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EFFECT OF MULTIPLE COLLAPSE MODES ON DYNAMIC FAILURE OF STRUCTURES WITH STRUCTURAL INSTABILITY

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In this paper, in order to establish the dynamic failure criteria of multi-degree structures with structural instability, the true dynamic ultimate strength of two-degree flexural systems with structural instability as well as the response parameters which determine these ultimate strength are investigated numerically. In particular, effects of the collapse mode shape on the dynamic ultimate strength and the amount of energy are examined.

As a result, it is revealed that the effective input energy governs the dynamic ultimate state of two-degree systems with structural instability and that the input energy depends on the collapse mode shape.

Furthermore, the method of estimating the effective input energy of two-degree systems is proposed from the results of the single-degree systems.

Keywords: *dynamic failure, structural instability, multiple collapse modes, effective input energy*

1. INTRODUCTION

When a dynamic lateral loading is applied to a column under the static axial compression which is smaller than the buckling load, the column may become unstable and collapse due to rapid development of the overall or local structural instability and the yielding of material. The behavior is mainly affected by the static load, and in the case where structures with such a static structural instability are subjected to dynamic loadings, most structures have a possibility of the above described collapse.

In the case of designing structures subjected to dynamic loadings as well as evaluating their ultimate strength, the dynamic loadings have been replaced by static loadings until several years ago. Therefore, in fact, the true dynamic ultimate strength of structures with structural instability is not known well. From this point of view, a number of investigations have been carried out to obtain the dynamic ultimate strength and the failure criteria of structures subjected to dynamic loadings by comparing the maximum strain energy absorption of structures with the input energy exerted by the dynamic loading.

One of earlier researches related to structures with structural instability was done by Kato and Akiyama[1),2)]. They analyzed single- and five-degree-of-freedom shear vibrating systems with the restoring characteristics deteriorated by the $P-\Delta$ effect. Then, they assumed that the static critical displacement, where the restoring force became zero, governed the dynamic ultimate state. Ishida and Morisako also investigated the dynamic collapse behaviors of single-degree systems with structural instability theoretically and numerically[3)]. Hirao *et al.* investigated seismic performance of steel piers using

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single-degree systems where the deteriorated restoring characteristics was incorporated on the basis of the energy concept[4]. Furthermore, Bernal showed some kinds of response spectra of single-degree systems with structural instability from the viewpoint of aseismic design[5]. However, the strain energy absorption up to the specified displacement is affected by the kinetic energy and the hysteretic energy, so the energy absorption up to the static critical displacement as well as the displacement are considered not to govern the true dynamic ultimate state.

Then the writers investigated the dynamic collapse of single-degree systems with structural instability[6]. As a result, it was revealed that the effective input energy, which was obtained by subtracting the energy dissipated by the hysteretic damping and other types of damping from the input energy exerted by the disturbing force, governed the dynamic ultimate state and that the system collapsed when the effective input energy exceeded the maximum strain energy absorption of the system. The displacement in the dynamic ultimate state was also found to be smaller than the static critical one.

However, a structure with multiple degrees of freedom is considered to absorb energy by the distributed multiple springs in the structure. Then the results obtained in the case of single-degree systems are not always applicable to the case of multi-degree systems.

Kato and Akiyama also showed a basic law which governed the distribution of damage in shear-type multi-structures under earthquakes[2),7),8)]. Ohno and Nishioka investigated the effect of the mass and stiffness on the energy distribution by the equivalent linearized method for multi-degree shear vibrating systems[9),10)]. However, the true dynamic ultimate strength of multi-degree structures with structural instability is not clear in these studies and the skeleton of the restoring characteristics of the system subjected to a static load which causes the structural instability depends on the static load, the mode shape and the structural properties. Then it is not easy to incorporate the restoring force-displacement relationship into the shear vibrating system. Therefore, in order to investigate the dynamic collapse behavior of structures with the deteriorated restoring characteristics affected by the several factors, it is better to employ the multi-degree flexural system considering the effect of the structural instability easily.

In this paper, in order to establish the dynamic failure criteria of multi-degree structures with structural instability, first, the true dynamic ultimate strength of two-degree systems with structural instability and the response parameters which determine this ultimate strength are investigated numerically. Second, the influences of the collapse mode shape, the static load and the frequency of the disturbing force on the dynamic collapse behavior and the amount of energy are examined. This system is the most fundamental model of multi-degree systems and consists of masses, rigid bars and rotational springs. Furthermore, the method of estimating the response parameters which govern the dynamic ultimate state of two-degree systems is proposed using the results of single-degree systems.

2. ELASTO-PLASTIC RESPONSE OF TWO-DEGREE SYSTEM WITH STRUCTURAL INSTABILITY

(1) Equation of motion

A two-degree-of-freedom system shown in Fig. 1 is considered here[11)]. The model consists of masses, rigid bars and rotational springs and a static load P_2 which causes the structural instability is applied only to the mass m_2 as shown in Fig. 1. Considering the equilibrium of moment about each rotational spring, the equation of motion is

$$\begin{aligned} l_1 m_1 \ddot{x}_1 + (l_1 + l_2) m_2 \ddot{x}_2 + R_1 &= l_1 f_1 + (l_1 + l_2) f_2 + P_2 x_2 \\ l_2 m_2 \ddot{x}_2 + R_2 &= l_2 f_2 + P_2 (x_2 - x_1) \end{aligned} \quad \cdots\cdots (1)$$

in which l_i, m_i, x_i, f_i and R_i $(i=1, 2)$ are the length of the rigid bar, the mass, the horizontal displacement of the mass, the disturbing force as shown in Fig. 1 and the restoring moment of the rotational spring, respectively. It is assumed

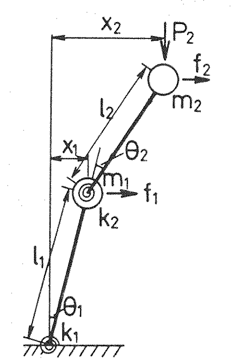


Fig. 1 2 DOF system with structural instability.

that there is no viscous damping in the system and that the rotational angle is small enough to neglect the geometrical nonlinearity. Eq. (1) is rewritten concisely in matrix form as

$$\boldsymbol{LM\ddot{x}}+\boldsymbol{R}-\boldsymbol{Px}=\boldsymbol{Lf} \qquad (2)$$

in which

$$\boldsymbol{L}=\begin{bmatrix} l_1 & l_1+l_2 \\ 0 & l_2 \end{bmatrix} \quad \boldsymbol{M}=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad \boldsymbol{\ddot{x}}=\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}$$
$$\boldsymbol{R}=\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} \quad \boldsymbol{P}=\begin{bmatrix} 0 & P_2 \\ -P_2 & P_2 \end{bmatrix} \quad \boldsymbol{x}=\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \quad \boldsymbol{f}=\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \qquad (3)$$

The restoring moment vector $\boldsymbol{R}$ is expressed as the product of the rotational spring constant matrix $\boldsymbol{k}$ and the angle of rotation vector $\boldsymbol{\theta}$ within the elastic range of the spring. Then

$$\boldsymbol{R}=\boldsymbol{k\theta} \qquad (4)$$

in which

$$\boldsymbol{k}=\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \quad \boldsymbol{\theta}=\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} \qquad (5)$$

The geometrical relationship between $\boldsymbol{x}$ and $\boldsymbol{\theta}$ yields

$$\boldsymbol{x}=\boldsymbol{L}^T\boldsymbol{\theta} \qquad (6)$$

Substituting Eqs. (4) and (6) into Eq. (2), the equation of motion, which considers the equilibrium of horizontal forces within the elastic range of the spring, is written as

$$\boldsymbol{M\ddot{x}}+\boldsymbol{L}^{-1}\{\boldsymbol{k}(\boldsymbol{L}^T)^{-1}-\boldsymbol{P}\}\boldsymbol{x}=\boldsymbol{f} \qquad (7)$$

The equation results in the same form as the general equation of motion by using the following notation :

$$\boldsymbol{k}'=\boldsymbol{L}^{-1}\{\boldsymbol{k}(\boldsymbol{L}^T)^{-1}-\boldsymbol{P}\} \qquad (8)$$

Therefore, time histories of responses can be obtained by using the step-by-step integration method, even if the system has the nonlinear restoring characteristics[12].

(2) Buckling load and vibrational mode shape

When a static load is applied only to the mass m_2 as shown in **Fig. 1**, the buckling load is obtained by setting the determinant of Eq. (8) equal to zero. If $l_1=l_2=l$ and $k_2=\eta k_1$, the buckling load becomes

$$P_{cr}=(k_1/l)(1+2\eta\pm\sqrt{1+4\eta^2})/2 \qquad (9)$$

Since only the lower value of P_{cr} is of practical interest, the minus sign should be used in Eq. (9). By using the notations $\alpha=P_2/P_{cr}$ $(0<\alpha<1)$ and $q=P_2l/k_1$, Eq. (9) becomes

$$q=\alpha(1+2\eta-\sqrt{1+4\eta^2})/2 \qquad (10)$$

The natural frequencies and their corresponding mode shapes of the system subjected to the static load are the eigenvalues and eigenvectors of Eq. (7), where the right-hand side equals zero.

Here, if it is assumed that the restoring characteristics is the ideal elasto-plastic one as shown in **Fig. 2** and that the yield rotational angles of both the springs are the same, there seems to be three fundamental mode shapes as shown in **Fig. 3** from the viewpoint of the dynamic collapse. The mode shape of type 1

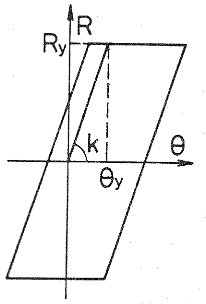
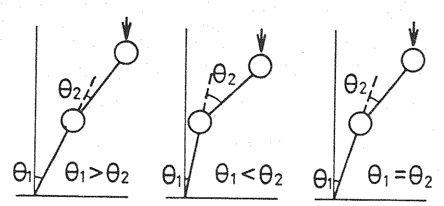


Fig. 2 Restoring characteristics of rotational spring.



(a) Type 1 (b) Type 2 (c) Type 3

Fig. 3 Collapse mode of 2 DOF system.

corresponds to the case where the yielding occurs in the lower rotational spring only and the system collapses after the yielding. The mode shape of type 2 corresponds to the case where the yielding occurs in the upper spring only. The mode shape of type 3 corresponds to the case where both the springs yield and thereafter the system collapses.

(3) Restoring characteristics of system

From the results in Ref. 6), it can be seen that for a single-degree system with the deteriorated restoring characteristics, the system collapses when the effective input energy exceeds the maximum strain energy absorption. However, a multi-degree system with structural instability has the multiple dynamic collapse modes and the skeleton of the restoring characteristics of the system depends on the multiple spring constants, the displacements of the masses and the static load. Therefore, it is difficult to obtain analytically the true restoring characteristics and the maximum strain energy absorption of multi-degree systems in the same manner as in the case of single-degree systems.

Then, in order to confirm the outline of the restoring characteristics of the two-degree systems with structural instability, the restoring characteristics of the system of type 1 is examined conceptionally as an example, when the restoring characteristics of the system is transformed into the one in a shear vibrating system on the basis of some assumptions.

From the second and third terms of Eq. (2), the restoring force $\boldsymbol{R}'$ of the system is derived as

$$\boldsymbol{R}'=\boldsymbol{L}^{-1}(\boldsymbol{R}-\boldsymbol{P}\boldsymbol{x}) \qquad (11)$$

In the case where $k_2=\eta k_1$, $x_2=\mu x_1$, $l_1=l_2=l$, $\theta_{y1}=\theta_y=x_y/l$, $P_2=qk_1/l$ and the restoring moment of rotational spring 1 only reaches the yield restoring moment R_{y1} $(=k_1\theta_y)$, the elements of the restoring force vector become

$$\boldsymbol{R}'=\frac{k_1}{l^2}\begin{Bmatrix} x_y-(\mu-2)(2\eta-q)x_1 \\ [(\mu-1)(\eta-q)-\eta]x_1 \end{Bmatrix} \qquad (12)$$

in which the restoring characteristics of rotational springs 1 and 2 are assumed to be the ideal elasto-plastic ones. If the restoring force of the system at the position of the lower mass, that is, the first row of Eq. (12) is set equal to zero, the displacement of the mass m_1 becomes the static critical one, x_{cr} and is expressed as

$$x_{cr}=x_y/\{(\mu-2)(2\eta-q)\} \qquad (13)$$

Fig. 4 shows the outlines of the restoring characteristics of the system of type 1 and these characteristics result in the ones of springs in a shear vibrating system. In the figure, while R_2' is proportional to x_1, the sum of the elasto-plastic strain energy E_1 and μE_2 is considered to become the maximum strain energy absorption E_{su}.

In this case, while it is assumed that only rotational spring 1 yields and x_2 is equal to μx_1, the latter assumption is satisfied only in a single natural mode of elastic vibration. Therefore, it is generally difficult to incorporate the true restoring characteristics of these types of structures into the shear vibrating systems, because the nonlinear responses as well as the properties of structures and loads must be clear in advance. Then, in the next chapter, the true dynamic ultimate strength of two-degree systems with structural instability and the parameters which determine the ultimate strength will be estimated numerically.

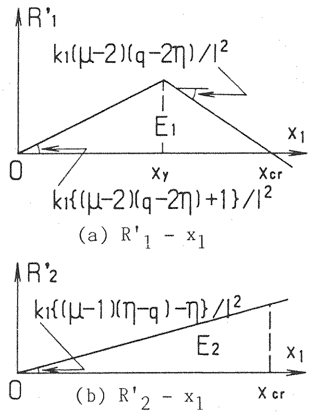


Fig. 4 Restoring characteristics of system (Type 1).

3. NUMERICAL RESULTS

(1) Parameters and properties of model

The restoring characteristics and maximum strain energy absorption of a two-degree system of type 1 with structural instability can be obtained analytically on the basis of some assumptions as mentioned above. However, it is difficult to generalize the analytic discussion to systems with various properties and their maximum strain energy absorptions can hardly be estimated directly, because the skeleton of the restoring characteristics of the system is affected by several factors and is not easily incorporated into the shear vibrating system. But, it is clear from the results of single-degree systems in Ref. 6) that the effective input energy, which is obtained by subtracting the energy dissipated by the hysteretic damping and other types of damping from the input energy exerted by the disturbing force, govern the dynamic ultimate state and that the effective input energy almost coincides with the maximum strain energy absorption.

Then, in this chapter, the effective input energy of two-degree systems is examined by the parametric analyses using a step-by-step integration procedure where the modified Newton-Raphson method and the Newmark β method ($\beta=1/4$) are combined[12]. In particular, this study focuses on the effects of collapse modes on the dynamic ultimate strength, so the effects of the frequency of the disturbing force, the spring constant and its yield rotational angle on the effective input energy is examined. For the Newmark β method, the time interval is set equal to 1/64 of the period of the disturbing force. The dynamic ultimate state is defined as the state at the time step after which the displacement diverges without subsequent application of the disturbing force[6]. The disturbing force due to the following harmonic support motion such as earthquake excitation is considered here :

$$f_i=-m_iZ\sin\omega t \quad (i=1,\ 2) \qquad (14)$$

in which Z and ω are the acceleration amplitude and the natural circular frequency of the disturbing force, respectively. The acceleration amplitude is given by the yield strength coefficient, which is defined by

$$\gamma=R_{y1}/Z\{m_1l_1+m_2(l_1+l_2)\} \qquad (15)$$

This coefficient denotes the ratio of the yield restoring moment of the rotational spring 1 to the moment at the support caused by the inertia force, which is assumed to be applied to the masses statically.

As mentioned in Article 2. (2), there are three fundamental mode shapes of collapse according to the properties of two-degree systems. Unless otherwise mentioned, the collapse behavior is investigated under the condition that $l_1=l_2=l$, $m_1=m_2$, $\theta_{y1}=\theta_{y2}=\theta_y$ and the circular frequency of the harmonic disturbing force is equal to the first natural circular frequency ω_1 of the system and that the static load is equal to a half of the buckling load in Eq. (9). Hereafter, this condition is called the reference condition and the magnitude of the static load is expressed by α, its ratio to the buckling load. Fig. 5 shows the properties, the natural circular frequencies and the mode shapes of each model, in which the rotational spring constant k_1 is fixed, whereas k_2 changes and $\alpha=0.5$. The first natural mode shapes in this figure correspond to the collapse modes in Fig. 3.

(2) Displacement response and restoring characteristics of system

Examples of time histories of the rotational angles of the springs in the cases of types 1 and 2 are shown in Figs. 6 (a) and (b), under the reference condition, where the circular frequency ω of the harmonic disturbing force is equal to the first natural circular frequency of the system and that the static load is equal to a half of the buckling load ($\alpha=0.5$). The ordinate shows the rotational angle of the spring normalized by the yield rotational angle θ_y and the abscissa shows the elapse time normalized by the period T of the disturbing force. The solid line corresponds to the rotational angle of spring 1 and the dashed line corresponds to that of spring 2. As shown in Fig. 6 (a), the rotational angle of spring 1 is greater than that of spring 2 in the initial elastic vibrating state as inferred from the first natural mode shape. Therefore, spring 1 yields first and thereafter the system collapses. On the other hand, in the case of type 2 shown in Fig. 6 (b), the rotational angle of spring 2 becomes greater and the system collapses. This implies that the collapse mode of type 2 is different from that of type 1.

Figs. 7 (a) and (b) show the restoring force-horizontal displacement relationships of the system of type 1 in the positions of masses m_1 and m_2, respectively. The ordinate shows the restoring force of the

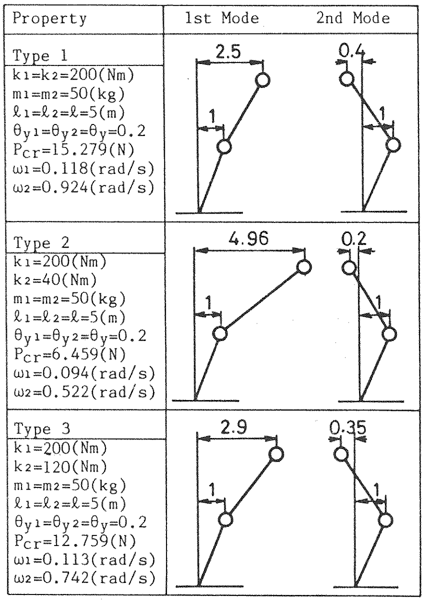


Fig. 5 Properties and mode shape of model.

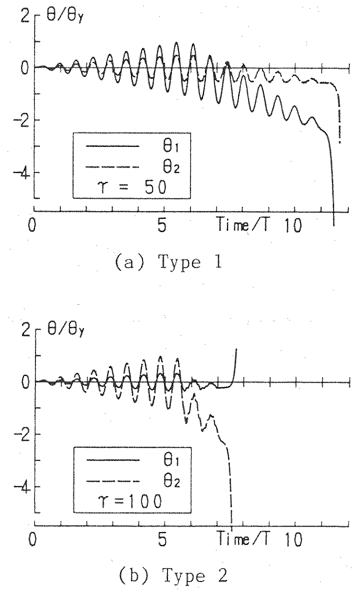


Fig. 6 Rotational displacement-time curve.

system and the abscissa shows the horizontal displacement normalized by the yield displacement x_y ($= l\theta_y$). Both the restoring characteristics in Fig. 7 show almost linear restoring force-displacement relationships until spring 1 yields, and the displacement of the mass m_2 is found to be proportional to that of the mass m_1. On the other hand, after the yielding of spring 1, both the restoring force-displacement relationships become complicated and the assumption that $x_2 = \mu x_1$ is not satisfied. However, both the envelopes of the relationships is found to become the deteriorated restoring characteristics. Then, if the magnitude of the horizontal restoring forces are always the same and if the forces are assumed to become zero simultaneously after the yielding of spring 1, the restoring characteristics obtained by Eq. (11) result in the ones shown by the dashed lines in Figs. 7 (a) and (b). These restoring characteristics agree comparatively with the envelopes of the time hystories of the restoring ones. Thus, two-degree systems with structural instability are considered to also have maximum strain energy absorption.

(3) Effective input energy and power up to dynamic ultimate state

Fig. 8 shows the effective input energy E_{ef} and the power S_f up to the dynamic ultimate state against the yield strength coefficient γ in the case of type 1 under the reference condition. The power is obtained by integrating the square of the value of the disturbing force at each time step from the initial to the ultimate state. Therefore, the power is considered to represent a kind of energy[13] and is employed to compare the relative magnitudes of the disturbing force taking into account its amplitude and duration. The abscissa shows the yield strength coefficient γ, which is inversely proportional to the amplitude of the disturbing force, and the ordinates show the effective input energy and the power. The circles show the effective input energy and the triangles show the power on a log scale. From this figure, it can be seen that the effective input energy little varies with γ, that is, the amplitude of the disturbing force and that the power does not greatly vary with γ.

Fig. 9 also shows the effective input energy E_{ef} and the power S_f against the yield strength coefficient γ under the same condition as that of Fig. 8, except that the frequency of the disturbing force is equal to the second natural frequency of the system. When $\gamma=0.2$, that is, the amplitude of the disturbing force is considerably large, the yielding occurs in spring 1 and the system collapses before the development of the second mode, because of impulsive application of the disturbing force. Therefore, the effective input

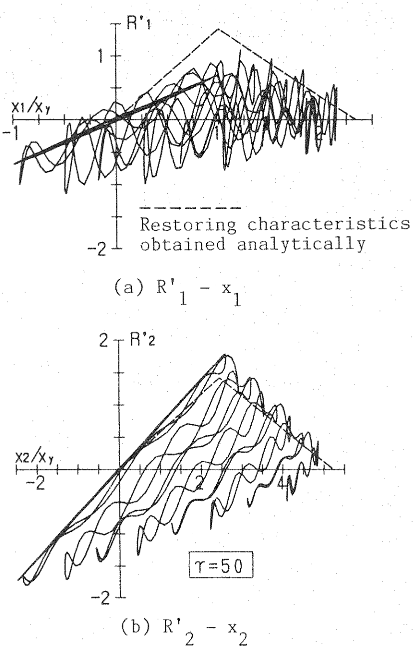


Fig. 7 Restoring force-horizontal displacement relationship (Type 1).

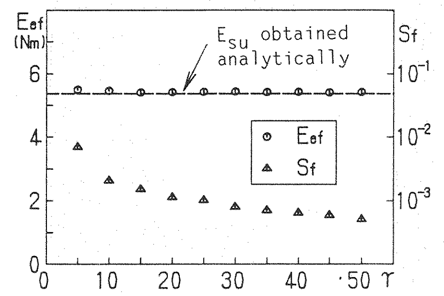


Fig. 8 Effective input energy and power versus yield strength coefficient (Type 1, $\omega=\omega_1$).

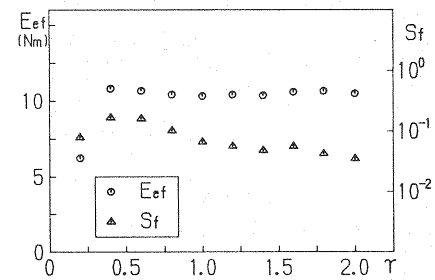


Fig. 9 Effective input energy and power versus yield strength coefficient (Type 1, $\omega=\omega_2$).

energy in the case of $\gamma=0.2$ becomes smaller than those in the other regions. However, when $\gamma>0.2$, the collapse mode is always the one where the yielding occurs in spring 2 and the effective input energy and the power are almost constant irrespective of γ.

According to Figs. 8 and 9, and the similar figures of other types which are not shown here because of space limitations, the effective input energy does not depend on the amplitude of the disturbing force and this implies that the effective input energy governs the dynamic ultimate state of the system. But, the amount of the effective input energy varies with the collapse mode. In Fig. 8, the dashed line corresponds to the maximum strain energy absorption E_{su}, which is estimated by the sum of the area bounded by the dashed lines and the abscissa in Figs. 7 (a) and (b). From Fig. 8, the effective input energy is found to be slightly larger than the maximum strain energy absorption and this result coincides with the results of single-degree systems. However, the maximum strain energy absorption can be obtained only on the basis of some assumptions and these assumptions are not always applicable to the system with different collapse mode.

(4) Effect of frequency of disturbing force on effective input energy

Fig. 10 shows the effective input energy E_{ef} by circles and the power S_f by triangles up to the dynamic ultimate state against the circular frequency ω of the disturbing force within the range of $0<\omega<2$ (rad/s), in the case of type 1. The abscissa shows the circular frequency of the disturbing force and the ordinates show the effective input energy and the power. In this figure, ω_1 and ω_2 are the first and second natural circular frequencies of the system, respectively. The effective input energy E_{ef} is nearly constant and the system collapses due to the yielding of spring 1 except in the vicinity of the second natural circular frequency. There, the collapse mode is the one where the yielding occurs in spring 2 and the effective input energy is greater than the one in the other regions. On the other hand, the power in the vicinity of the first natural circular frequency is considerably smaller than the power in the other regions.

Consequently, it is very likely that a two-degree system collapses in a mode shape which is developed

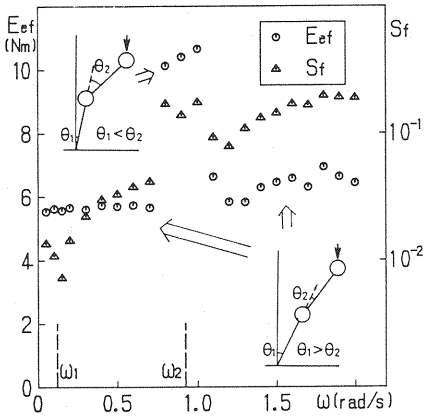


Fig. 10 Effective input energy and power versus circular frequency of disturbing force (Type 1).

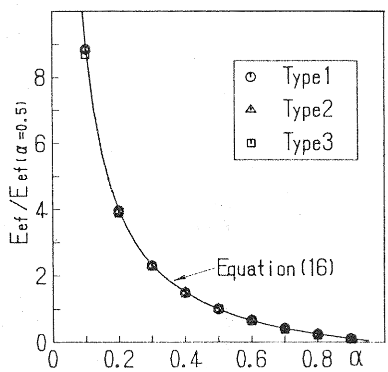


Fig. 11 Effective input energy versus static load.

from the first natural mode shape and under the disturbing force whose frequency is near the first natural one.

(5) Effect of static load on effective input energy

Fig. 11 shows the relationship between the effective input energy E_{ef} up to the dynamic ultimate state and the magnitude of the static load in the cases of types 1, 2 and 3, when the frequency of the disturbing force is equal to the first natural frequency. The abscissa shows the magnitude of the static load by α, the ratio to the buckling load, and the ordinate shows the effective input energy normalized by the one in the case of $\alpha=0.5$. The circles, the triangles and the squares correspond to types 1, 2 and 3, respectively. The effective input energy increases rapidly as the static load decreases. However, if the systems have the same static load ratio, the normalized effective input energy in three types have no significant difference. The relationship between the effective input energy of single-degree systems and the static load ratio (estimated by the ratio to the buckling load of the single-degree system) is also shown by the solid line in this figure. The relationship is obtained from Ref. 6) and written as

$$E_{ef}=E_{su}=E_y(1-\alpha)/\alpha \quad (16)$$

in which E_y is the maximum elastic strain energy. Eq. (16) gives a good prediction of the relationship between the effective input energy of two-degree systems and the static load ratio.

Then, if the effective input energy of the system subjected to the static load of an arbitrary magnitude is known, the effective input energy of the system subjected to the static load of a different magnitude can be predicted from this figure.

(6) Effect of rotational spring constant ratio on effective input energy

One of the parameters which characterize a two-degree system is the ratio between the rotational spring constant k_2 and k_1. Then, Fig. 12 shows the relationship between the effective input energy E_{ef} up to the dynamic ultimate state and the ratio η $(=k_2/k_1)$ under the reference condition. In this case, k_1 is fixed ($k_1=200$ Nm). The variation of the ratio η corresponds to the variation of the stiffness between the upper part and the lower part of a structure such as a tower. The ordinate shows the effective input energy E_{ef} and the abscissa shows the ratio η. The circles indicate the effective input energy obtained numerically for two-degree systems. When $\eta>0.6$, θ_1 is greater than θ_2 in the first natural mode shape and the system collapses as a result of the yielding of rotational spring 1. On the other hand, when $\eta<0.6$, θ_2 is greater than θ_1 in the first natural mode shape and the system collapses as a result of the yielding of rotational spring 2. Though both the rotational springs yield in the case of $\theta=0.6$, the concentration of yielding occurs actually in one of the two springs. Therefore, the relationship between the effective input energy and the spring constant ratio η is found to be characterized by the two curves. Then, for the two types of

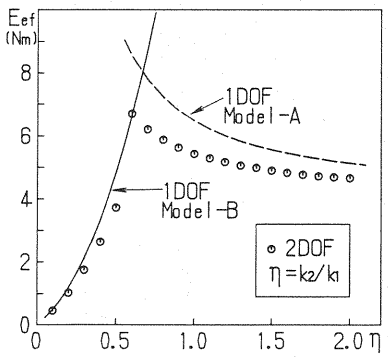


Fig. 12 Effective input energy versus rotational spring constant ratio (Type 1).

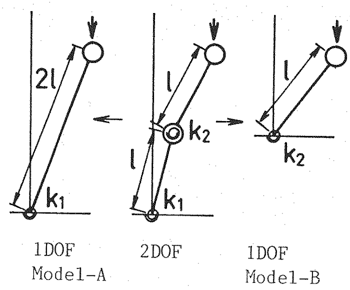


Fig. 13 Simplified 1 DOF model.

single-degree systems which are obtained by simplifying the two-degree system, the relationship between the effective input energy obtained by Eq. (16) and the ratio η is also shown in Fig. 12. One of the simplified single-degree systems has only the lower rotational spring and the other has only the upper rotational spring as shown in Fig. 13. Hereafter, the former is called Model-A and the latter is called Model-B. In Fig. 12, the dashed line shows the effective input energy estimated by Model-A and the solid line shows the energy estimated by Model-B. In this case, the static load ratio α is given according to the corresponding single-degree system. The effective input energy of the two-degree system is approximately predicted by the modeled single-degree systems, that is, Model-A for the range of $\eta > 0.6$ and Model-B for the range of $\eta \leqq 0.6$.

(7) Effect of yield rotational angle ratio on effective input energy

An important parameter other than the spring constant ratio with respect to the restoring characteristics is the ratio between the yield rotational angles of springs 1 and 2. Fig. 14 shows the relationship between the effective input energy up to the dynamic ultimate state and the yield rotational angle ratio ξ for the system of type 1 under the reference condition except for the yield rotational angle ratio. The ratio ξ is defined as the ratio of the yield rotational angle of spring 2 to that of spring 1, that is, $\xi = \theta_{y2}/\theta_{y1}$ (θ_{y1} is fixed). The ordinate shows the effective input energy E_{ef} and the abscissa shows the yield rotational angle ratio. The circles show the effective input energy obtained numerically for two-degree systems. In the range of $\xi \geqq 0.6$, the effective input energy does not change with the ratio ξ. This implies that in this range, the system collapses as a result of the yielding of spring 1 and that spring 2 does not yield. On the other hand, in the range of $\xi < 0.6$, the system collapses as a result of the yielding of spring 2. Therefore, the effective input energy decreases with reduction of ξ, because the maximum strain energy absorption of the system is considered to decrease with reduction of ξ. The effective input energy estimated from Model-A and Model-B is also shown in Fig. 14. From this figure, it is also found that the effective input energy of two-degree systems can be practically estimated by using simplified single-degree systems in the case where the yield rotational angle ratio changes.

(8) Effect of mass ratio on effective input energy

For systems with the same properties as the system of type 1 except for the mass, Fig. 15 shows the relationship between the effective input energy up to the dynamic ultimate state and τ, the ratio of the mass m_2 to the mass m_1. Here, m_1 is fixed, whereas m_2 changes. The ordinates show the effective input energy E_{ef} and the power S_f. The abscissa shows the mass ratio τ ($= m_2/m_1$). The circles show the effective input energy and the triangles show the power. According to the figure, the effective input energy does not depend on the mass ratio τ. Because the natural mode shape, that is, the collapse mode shape does not vary with variation of the mass in the range treated here. However, the power up to the dynamic ultimate state tends to decrease with increase of the mass ratio τ. This point should be investigated to establish the

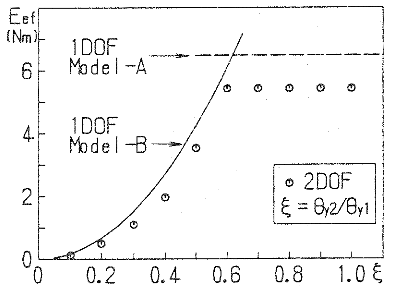


Fig. 14 Effective input energy versus yield rotational angle ratio (Type 1).

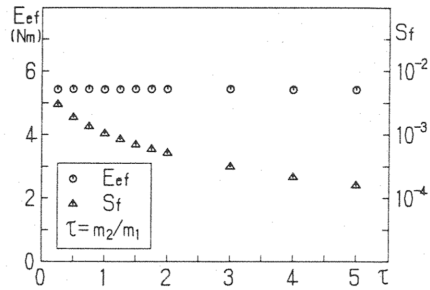


Fig. 15 Effective input energy versus mass ratio (Type 1).

dynamic failure criteria of structures with structural instability in relation to the magnitude of a disturbing force in the future.

4. CONCLUSIONS

In order to study the fundamental dynamic collapse behaviors of multi-degree-of-freedom structures with structural instability, the behaviors of two-degree flexural systems are investigated mainly by the numerical method. The system consists of two masses, two rigid bars, two rotational springs and is subjected to a disturbing force and a static load. Special attention is paid to the effects of the collapse mode on the dynamic ultimate strength, so the properties of structures and loads on the effective input energy are examined. In this study, the restoring characteristics of the rotational spring is an ideal elasto-plastic one and the viscous damping is not taken into account.

The main findings through this study are summarized as follows :

(1) When a two-degree system with structural instability is subjected to a disturbing force and collapses, the effective input energy governs the dynamic ultimate state of the two-degree system in the same manner as the case of a single-degree system. The effective input energy is obtained by subtracting the energy dissipated by the hysteretic damping from the input energy exerted by the disturbing force.

(2) The effective input energy of the two-degree system up to the dynamic ultimate state increases rapidly with decreases of the magnitude of the static load, which causes the structural instability. This tendency is the same as the case of a single-degree system. Therefore, for example, if the effective input energy of the system subjected to the static load equal to a half of the buckling load is known in advance, it is easy to estimate the effective input energy of the system subjected to the static load of a different magnitude.

(3) The effective input energy of the two-degree system up to the dynamic ultimate state little depends on the masses of the system and the magnitude of the disturbing force. On the other hand, it depends on the collapse mode of the system. However, it is very likely that the system collapses in a mode shape which is developed from the first natural mode shape and under the disturbing force whose frequency is near the first natural frequency.

(4) Though the effective input energy of the two-degree system up to the dynamic ultimate state varies with the properties of the springs, the energy can be approximately estimated by the energy of a simplified single-degree system which is assumed from the collapse mode shape of the two-degree system.

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numerical analyses in this study.

APPENDIX ESTIMATION OF NONLINEAR RESTORING FORCE

The equation of motion for the system is written from Eqs. (2) and (11) in matrix form as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{R}'=\boldsymbol{f} \quad \cdots\cdots (A\cdot 1)$$

The equation is solved by a step-by-step integration method, where the modified Newton-Raphson method and the Newmark β method are combined, and the process of the solution is shown in Ref. 12). The process of estimating the restoring force vector $\boldsymbol{R}'$ of the system from the horizontal displacement vector $\boldsymbol{x}$ of the mass at an arbitrary time is briefly described in the following.

First, the rotational angle vector $\boldsymbol{\theta}$ of the spring is obtained from the horizontal displacement vector by using Eq. (6). Second, the restoring moment vector $\boldsymbol{R}$ of the spring is calculated from the rotational angle vector $\boldsymbol{\theta}$ according to the assumed restoring characteristics.

For example, if the restoring characteristics of a rotational spring is an ideal elasto-plastic one such as shown in Fig. A・1, an element R_i of the restoring moment vector $\boldsymbol{R}$ is obtained from the corresponding rotational angle θ_i. The restoring moment vector $\boldsymbol{R}$ is composed of the elements obtained above. Then the restoring force vector $\boldsymbol{R}'$ of the system is obtained from Eq. (11).

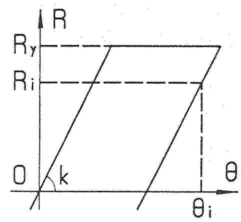

Fig. A・1 Estimation of restoring moment from corresponding rotational angle.

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