FRACTURE ANALYSES OF MEDIA COMPOSED OF IRREGULARLY SHAPED REGIONS BY THE EXTENDED DISTINCT ELEMENT METHOD

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Analysis of a medium which consists of irregularly shaped regions is a major problem in the DEM studies. A new method for solving this problem is proposed. We developed the Extended Distinct Element Method (EDEM) and applied it to problems of fracture behavior in media composed of irregularly shaped regions. An aggregate of circular (2-D) or spherical (3-D) elements connected by pore-springs constitutes an irregularly shaped region, and is called the "Combined Discrete Element (CDE)". The concept of the CDE has greatly broadened application of the EDEM as the behavior of an anisotropic medium composed of variously shaped regions can be easily studied; moreover, cracks within the CDE as well as complete fracture of the CDE can be simulated automatically.

Keywords : *Distinct Element Method, Extended (or Modified) Distinct Element Method, Combined Discrete Element, Fracture Analysis, Computer Simulation*

1. INTRODUCTION

When an objective medium has strong discontinuity and/or heterogeneity, it is difficult to analyze its behavior by procedures such as the Finite Element Method (FEM) in which a medium is assumed to be continuous. Consequently, numerical methods in which the objective medium is considered an assembly of many independent small elements have been developed to deal with that problem[1]~[4]. The Distinct Element Method (DEM) is one such method that is based on the assumptions that each individual element satisfies the equations of motion and that the transmission of force between elements follows the law of action and reaction. The equations of motion for each element are solved by step-by-step numerical integration in the time domain. The DEM allows us to analyze numerically problems in which the contact between elements changes at any moment and compensates for the weak points of continuous analysis methods. Because this method will be realistic only with major improvements to computer systems, there are many problems still to be solved. Analysis of media composed of irregularly shaped regions is one major problem. We here propose a new idea by which this problem is easily solved.

In early versions of the DEM, polygonal elements were often used, the forces acting

between these elements being assumed to be transmitted through contact between corners or edges. The use of polygonal elements required extended computations because of the complexity in judging the contact between elements. The total number of elements that could be used for simulation was limited by the amount of CPU time available. To solve this problem, circular (2-D)[1],[5] and spherical (3-D)[6] elements were introduced because of the simplicity of their contact conditions. Although it is easy to judge the contact between circular or spherical elements, it is difficult to consider the anisotropy of the medium brought about by the shape of the elements used. In 2-D problems, analyses have been made that use elements of noncircular, but simple geometrical shapes (elliptical[7], triangular[8], square[1]). In reality, media that have regions made up of such shapes are unusual; moreover, much more computation time is needed than when circular elements are used.

A medium consisting of independent discontinuous elements has some continuity due to the bonding effect of the pore material. Iwashita and Hakuno[9],[10] proposed an additional nonlinear spring (Iwashita's pore-spring) which represents the effect of clay between gravels and made a DEM analysis of the dynamic behavior of ground. Although introduction of Iwashita's pore-spring brought some continuity to the DEM model, neither the transmission of the moment nor rotation of the element produced by the effect of the pore-springs was considered because Iwashita assumed that clay does not transmit the moment. He also assumed that the force from the pore-spring acts on the elements in a direction such that the relative displacement vector between two elements connected by the pore-spring would be

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the same as in the initial situation. The rotational behavior of all the elements connected by the pore-spring, as one body, could not be analyzed; therefore, starting with Iwashita's model, Meguro and Hakuno[11] established a new pore-spring model (Meguro's model) in which both transmission of the moment and rotation of the elements are considered. In this model, an aggregate of elements connected by pore-springs rotates as one body. Combining the DEM with this new pore-spring, they developed a new simulation program that uses arrays of different material parameters and called it the "Extended Distinct Element Method (EDEM)" or "Modified Distinct Element Method (MDEM)". With this method, Meguro and Hakuno analyzed the fracture behavior of concrete structures[12] as well as nonlinear soil-structure interaction[13]. They showed that the conventional belief that the DEM is applicable only to soil (geotechnical engineering) is wrong, and pointed out the usefulness of the EDEM for other materials.

A series of behaviors (from the continuous to perfect discrete stage) can be analyzed, and the material, structural and geometrical nonlinearities expressed automatically in the EDEM. The effect of pore material is modeled by pore-springs, and an aggregate of elements connected by pore-springs behaves as a single body both in translation and rotation. Therefore, the behavior of media composed of irregularly shaped regions should be representable by the EDEM.

A new idea, which enables us to use the EDEM to analyze media composed of variously shaped regions, is proposed: An irregularly shaped region is taken as an aggregate of circular (2-D) or spherical (3-D) elements for which judgment of the contact conditions is simple. This aggregate of elements is called the "Combined Discrete Element (CDE)". The pore-springs between the circular or spherical elements inside the CDE hold the elements together. We have used the EDEM to simulate two dimensional problems of the fracture behavior of media composed of square and irregularly shaped regions with CDEs corresponding to the respective shapes.

The numerical results obtained in the study reported here confirm that introduction of the CDE to the EDEM makes it easy to simulate the fracture behavior (from the continuous to perfect discrete stage) of media composed of variously shaped regions. Moreover, this new idea can be readily applied to three dimensional problems by replacing circular elements with spherical ones. This method has wide application and great usefulness in various fields of engineering.
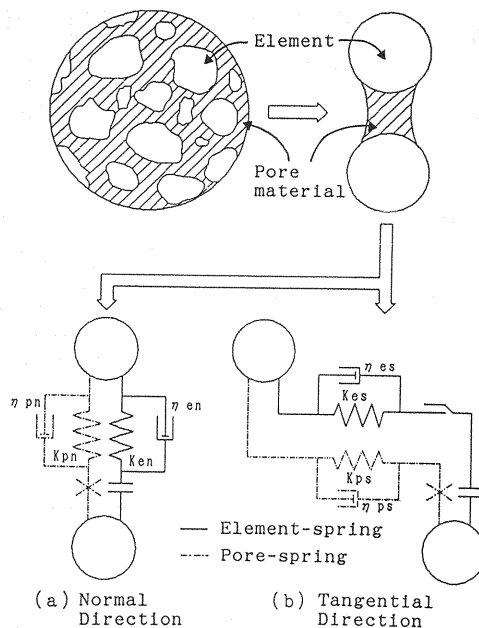

Fig.1 EDEM modeling of the medium

2. EXTENDED DISTINCT ELEMENT METHOD (EDEM)

The Extended Distinct Element Method (EDEM) is a numerical method applicable both to homogeneous and perfect discrete media and to complex, heterogeneous and continuous media. Whereas, the objective medium of the conventional DEM was limited to one that was homogeneous and perfectly discrete, the EDEM can deal with composite and continuous media. **Fig.1** shows the modeling of a medium for EDEM model. The EDEM was developed by the introduction of modified pore-springs and arrays of different material parameters to extend the applications of the DEM. The pore-spring used was established by Meguro based on Iwashita's model. With this pore-spring, rotation of the elements and the transmission of moment can be taken into account; moreover, an aggregate of elements connected by pore-springs rotates as one body.

The equations of motion of an element, i, having the mass, m_i, and the moment of inertia, I_i, are

$$m_i \ddot{u} + C_i \dot{u} + F_i = 0 \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

$$I_i \ddot{\phi} + D_i \dot{\phi} + M_i = 0 \cdots\cdots\cdots\cdots\cdots (2)$$

in which F_i is the sum of all the forces acting on the particle; M_i the sum of all the moments acting on it; C_i and D_i the damping coefficients; u the displacement vector; and ϕ the rotational displacement.
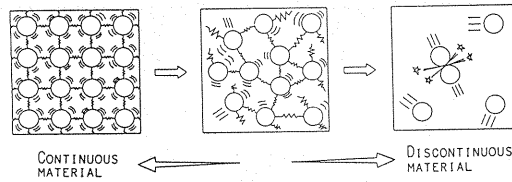
Because two kinds of force act on an element

Fig.2 Behavior of the EDEM model

Table 1   Parameters of the EDEM simulation

The element-spring	spring constant (normal), k_{en} (N/m)	3.0×10^5	The element-spring	spring constant (tangential), k_{es} (N/m)	1.5×10^5
	damping coefficient (normal), η_{en} (Ns/m)	2.0×10^2		damping coefficient (tangential), η_{es} (Ns/m)	1.0×10^2
The pore-spring	spring constant (normal), k_{pn} (N/m)	3.0×10^5	The pore-spring	spring constant (tangential), k_{ps} (N/m)	1.5×10^5
	damping coefficient (normal), η_{pn} (Ns/m)	0.0		damping coefficient (tangential), η_{ps} (Ns/m)	0.0
critical tensile strain of the pore-springs (normal), β		1.50	cohesion of the EDEM model C_{EDEM} (N)		7.0×10^6
density of the element ρ (kg/m³)		1.0×10^2	friction coefficient μ		1.00
time increment Δt (s)		5.0×10^{-4}			

(the force received from all the elements in contact and the force of all the pore material surrounding it), F_i and M_i are expressed as eqs. (3) and (4);

$$F_i = F_{ie} + F_{ip} + m_i (G + a) \cdots\cdots\cdots\cdots\cdots (3)$$

$$M_i = M_{ie} + M_{ip} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

in which F_{ie} is the sum of all the force vectors from all the elements in contact, and F_{ip} the force from all the pore material surrounding the element. M_{ie} and M_{ip} are the respective sums of all the moments of all the elements in contact and of all the pore material surrounding the element. These forces are obtained from the deformation of the element and pore-springs set in the normal and tangential directions. In eq. (3), G is the acceleration due to gravity and a the external acceleration acting on it. As the fracture criterion of the pore-spring in the normal direction, a critical tensile strain (β) is specified, and in the tangential direction, Coulomb's equation is used. The time histories of u and ϕ are obtained step-by-step in the time domain by the numerical integration of equations (1) and (2).

In the EDEM simulation, an aggregate of elements connected by pore-springs that behaves as one body gradually becomes nonlinear as the pore-springs are destroyed. At first, the EDEM model behaves as a continuous medium; but, as pore-springs are destroyed according to the fracture criteria, it gradually loses continuity, and finally behaves as a perfect discrete medium (Fig.2). This

means material, structural and geometrical non-linearities can be readily incorporated and simulated as well as phenomena in which every element separates and moves widely after destruction of the pore-springs, also that a new stress field formed by contacts between elements which were not previously in contact can be analyzed.

Examples of EDEM simulation are shown in Fig.3, and the parameters used in Table 1. Case 1 shows the analytical results when horizontal acceleration (5 G) is applied to a single element (m = 78.5 kg) at the top of a column model whose base is connected to the ground. Cases 2 and 3 show results when horizontal impulsive acceleration (10G) acts on an element (m = 78.5 kg) at the top of a model whose base is not connected to the ground. Acceleration due to gravity is considered in cases 1 and 3, but not in case 2. The result for case 2 is applicable to the simulation of space structures. Generally, the behavior seen in cases 2 and 3 is expressed by calculating the movement of the center of gravity in the translational direction and the moment about the center, but in the EDEM simulation, this behavior is expressed simply by calculating the equations of motion of each element, the whole medium being analyzed automatically. Vibration, deformation, separation and collision during movement also can be simulated automatically.
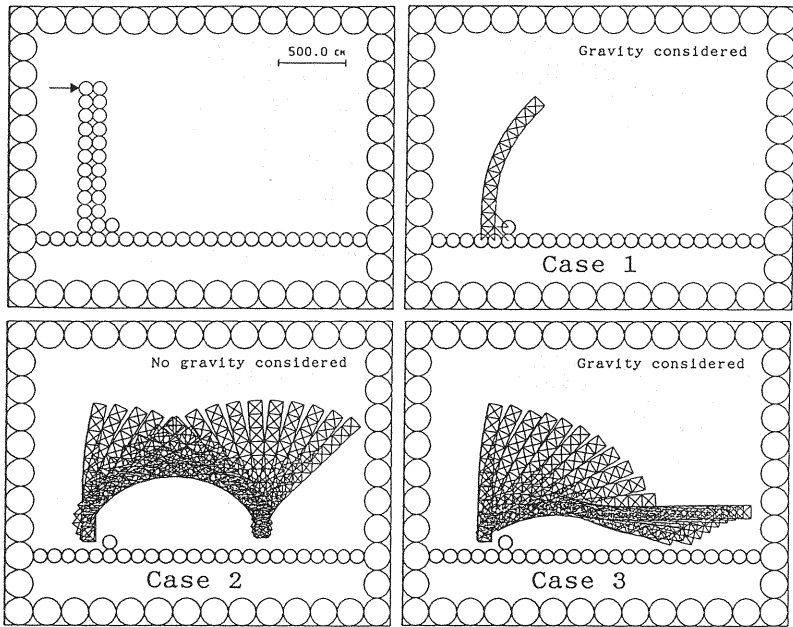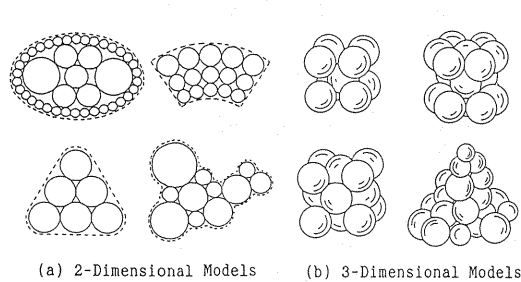
Fig.3 Examples of EDEM simulation

Fig.4 Examples of CDEs composed of circular (2-D)
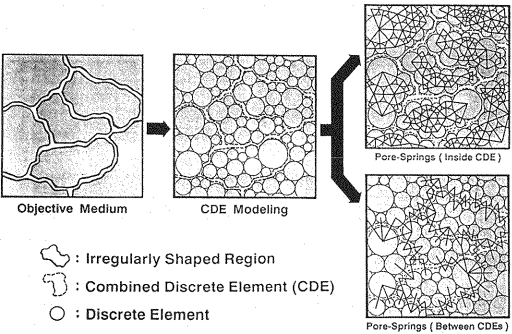and spherical (3-D) elements

Fig.5 EDEM modeling of the medium using CDEs

3. EXPLANATION OF THE BEHAVIOR OF AN IRREGULARLY SHAPED REGION USING CIRCULAR (2-D) OR SPHERICAL (3-D) ELEMENTS

Elements connected by pore-springs behave as one body both in translation and rotation (**Fig.3**). Thus, by connecting circular (2-D) or spherical (3-D) elements properly by pore-springs, it is possible to analyze the behavior of a medium that consists of regions of various shapes.

This aggregate composed of independent and discontinuous elements is called the "Combined Discrete Element (CDE)". We can make a CDE of practically any shape in 2-D or 3-D by combining elements that have several different radii (**Fig.4**). But, because the CDE is a combination of circular

(2-D) or spherical (3-D) elements, and has no sharp corners, there is a question as to whether media composed of polygonal (2-D) or polyhedral (3-D) shaped regions can be analyzed by the EDEM combined with CDEs. This question is answered by setting two kinds of pore-springs both between CDEs and inside individual CDEs during the modeling of the objective medium (**Fig.5**). (The strength of the pore-springs inside the CDE usually is greater than that of pore-springs set between CDEs.) **Table 2**, in which the fracture properties of the actual medium are compared with those of the conventional DEM with polygonal elements and the EDEM using CDEs, shows why.

The actual medium is composed of irregularly shaped regions and, in a small deformation range, each region is given resistance to deformation by the effect of the sharp corners of the regions

Table 2 Comparison of the characteristics of the conventional DEM with polygonal elements, the EDEM with CDEs and an actual medium

	Actual medium	DEM with polygonal elements	EDEM with CDEs
Shape of the region	The actual medium is composed of regions of irregular shape.	Because of complexity in judging contact conditions, only elements (regions) with simple geometrical shapes can be used.	Regions of various shapes can be used by combining circular (2D) or spherical (3D) elements of different radii for which judgment of the contact conditions is very simple.
Behavior Small deformation range	Every region is given resistance to deformation because of the effect of the sharp corners of the surrounding regions. Because of the stress concentration, the sharp corners gradually are destroyed and the shape of a region becomes round.	Every polygonal element is given resistance to deformation because of the effect of the sharp corners of the surrounding elements. Although concentrated stress occurs at the sharp corners, because the shape of the elements is unchangeable, the corners do not break; hence, the effect of corners often is overestimated as compared with the behavior of the actual medium.	The effect of the irregular shape of the region is expressed by the shape of the CDE itself and the pore-springs set between CDEs. A CDE of irregular shape is given resistance to deformation because of the effect of the pore-springs between this CDE and the surrounding CDEs. As the pore-springs between the CDEs are destroyed because of concentrated stress, the shapes of individual CDEs become round.
Large deformation range	In the large deformation range, cracks within regions and fracture of the regions themselves occur.	It is very difficult to model the occurrence of cracks within elements and the fracture of the region itself.	As the pore-springs inside the CDE are destroyed, cracks within the CDE and fracture of the CDE can be simulated automatically.

surrounding it. But, when external force acts on the medium and deforms it on a large scale, the corners of the regions are destroyed and the shapes become rounded by the concentrated stresses. When a larger external force acts, cracks are produced within the region or the entire region fractures because of the stress concentration and large deformation inside the region.

With the conventional DEM and polygonal elements, because judgment of contact conditions is complicated, only elements that have simple geometrical shapes can be used; but, such a medium is unusual. In a small deformation range, each element resists deformation because of its sharp corners; but, as deformation becomes greater, the effect of sharp corners or edges is overemphasized because the shape does not change even when a large external force acts on the medium and deformation is marked. Moreover, it is not possible to simulate the fracture of an element itself.

The EDEM that includes CDEs allows media composed of various shapes to be used in both 2-D and 3-D problems. In a small deformation range, in which the pore-springs between CDEs remain intact, each CDE is given resistance to deformation because of the pore-springs between CDEs. This resistance corresponds to that conferred by the sharp corners or edges of the regions. As deformation of the medium increases and the pore-springs set between the CDEs are destroyed, the shapes of the CDEs become rounded. When a very large external force acts on the medium, concentrated stress and marked deformation are produced

inside the CDE, and the pore-springs set inside it are destroyed. Therefore, the development of cracks within the CDE and the fracture of the CDE are simulated automatically.

A series showing the behavior of the EDEM model with CDEs is simulated automatically, and it corresponds well to the behavior of the actual medium. In addition, the CDE is readily introduced to the EDEM simulation program because the algorithm (contact conditions, flow of the analysis, etc.) of the EDEM with CDEs is the same as for the conventional EDEM, a most important strong point (**Fig.6**).

4. NUMERICAL RESULTS

Using this newly proposed method, we analyzed the fracture behavior of media composed of square and irregularly shaped regions. In the two dimensional EDEM simulation, behavior was analyzed with CDEs made up of square and irregular shapes by combining circular elements. The contact conditions for these CDEs are much simpler than those for polygonal or polyhedral elements.

(1) FRACTURE ANALYSES OF MEDIA COMPOSED OF SQUARE-SHAPED REGIONS

Using square-shaped CDEs made up of 4 or 8 circular elements, we modeled 2 types of structures (**Fig.7**), and analyzed the fracture process in media composed of those square-shaped regions. The parameters used are listed in **Table 3**. The conditions of analysis were that both models were located on a slope with an angle of 30 degrees.
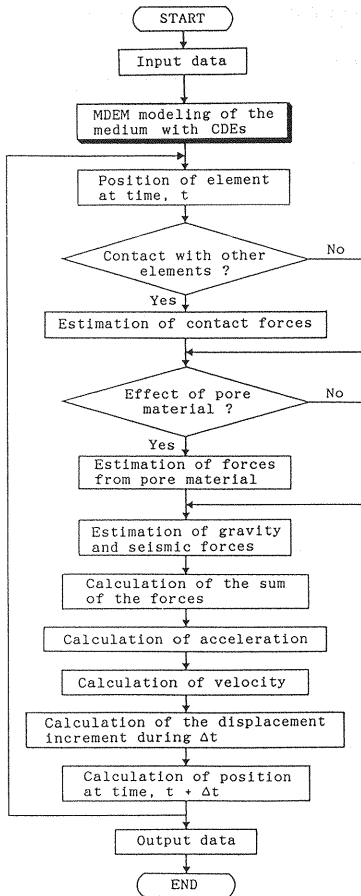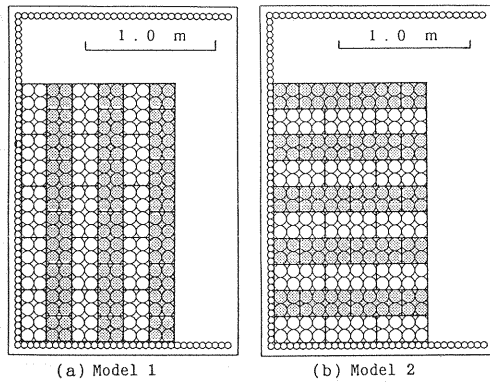
Fig.6 Flow of EDEM simulation with CDEs

(a) Model 1   (b) Model 2

Fig.7 Models of media composed of square CDEs

Table 3 parameters of the EDEM simulation

The element-spring	spring constant (normal), k_{en}(N/m)	6.0×10^7	The element-spring	spring constant (tangential), k_{es}(N/m)	1.5×10^7
	damping coefficient (normal), η_{en}(Ns/m)	3.0×10^3		damping coefficient (tangential), η_{es}(Ns/m)	7.0×10^2
The pore-spring	spring constant (normal), k_{pn}(N/m)	1.2×10^7	The pore-spring	spring constant (tangential), k_{ps}(N/m)	3.0×10^6
	damping coefficient (normal), η_{pn}(Ns/m)	0.0		damping coefficient (tangential), η_{ps}(Ns/m)	0.0
critical tensile strain of the pore-springs (normal), β		1.50	cohesion of the EDEM model C_{EDEM} (N)		7.0×10^6
density of the element ρ (kg/m³)		2.5×10^3	friction coefficient μ		1.00
time increment Δt (s)		1.0×10^{-4}			

No pore-spring used between CDEs.

The results for the two different models are shown in **Figs.8** and **9**. In both, the square-shaped CDEs composed of 4 or 8 circular elements behave as one body. A comparison of the two figures shows the difference in fracture mode produced by the different arrangements of the CDEs; moreover, careful observation reveals that movements for each CDE such as rotation, free fall, collision, friction, sliding and toppling are expressed automatically.

(2) FRACTURE ANALYSIS OF A MEDIUM COMPOSED OF IRREGULARLY SHAPED REGIONS

Using the model shown in **Fig.10**, we analyzed the fracture process of a medium composed of irregularly shaped CDEs. The parameters are given in **Table 4**. The model consisted of 130 circular elements and 48 wall elements, a total of 178 elements. Initially, 22 CDEs, each composed of 3 to 11 circular elements, and 19 independent circular elements constituted the model.

Numerical results are shown in **Fig.11**. Analysis was made under the conditions of the model being on a slope with an angle of 30 degrees. The rotation, free fall, collision, friction, sliding and toppling of each irregularly shaped CDE were simulated automatically. At stage 7, cracks appear inside the CDE, and it breaks into 2 parts. With the conventional DEM, neither the development of cracks inside an element nor the fracture of that element can be simulated, nor can the behavior of media composed of irregularly shaped regions. The CDE concept enables us to analyze the phenomena of cracks inside a CDE and the fracture of that CDE.

When pore-springs are set between CDEs, within a small deformation range each CDE resists deformation because of their effect. After these pore-springs are destroyed by an applied force, each CDE behaves as a single body with round corners. When an external force acts on the model and concentrated stress or marked deformation occurs within a CDE, the destruction of pore-
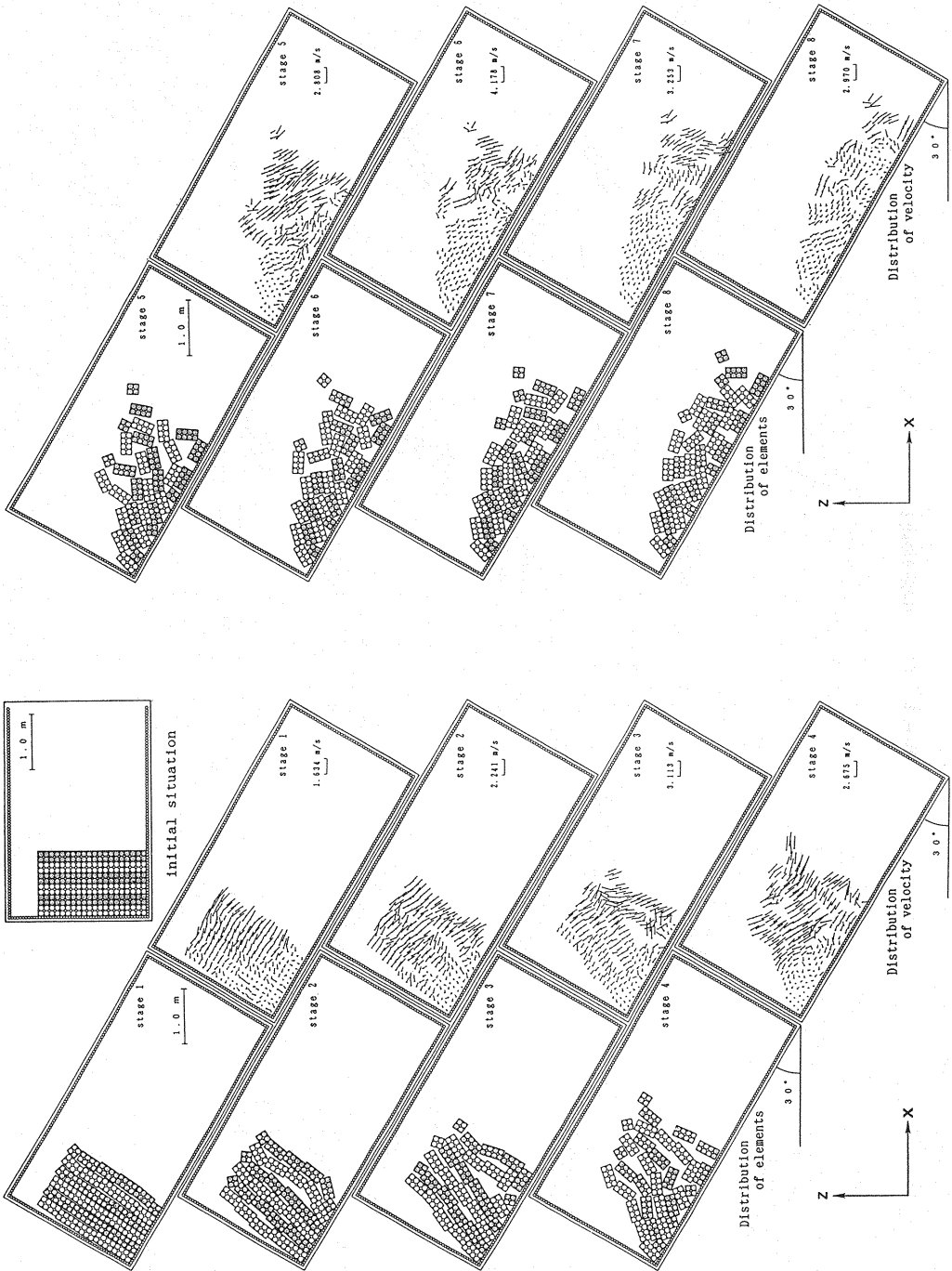
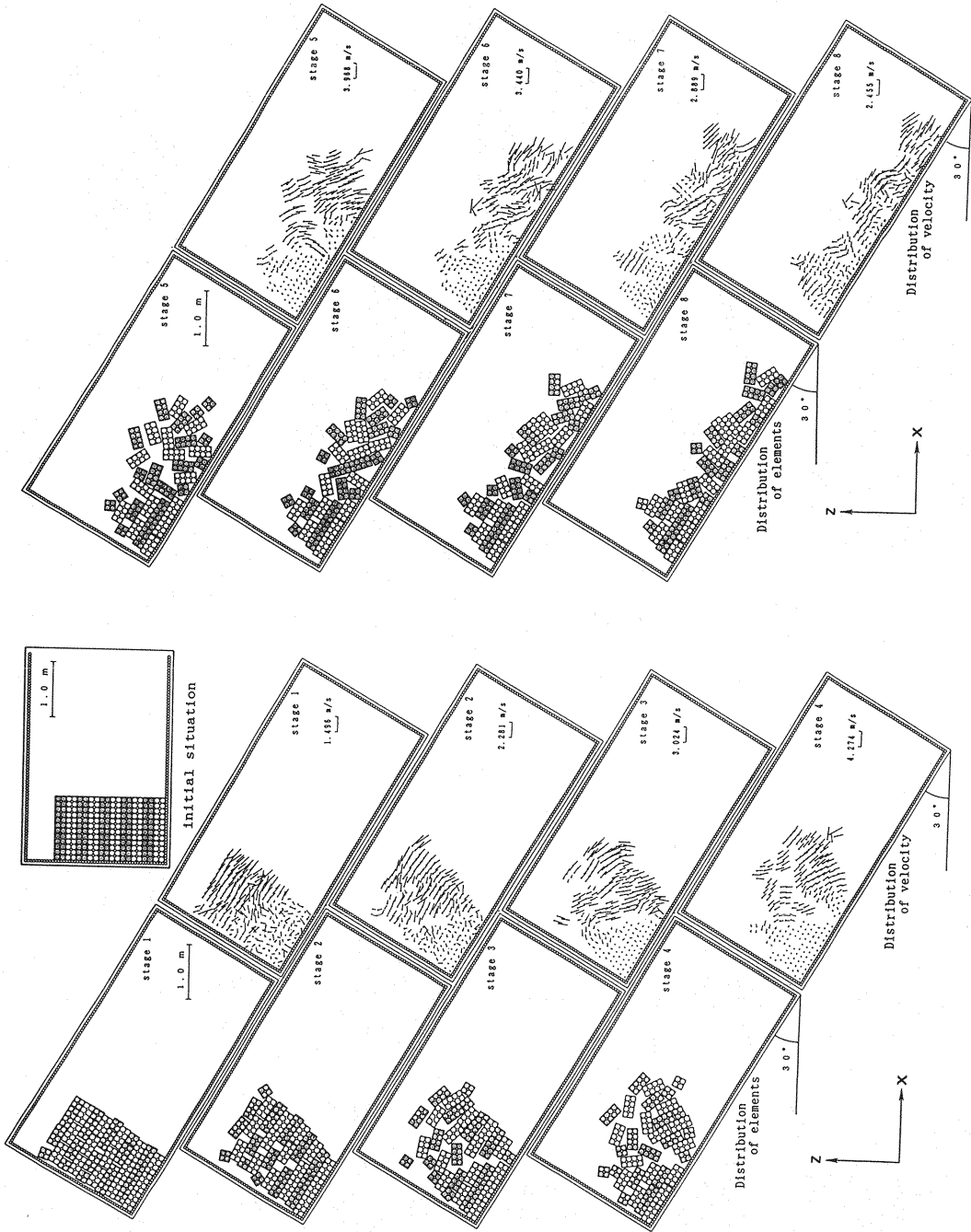**Fig.8** Fracture process of a medium composed of square CDEs (Model 1)

Fig.9 Fracture process of a medium composed of square CDEs (Model 2)

(a) Distribution of elements

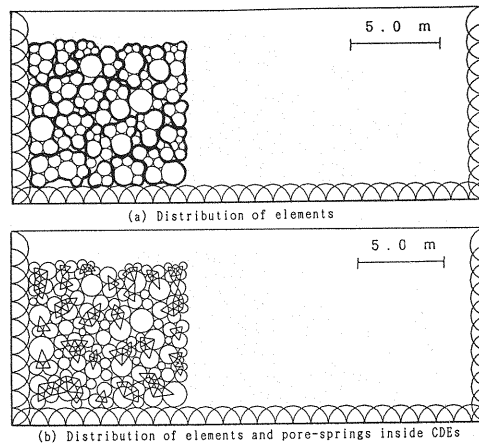(b) Distribution of elements and pore-springs inside CDEs

Fig.10 Model of a medium composed of irregularly shaped CDEs

Table 4 Parameters of the EDEM simulation

The element-spring	spring constant (normal), ken(N/m)	9.0×10^7	The element-spring	spring constant (tangential), kes(N/m)	4.5×10^7
	damping coefficient (normal), η en(Ns/m)	3.0×10^3		damping coefficient (tangential), η es(Ns/m)	7.0×10^2
The pore-spring	spring constant (normal), kpn(N/m)	4.5×10^7 (4.5×10^5)	The pore-spring	spring constant (tangential), kps(N/m)	2.25×10^7 (2.25×10^5)
	damping coefficient (normal), η pn(Ns/m)	0.0 (0.0)		damping coefficient (tangential), η ps(Ns/m)	0.0 (0.0)
critical tensile strain of the pore-springs (normal), β		1.50 (1.005)	cohesion of the EDEM model C_{EDEM} (N)		1.0×10^6 (1.0×10^4)
density of the element ρ (kg/m³)		2.5×10^3	friction coefficient μ		1.00 (1.00)
time increment $\triangle$ t (s)		5.0×10^{-5}			

The parameters given are for the pore-springs inside a CDE; those in parentheses are parameters for pore-springs between CDEs.

springs set inside the CDE produces cracks within the CDE then fracture of the CDE. With the proposed EDEM that uses CDEs, a series of such behavior can be simulated automatically.

5. CONCLUSIONS

Analysis of media composed of irregularly shaped regions is a major problem in DEM studies. We here have proposed a new idea by which this problem can be easily solved.

Application of the EDEM has been greatly extended to include complex, heterogeneous and continuous media modeled as aggregates of elements connected by pore-springs that behave as single bodies in translation and rotation. This enables us to use the EDEM to analyze a medium composed of various shaped regions. Moreover, the fracture behavior of media that consists of various shaped regions can be shown by circular (2-D) or spherical (3-D) elements which require only simple judgment of contact conditions. Using 2-D

EDEM simulation, we analyzed the fracture behavior of media composed of square- and irregularly shaped regions. In that analysis, an irregularly shaped region was treated as an aggregate of circular elements, which we have termed the "Combined Discrete Element (CDE)". The forces produced by the pore material between the circular elements inside a CDE are transmitted by the pore-springs between the elements.

The numerical results obtained confirm that the process of fracture of a medium composed of irregularly shaped regions can be analyzed by the EDEM with CDEs. Although there are still some areas to refine, such as effective CDE modeling of the objective medium, this new CDE technique has considerably extended the application of the EDEM.

Major merits of the new method :

＊ Introduction of the concept of the CDE to the EDEM enables us to simulate the behavior (from the continuous to perfect discrete stage) of a
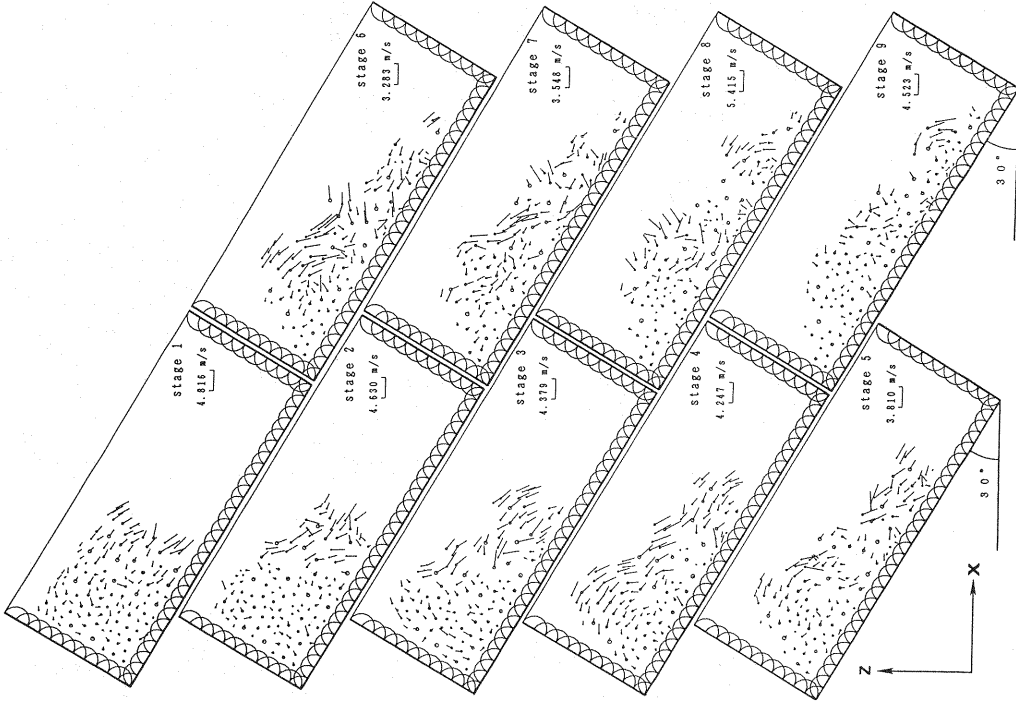
Fig.11 (b) Fracture process of a medium composed of irregularly shaped CDEs (Distribution of velocity)
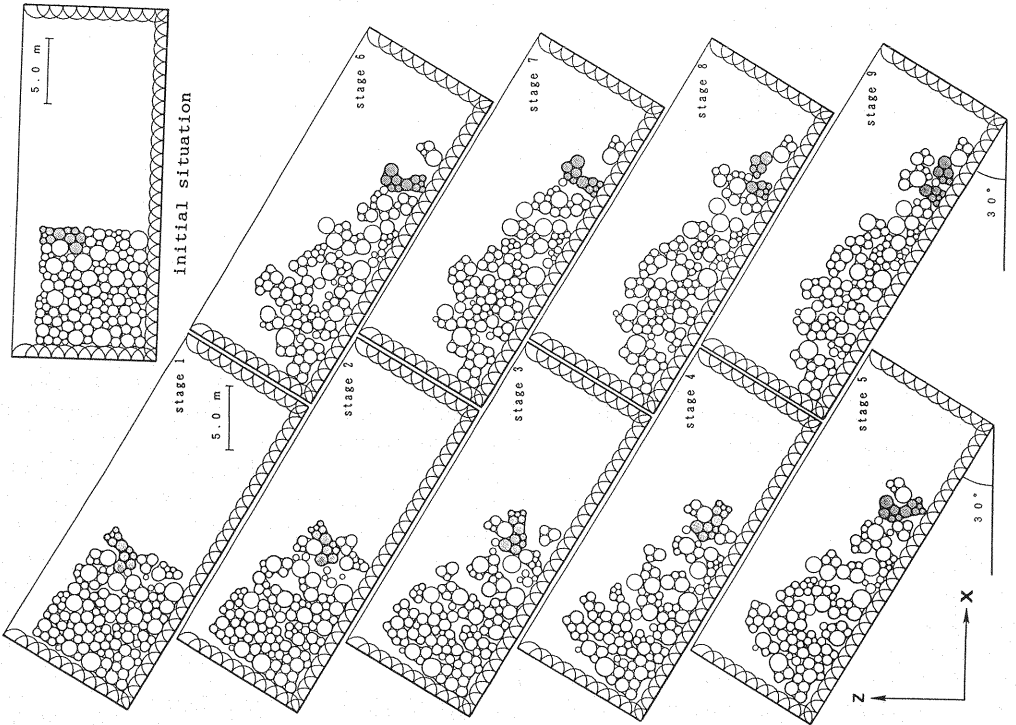
Fig.11 (a) Fracture process of a medium composed of irregularly shaped CDEs (Distribution of elements)
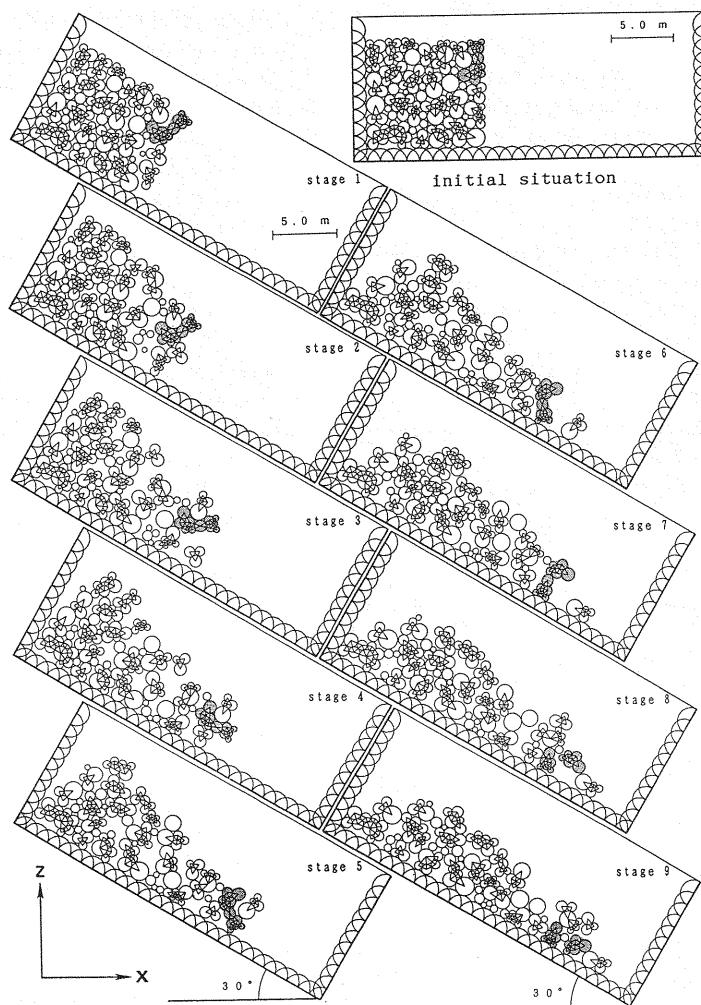
Fig.11 (c) Fracture process of a medium composed of irregularly shaped CDEs (Distribution of elements and pore-springs inside the CDEs)

medium composed of variously shaped regions by the use of circular (2-D) or spherical (3-D) elements.

* Because only simple judgment of contact conditions is required, computation time can be reduced when CDEs composed of circular or spherical elements are used rather than complicated polygonal or polyhedral elements.

* Variously shaped CDEs, for both two and three dimensional problems, are made by combining elements of different radii.

* Movements such as rotation, free fall, collision, friction, sliding and toppling of individual CDEs are analyzed automatically.

* Cracks within the CDE and its fracture also are simulated automatically.

* Use of CDEs of irregular shape allows

expression of the anisotropy of the medium due to the effect of the shapes of the regions that make up the medium.

* In addition, the CDE is readily introducible to the EDEM simulation program because the algorithm for the two methods is the same.

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