

Consequently, if ω_B is not near ω_A, the duration T_d up to the dynamic ultimate state under the combined disturbing force $Z_A \sin\omega_A t + Z_B \sin\omega_B t$ is nearly equal to the one under the disturbing force $Z_A \sin\omega_A t$, because the non-resonance disturbing force has little effect on the dynamic collapse of the system. On the other hand, if ω_B is close to ω_A, the duration T_d under the combined disturbing force $Z_A \sin\omega_A t + Z_B \sin\omega_B t$ is nearly equal to the one under the disturbing force $(Z_A+Z_B)\sin\omega_A t$. Therefore, the effect of the combined disturbing force is considered to be replaced by the effect of an equivalent resonance one by using the coefficient ϕ as follows:

$$Z_A \sin\omega_A t + Z_B \sin\omega_B t = (Z_A+\phi Z_B)\sin\omega_A t \qquad \cdots\cdots(8)$$

Then, when the system is subjected to the equivalent disturbing force defined in Eq.(8) and the dynamic ultimate state is just reached, if the strength power S in Eq.(6) is assumed to be equal

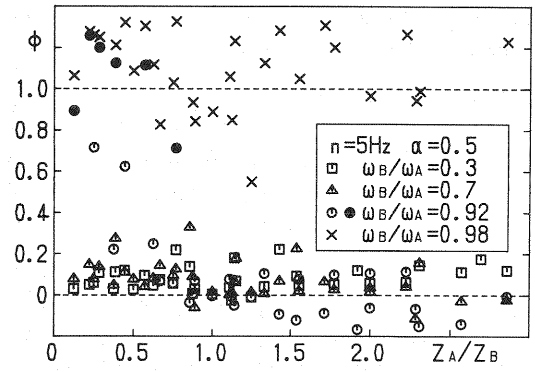


Fig.6 Relation between Z_A/Z_B, ω_B/ω_A and ϕ

to the disturbing force power L in Eq.(7), the equivalent amplitude Z can be obtained as follows:

$$Z=\frac{4E_{SU}}{mX_m\{2\omega_A T_d-\sin(2\omega_A T_d)\}}(=Z_A+\phi Z_B) \qquad \cdots\cdots(9)$$

Therefore, the coefficient ϕ, by which the amplitude of the non-resonance disturbing force is multiplied, can be obtained by the reverse calculation. **Fig.6** shows the relation between Z_A/Z_B, ω_B/ω_A and ϕ obtained from Eq.(9). From the figure, it can be seen that ϕ scatters mostly around zero or unity. Namely, ϕ is nearly equal to zero, when ω_B/ω_A is much less than unity, and ϕ is nearly equal to unity as ω_B/ω_A approaches unity. However, when ω_B/ω_A is equal to 0.92, ϕ is distributed around both zero and unity.

Then to investigate the reason of the above result, the power L_{AB} of the combined disturbing force written in the following is examined:

$$L_{AB}=L_A+L_B+2L_C \qquad \cdots\cdots(10)$$

in which

$$L_{AB}=\int_0^{T_d}(Z_A \sin\omega_A t+Z_B \sin\omega_B t)^2 dt$$

$$L_A=\int_0^{T_d}(Z_A \sin\omega_A t)^2 dt$$

$$L_B=\int_0^{T_d}(Z_B \sin\omega_B t)^2 dt$$

$$L_C=\int_0^{T_d}(Z_A \sin\omega_A t\cdot Z_B \sin\omega_B t)\,dt$$

According to the equations, the power L_{AB} is composed of the term L_C, which is the product of the resonance component and the non-resonance one, in addition to L_A and L_B. Then the power depends on the term L_C and the input energy exerted by the combined disturbing force is considered to be also affected by the term.

Thus to investigate the effect of the term L_C on the power, the following power ratio λ is introduced:

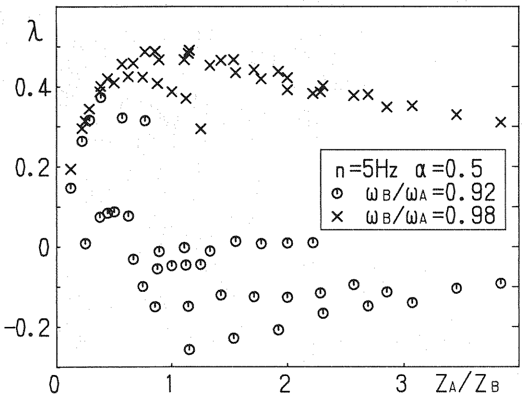


Fig.7 Relation between λ and Z_A/Z_B

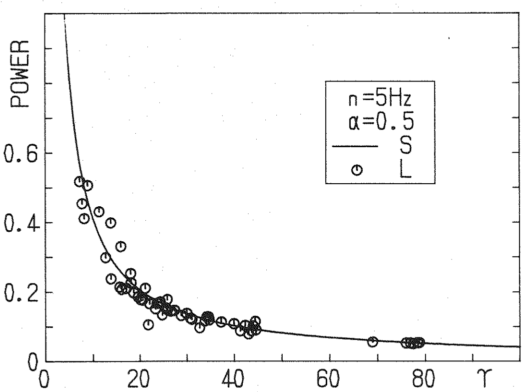


Fig.8 Relation between L, S and γ under combined disturbing force

$$\lambda=2L_C/L_{AB} \quad \cdots\cdots(11)$$

Fig.7 shows the relation between the power ratio λ and Z_A/Z_B for $\omega_B/\omega_A=0.92$ and 0.98. From a comparison of **Fig.6** with **Fig.7**, it is evident that the value of ϕ is scattered around unity for $\lambda \geqq 0.1$ and around zero for $\lambda<0.1$. In **Fig.6**, the closed circles correspond to the case where ω_B/ω_A is equal to 0.92 and λ is greater than 0.1. The similar tendency is confirmed in the case where $\omega_B/\omega_A>1$, but the case is omitted in this paper.

(2) Evaluation of equivalent amplitude

From the results of the extensive parametric study based on the above considerations, it is revealed that the non-resonance disturbing force can be replaced by an equivalent resonance one and that the value of ϕ in Eq.(8) is considered to be determined by the following condition corresponding to the ratio of the non-resonance frequency to the resonance one.

Namely, ϕ is equal to unity for $|1-\omega_B/\omega_A| \leqq 0.02$, $\phi=0.02$ for $|1-\omega_B/\omega_A|>0.16$ and for $0.02<|1-\omega_B/\omega_A| \leqq 0.16$, ϕ is set equal to 0.02 or unity according to the value of λ. Here ϕ is set equal to not zero, but 0.02, because the system may collapse under the non-resonance disturbing force of larger amplitude, even if its frequency ω_B is not near ω_A. The values of 0.16 and 0.02 are determined by the method of trial and error so that the power L is more correlated to the power S.

Fig.8 shows the relation between the equivalent amplitude Z and the disturbing force power L obtained from Eq.(7), when the system is subjected to the combined disturbing force and collapses. A part of data in **Fig.4**(a) is employed to depict the plot in **Fig.**8, where ω_B/ω_A is ranged from 0.3 to 0.98. The ordinate shows the power and the abscissa shows the yield strength coefficient γ. The strength power S obtained from Eq.(6) is also plotted in this figure. It will be noted that the disturbing force power L up to the dynamic ultimate state agrees well with the strength power S.

4. EVALUATION OF DYNAMIC COLLAPSE UNDER RANDOM EXCITATION

When a disturbing force composed of the harmonic component with the resonance frequency and another harmonic component is applied to the system, it is possible to evaluate the dynamic collapse of the system by replacing the combined disturbing force with the resonance disturbing force of an equivalent amplitude. In this case, the equivalent resonance disturbing force does not have the same power L as the combined disturbing force does, but only the duration up to the dynamic ultimate state under the equivalent disturbing force agrees with the one under the combined disturbing force.

Then, the efficiency of the above technique must be investigated by applying it to the case where the system is subjected to the random excitation and collapses.

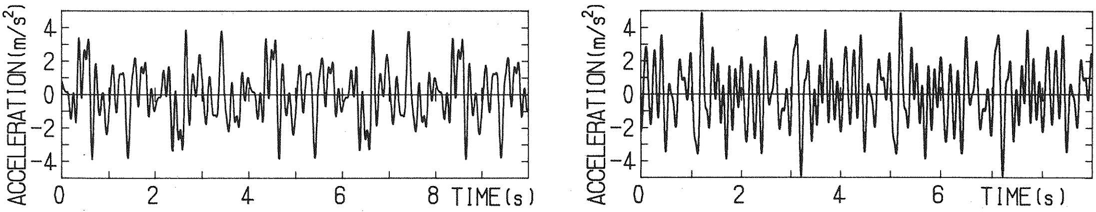
(1) Generation of random excitation

The random excitation employed in the analysis is generated by combining the random variables of the amplitude, the frequency and the phase angle. Five artificial random accelerograms are generated here. **Fig.9** shows examples of the random accelerograms and **Fig.10** also shows the corresponding Fourier amplitude spectra. In **Fig.10**, the ordinate shows the Fourier amplitude spectra normalized by its maximum value.

Here, a parametric study is carried out by changing a set of the natural frequency n and the static load ratio α of the system, and the amplitude of the random accelerograms.

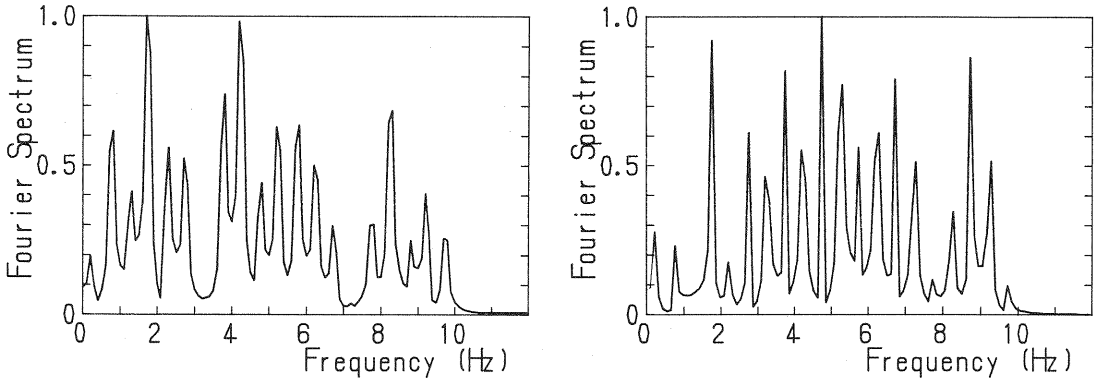
(2) Effect of phase angle

In the case where the combined harmonic



(a) Sample No.1 (b) Sample No.2

Fig.9 Examples of artificial accelerogram



(a) Sample No.1 (b) Sample No.2

Fig.10 Fourier amplitude spectrum of artificial accelerogram

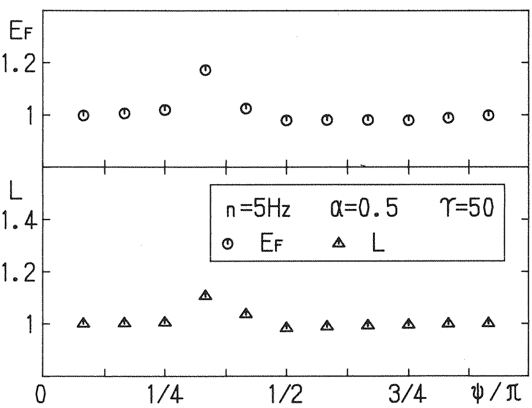


Fig.11 Relation between E_F, L and ψ

disturbing force is applied to the system, the phase angle is not taken into account. However, each harmonic component of the random excitation is considered to have a different phase angle.

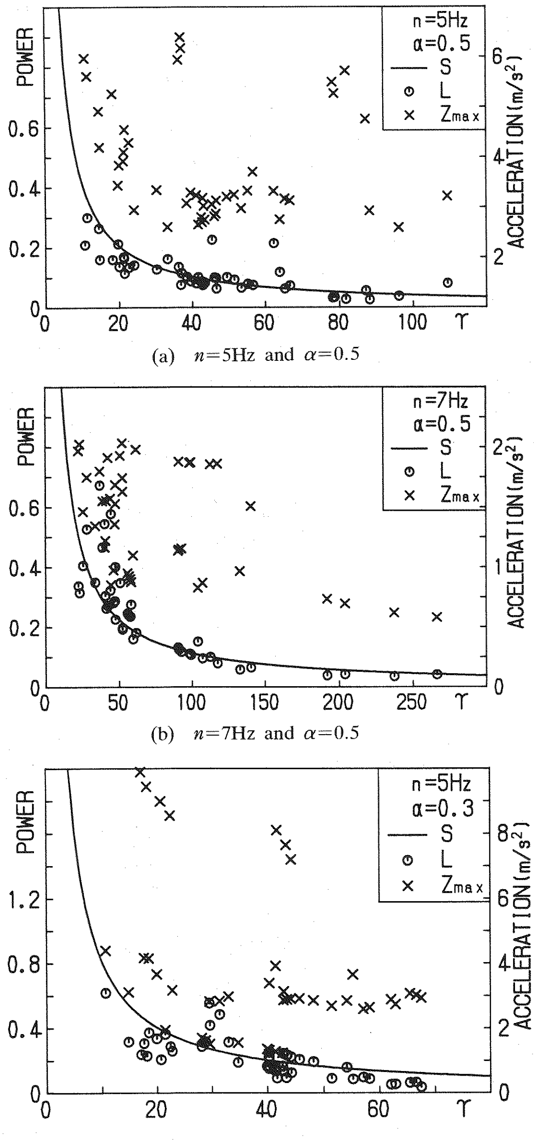
Then, when the system is subjected to a harmonic disturbing force with the resonance frequency and an arbitrary phase angle, the input energy exerted by the disturbing force and its power L against the phase angle ψ are shown in **Fig.11**. The ordinates show the input energy and the power normalized by the value for $\psi=0$, and the abscissa shows the phase angle. If the system has some hysteretic energy loss, the value of the input energy becomes larger and then the power L also becomes larger. However, maximum differences in the input energy and the power affected by the phase angle is less than 20%. Therefore, the effect of the phase angle of the random excitation is not taken into account in the parametric study.

(3) Application of evaluation procedure of dynamic collapse under random excitation

Here, the technique described in chapter 3, which evaluates the dynamic collapse of the system subjected to the combined harmonic disturbing force, is applied to the case where the system is subjected to the random excitation. Firstly, each harmonic component is obtained by the Fourier transform technique such as FFT. Secondly, the corresponding coefficient ϕ is selected according to the ratio of its frequency to the resonance frequency. Thirdly, the equivalent amplitude Z is given by

$$Z=\phi_1 Z_1+\phi_2 Z_2+\cdots \qquad (12)$$

In calculating the equivalent amplitude, the harmonic component in the range $\omega/\omega_A>2$ and except in the vicinity of the resonance frequency, the harmonic one, whose amplitude is smaller than one-fifth of the maximum amplitude, are ignored, because these harmonic components have little



(a) n=5Hz and α=0.5

(b) n=7Hz and α=0.5

(c) n=5Hz and α=0.3

Fig.12 Relation between L, S, Z_{max} γ under random excitation

effect on the dynamic collapse of the system. **Fig.12** shows the relations between the disturbing force power L, the strength power S and the equivalent amplitude Z. The abscissa shows the yield strength coefficient corresponding to Z and the ordinate shows the strength power S by a solid line and the disturbing force power L by circles. Whereas the strength power is obtained by substituting Z and the structural properties into Eq.(6), the disturbing force power L is obtained by substituting the duration T_d and Z into Eq.(7). The peak acceleration Z_{max} of the applied random excitation is also shown in this figure by crosses.

Table 1 Values of coefficient ϕ_i

Range of Frequency Ratio	Range of Power Ratio	Coefficient ϕ_i
$0.98 \leqq \omega_i/\omega_A \leqq 1.02$	――	1
$0.84 \leqq \omega_i/\omega_A < 0.98$ or $1.02 < \omega_i/\omega_A \leqq 1.16$	$\lambda \geqq 1$	1
	$\lambda < 1$	0.02
$\omega_i/\omega_A < 0.84$ or $1.16 < \omega_i/\omega_A \leqq 2.0$	――	0.02
$2.0 < \omega_i/\omega_A$	――	0

These figures correspond to the cases that the system has a different natural frequency n and a different static load ratio α. From these figures, it is clear that Z_{max} is not correlated to γ, but that the disturbing force power L is distributed around the strength power S, irrespective of γ. This implies that the dynamic collapse can be roughly evaluated by the procedure, when the system is subjected to a random excitation of an arbitrary duration and any frequency components.

(4) Summary of evaluation procedure of dynamic collapse

A procedure of evaluating the dynamic collapse of an SDOF system with the degrading restoring force characteristics is summarized here, when the system is subjected to the random excitation.

Firstly, an acceleration amplitude of each harmonic component for an earthquake excitation with an arbitrary duration is obtained by the Fourier transform technique. In this case, the frequency of each harmonic component is normalized by the natural frequency of the system in advance. According to the frequency ratio, the coefficient ϕ_i, which translates the amplitude of an arbitrary harmonic component into the one of resonance component, is determined from the condition shown in **Table 1**.

The equivalent amplitude is calculated using Eq.(12) and then the strength power S and the disturbing force power L are calculated using the following equations :

$$S=\frac{Z}{mX_m\omega}E_{SU} \cdots\cdots (13)$$

$$L=\int_0^{T_d}(Z\sin\omega t)^2 dt \cdots\cdots (14)$$

By comparing the power L with S, whether the system collapses or not is determined as follows :

$S>L$: survive

$S \leqq L$: collapse

Consequently, it is possible without using any dynamic analysis to evaluate the dynamic collapse of the SDOF system with degrading restoring force characteristics which is subjected to the random excitation.

However, in the range of this paper, the coefficient ϕ is determined so that the disturbing force power L is distributed around the strength power S on the average. Therefore, while the proposed method judges the system not to collapse, actually the system may collapse under the random excitation. Then, for more practical design method, it may be necessary to select the coefficient ϕ so that the disturbing force power L is always conservative, when the system is subjected to the random excitation and collapses.

5. CONCLUSIONS

The objective of this paper is to construct a method of evaluating the dynamic collapse of an SDOF system with the degrading restoring force characteristics under random excitation, from a standpoint of establishing the dynamic strength design method.

Firstly, an extensive parametric study was carried out to investigate the duration up to the dynamic ultimate state of the system and the power, when the system was subjected to a combined harmonic disturbing force, which was composed of a harmonic disturbing force with resonance frequency and another with other frequency. Secondly, according to these results, a method of evaluating the dynamic collapse of the system subjected to a harmonic disturbing force was extended to the case where the system was subjected to the random excitation by replacing the random excitation with a harmonic disturbing force of the resonance frequency. Finally, its efficiency was confirmed by applying the proposed technique to the case where the system was subjected to artificial random excitations.

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