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HIGHER-ORDER THEORIES FOR FREE VIBRATION ANALYSIS OF CIRCULAR RINGS

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We have proposed the most general higher-order equations of three-dimensional static and dynamic theories for a circular cylindrical shell by expanding the displacement into infinite Power series with respect to the radial coordinate of the shell.

In this paper, the shell theories by our reduction are applied to the free ring analysis, and the natural frequencies of vibration of free rings are computed. The theoretical results are compared with the results of various theories and with experimental data.

Keywords : circular ring, higher-order theories, free vibration

1. INTRODUCTION

Shear deformation beam theories of Timoshenko-type to the circular ring, are studied by some investigators such as Philipson[1], Seidel[2], Rao[3], Wang[4,5], and Mohamed[6]. But, these are on the bases of a presumption that the shear strain along the thickness of the ring has constant distribution along the thickness, therefore this is a disadvantage which involves the unrigorous process of determining the unknown shear coefficients. Furthermore, it is pointed out that the errors in natural frequencies obtained from these theories increase according to the thickness of rings.

In this paper, an attempt is made to apply higher-order shell theories previously published by the present authors, to the problem of circular ring, for the examination of the accuracy of circular ring analysis. Higher-order theories of circular rings can be derived by neglecting the x-coordiante terms (axial terms of the shell) and the partial differential terms with x-coordinate, by use of our two-dimensional shell theory obtained by expanding the three displacements into infinite Power series with respect to the radial coordinate of the shell. Several theories above mentioned can be deduced by the employment of the lower-order terms in our theory.

Next, in order to confirm the accuracy of the present theory, natural frequencies of vibration of free rings are computed. And, the theoretical results are compared with those of various theories and with experimental data.

We define the notations as follows : $h(=2b)$ is the thickness of a ring, R is the ring radius of center line, z is the radial coordinate, θ is the circumferential coordinate, $u_\theta^{(n)}$ and $w^{(n)}$ are the displacement coefficients of the n-th order, β is an angular deformation due to shear, ψ is a total slope of deflection

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curve, k' is the cross-sectional shape factor, A is the area of cross section of ring, I is the moment of inertia of cross section of ring, and $\hat{\sigma}_\theta^{(n)}$ are stress coefficients of the n-th order.

2. EMPLOYMENT OF THE LOWER-ORDER TERMS IN THE HIGHER-ORDER THEORY AND VARIOUS THEORIES OF CIRCULAR RING

The approximate equations can be derived by neglecting the x-coordinate terms (axial terms of the shell) and the partial differential terms with x-coordinate, and by the employment of the lower-order terms, in the previous paper of our shell theories[12].

For reasons of explanation, the higher-level approximate equation is firstly explained.

(1) 3-order type theory

The terms of order $z^0 \sim z^3$ are adopted with tangential displacement u, and terms of order z^0 with radial displacement w.

$$\left.\begin{aligned} &u = u^{(0)} + z u^{(1)} + z^2 u^{(2)} + z^3 u^{(3)}, \\ &w = w^{(0)} \equiv w_0. \end{aligned}\right\} \qquad (1)$$

(2) Levinson type theory[9]

By combining eq. (1) with the following boundary condition with shear load on the upper and the lower surfaces of the shell $(z = \pm b)$.

$$\gamma_{\theta z|z=\pm b} \equiv \frac{1}{G}\tau_{\theta z}^{\pm}. \qquad (2)$$

$u^{(2)}$ and $u^{(3)}$ can be finally written as

$$\left.\begin{aligned} u^{(2)} = &-\frac{u^{(0)} - w_{0,\theta}}{3R^2 - b^2} + \frac{R}{3R^3 - b^2}u^{(1)} + \frac{1}{4bG}\cdot\frac{1}{3R^2 - b^2}\{(R+b)(3R-2b)\tau_{\theta z}^{+} \\ &-(R-b)(3R+2b)\tau_{\theta z}^{-}\}, \\ u^{(3)} = &\frac{R}{b^2}\cdot\frac{u^{(0)} - w_{0,\theta}}{3R^2 - b^2} - \frac{R^2}{b^2}\cdot\frac{1}{3R^2 - b^2}u^{(1)} + \frac{1}{4b^2G}\cdot\frac{1}{3R^2 - b^2}\{(R+b)(2R-b)\tau_{\theta z}^{+} \\ &+(R-b)(2R+b)\tau_{\theta z}^{-}\}. \end{aligned}\right\} \qquad (3)$$

(3) Levinson-Voyiadjis-Baluch type theory (L. V. B theory)[10]

By use of axial force $N = \hat{\sigma}_\theta^{(0)} = \int_{-b}^{b} \sigma_\theta dz$ and bending moment $M = \hat{\sigma}_\theta^{(1)} = \int_{-b}^{b} z\sigma_\theta dz$, and by taking into the method used in (2), $u^{(2)}$ and $u^{(3)}$ can easily be found.

(4) Mohamed type theory[6]

By use of the condition $u^{(2)} = u^{(3)} = 0$, eq. (1) can be deduced to

$$u = u^{(0)} + z u^{(1)}, \quad w = w_0. \qquad (4)$$

Mohamed has expressed equations of motion by using of the shear deformation and the slope due to bending. The relations among $u^{(1)}$, β and ψ can be written as

$$\psi = u^{(1)} - \beta. \qquad (5)$$

(5) Waltking type theory[7]

The theory without consideration of the shear deformation along the thickness and the rotary inertia is derived from (4) by setting the following equation.

$$u^{(1)} = \frac{1}{R}\left(u^{(0)} + \frac{\partial w_o}{\partial \theta}\right). \qquad (6)$$

(6) Love type theory (Thin ring theory)[8]

The condition of inextension of the center line of ring is used in this theory. Namely, this theory uses the following equation instead of eq. (6).

$$u^{(1)} = \frac{1}{R}\left(u^{(0)} + \frac{\partial^2 u^{(0)}}{\partial \theta^2}\right). \qquad (7)$$

Table 1 Coefficients in Eq. (9).

Theory	A_1	A_2	A_3	A_4
Thin ring	—	—	n^2+1	$-n^6+2n^4-n^2$
Waltking	—	$-S_2$	$n^2+1+n^4S_2^2+n^2S_2$	$-n^6+2n^4-n^2$
Rao Wang	—	$-n^2S_2^2S_1-S_2^2S_1$	n^2+1 $+n^4S_2+n^4S_2S_1-2n^2S_2+n^2S_2S_1+S_2$	$-n^6+2n^4-n^2$
Mohamed	$S_2^3S_1$	$-S_2-2n^2S_2S_1-S_2^2S_1-n^2S_2^2-S_2^2$	n^2+1 $+2n^4S_2+n^4S_2S_1-n^2S_2+n^2S_2S_1+S_2$	$-n^6+2n^4-n^2$

3. NUMERICAL RESULTS

For examination of the accuracy of various theories, numerical results of some lower natural frequencies for circular free ring are computed. For example, the solution of 3-order type theory (1) mentioned in the previous section may be assumed by the following modal form.

$$\left.\begin{aligned} \begin{Bmatrix} u^{(0)} \\ \vdots \\ u^{(3)} \end{Bmatrix} &= \begin{Bmatrix} a_0 \\ \vdots \\ a_3 \end{Bmatrix} e^{i\omega t}\sin n\theta, \\ w_0 &= b\, e^{i\omega t}\cos n\theta. \end{aligned}\right\} \qquad (8)$$

where, a_0, ……, b represent the amplitudes of displacement coefficient, ω the natural circular frequency of vibration, n the number of complete waves in a circumference.

The foregoing equations mentioned by (4)～(6) of the previous section can be written as

$$A_1K_1^3+A_2K_1^2+A_3K_1+A_4=0. \qquad (9)$$

where,

$$K_1=\frac{\rho AR^4}{EI}\omega^2. \qquad (10)$$

The coefficients A_1, ……, A_4 in eq. (9) for various theories are shown inTable 1. S_1 and S_2 of the table can be expressed as follows.

$$S_1=\frac{E}{k'G},\quad S_2=\frac{I}{AR^2}. \qquad (11)$$

The values of natural frequencies of vibration depend on geometrical dimension, material factor and mode number. The following ring geometric and material parameters are adopted in this paper.

$h/R=0.1\sim0.8$, $n=2, 3$, $\nu=0.25$; $k'=5/6$.

In Fig. 1, the results are also compared with those of Seidel and with the experimental values of Kuhl[11] in case of mode number $n=2$. Since the results obtained in case of $n=3$ are roughly similar with those in case of $n=2$, the figure of $n=3$ are omitted in this paper.

From the results shown in Fig. 1, the following tendencies of accuracy for various theories may be obtained.

(1) The errors of natural frequencies obtained from Levinson type theory are large, and agree roughly with the errors of thin ring theory.

(2) The accuracy of Levinson-Voyiadjis-Baluch type is higher than that of Levinson type but lower than that of the other theories ; which are those of Seidel, Mohamed

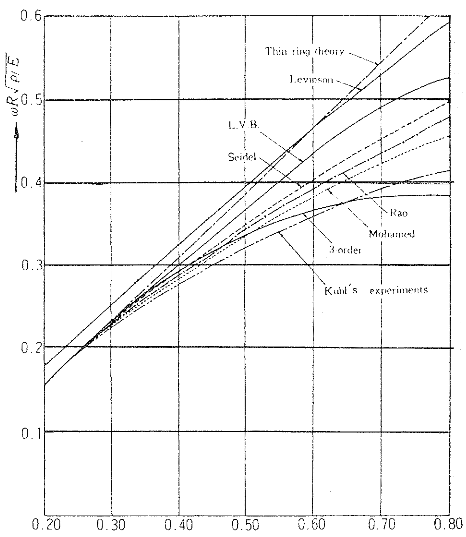

Fig. 1 Comparison of frequencies of a free ring (for $k=5/6$, $\nu=0.25$, $n=2$).

and 3-order.

(3) In case of thinner ring ($0<h/R<0.25$), a small difference can be recognized between thin ring theory and various ring theories. For $h/R \geqq 0.25$ the error of thin ring theory becomes large.

(4) In case of thick ring ($0.25<h/R<0.50$), the results of Mohamed are closer to the experimental values than those of the other theories. The following orders of the accuracy are Rao, Seidel and 3-order type.

(5) For $h/R \geqq 0.50$, the results of 3-order type theory are closer to those of the experimental values.

4. CONCLUDING REMARKS

In this paper, the higer-order shell theories published in the previous paper have been applied to the problems of circular free ring and deduced to several approximate theories by the adequate employment of the lower-order terms. At the same time, the characteristics of accuracy are examined numerically by computing the natural frequencies of vibration of free ring and by comparing our theories with the other theories.

The results are summarized at the end of preceeding section. In conclusion, for the case of thick ring ($h/R \geqq 0.50$), the results of 3-order type theory are closer to those of the experimental values. And now, our next task will be the comparative study of the results of theories with the theoretically exact values.

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