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RELIABILITY BASED ECONOMIC EVALUATION OF STRUCTURES CONSIDERING THE LIFE TERM

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The reliability based economic evaluation of structures is usually carried out to minimize the expected total life cost of structures, including initial cost C_i and failure cost $p_f C_f$. The minimization is performed using the principle of minimum total cost. But the expected total loss cost which is to be minimized lacks consideration of serviceability or life term of structures. In this paper, interest rates are considered into the evaluation of expected total loss cost in order to translate the cost of failure to present worth for comparison with the initial cost. In order to calculate the expectation of reliability during the life term of structures, a safety reduction function is also proposed.

Keywords: *optimum, reliability, cost, interest rate, life term*

1. INTRODUCTION

Civil engineering structures are usually expected to remain safe and fully serviceable to the public, during their design life. Also, civil engineering structures are generally expected to be economical to have an optimal total life cycle cost. Thus, the evaluation of cost-effectiveness is an important area of study. At this point, it should be evident that there is a trade-off between the cost and safety of a structure. Therefore, there is a need to optimize both cost and safety in a structure.

In this paper, the reliability-based economic evaluation of structures is performed utilizing so called the principle of minimum total cost. In order to evaluate the total cost of a structure, the interest rate during the service life of the structure is considered so that translation of the cost of failure to a present worth value can be achieved. The concept of expected serviceability or design life term of structures is also introduced as well as the safety reduction function. The safety reduction function is used to estimate the mean safety level during the life term or serviceability term of structures.

2. OPTIMUM SAFETY INDEX

The safety index derived from the First-Order Second-Moment is generally shown as[1)~3)]:

$$\beta=\frac{\bar{\theta}-1}{\sqrt{\bar{\theta}^2 V_R^2+V_S^2}} \quad (1)$$

The safety index can be alternatively written:

$$\beta=\frac{\ln \bar{\theta}}{\sqrt{V_R^2+V_S^2}} \quad (2)$$

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in which $\overline{\theta}=\overline{R}/\overline{S}$=the central safety factor, $\overline{R}$, $\overline{S}$, V_R, V_S=the means and coefficients of variation (c. o. v.) of strength R and load effect S, respectively. To facilitate the expression, Eq. (2) is adopted as the safety index formulation, hereafter.

The expected total loss cost is expressed by[4)~6)]:

$$E[C_T]=C_I+p_f C_f \quad (3)$$

in which $E[C_T]$=the expected total loss cost, C_I=the initial loss cost, and C_f=the loss cost due to failure of structures.

The failure probability p_f can be obtained by:

$$p_f=\Phi(-\beta) \quad (4)$$

in which $\Phi(\cdot)$=the cumulative distribution function of the standard normal variable.

The structures which are designed by the failure criteria $Z=R-S=0$ are optimized using the requirement that total cost should be minimized.

According to this requirement, the optimum safety index can be obtained from the equation as follows[4),7)]:

$$\frac{\partial E[C_T]}{\partial \beta_{|\beta_0}}=\frac{\partial C_I}{\partial \beta}+C_f\frac{\partial p_f}{\partial \beta}=0 \quad (5)$$

in which β_0=the optimum safety index.

The optimum safety index β_0 can be obtained, principally, using Eq. (5). But this is very hard to do[8)] because the failure cost is difficult to estimate. So, to avoid difficulty, failure cost C_f is assumed to be given as the ratio to initial cost C_I, here.

3. EVALUATION OF EQUIVALENT VALUE

The expected total loss cost, Eq. (3), lacks consideration of serviceability or life terms of structures. To estimate the economic value of structures, it is desirable that the time factor should be taken into consideration. Thus, design life N_0 and expected serviceability life N of structures are introduced. Design life is usually determined at the time when structures are designed by considering the return periods of maximum load. After the construction, structures are going to deteriorate gradually, for many reasons and eventually be replaced N years after construction due to failure or unusability. Generally, the relation between N_0 and N can be shown to be:

$$N=\lambda N_0, \quad (0\leq\lambda\leq 1) \quad (6)$$

in which λ is a constant.

Therefore, the safety index β after N year from construction can be proposed as:

$$\beta\simeq A_F\beta_I \quad (7)$$

in which A_F=attenuation factor.

$$\beta_I=\frac{\ln\overline{\theta}_I}{\sqrt{V_R^2+V_S^2}} \quad (8)$$

in which $\overline{\theta}_I=\overline{R}_I/\overline{S}$, $\overline{R}_I$=the mean strength at initial design condition, $\overline{S}$=the mean load effect during N and V_R, V_S=the c. o. v. s of strength and load effect, respectively.

Many types of function can be assumed for A_F depending on the particular purpose of study. However, as shown in Fig. 1, an exponential type function is proposed as the safety reduction function.

It takes the form as:

$$\beta\simeq\beta_I\exp(-\alpha N/N_0) \quad (9)$$

in which α=the constant showing the degree of deterioration. α is assumed to vary between 0.5 and 1.0, in this paper.

Usually, N_0 is 30 to 50 years for concrete structures. The p_f value in Eq. (3) and (4) is considered to be the expected value of p_f through N- year.

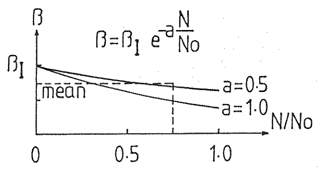


Fig. 1 Attenuation Function.

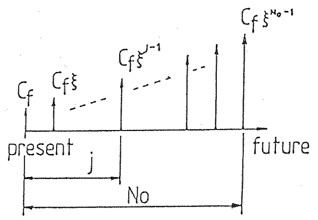


Fig. 2 Cash Flow Diagram.

Using Eq. (9), p_f can be obtained as:

$$\tilde{p}_f=\Phi(-\tilde{\beta}) \quad (10)$$

in which

$$\tilde{\beta}=\frac{1}{\alpha N}\int_0^{\alpha N}\beta_I \exp\left(-\frac{t}{N_0}\right)dt=\frac{N_0}{\alpha N}\left(1-\exp\left(-\frac{\alpha N}{N_0}\right)\right)\beta_I \quad (11)$$

To calculate the equivalent value of failure cost, the cash flow diagram of final worth of C_f is assumed as Fig. 2.

The year j when failure occurs or replacement is needed is a random variable, so the expected value of C_f through j is calculated as:

$$\tilde{C}_f=C_f(1+\xi+\xi^2+\cdots\cdots+\xi^{j-2}+\xi^{j-1})/j=C_f(\xi^j-1)/j(\xi-1), \quad (j=1,\cdots,N+1) \quad (12)$$

in which $\xi=1+i$ and i=the interest rate.

Furthermore, the initial cost C_I can be assumed as follows[9].

$$C_I=a(1+b\theta_I) \quad (13)$$

in which a, b are constants. Finally, the expected total loss cost, Eq. (3), is rewritten using Eq. (10), Eq. (12) and Eq. (13) as:

$$E[C_T]=a(1+b\bar{\theta}_I)+\tilde{p}_f C_f(\xi^j-1)/j(\xi-1), \quad (j=1,\cdots,N+1) \quad (14)$$

Using Eq. (5), the optimum safety index β_0 is found to be:

$$\beta_0=\frac{-V_\theta+\sqrt{V_\theta^2+2A^2\ln(A\cdot C_f^*\cdot\phi/(\sqrt{2\pi}\,V_\theta))}}{A^2} \quad (15)$$

in which

$$A=\frac{N_0}{\alpha N}\left(1-\exp\left(-\frac{\alpha N}{N_0}\right)\right) \quad (16)$$

$$C_f^*=C_f/a\cdot b \quad (17)$$

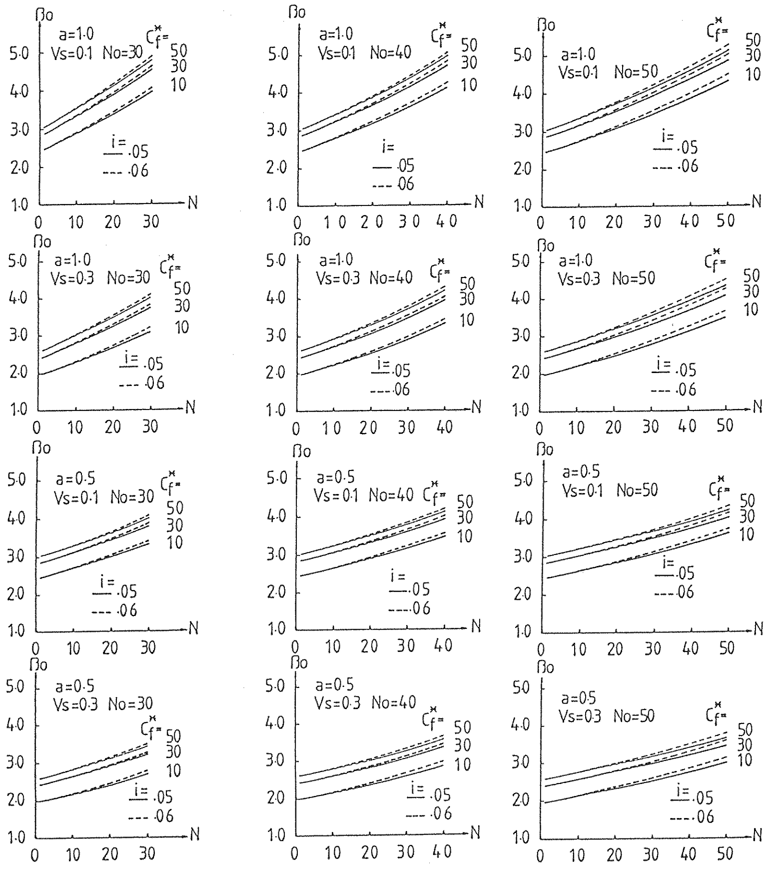
$$\phi=(\xi^j-1)/j(\xi-1) \quad (18)$$

$$V_\theta=\sqrt{V_R^2+V_S^2} \quad (19)$$

4. NUMERICAL EXAMPLES AND AN APPLICATION

(1) Examples

In Fig. 3(a)-3(c), the optimum safety index β_0 for N is shown. Fig. 3 is composed of the combination of three parameters $\alpha=0.5$, 1.0 and $V_S=0.1$, 0.3 and $N_0=30$, 40, 50, respectively. In these combinations, Fig. 3(a), (b), (c) corresponds to $N=30$, 40, 50, respectively when other combinations are equal. The c. o. v. of strength, V_R, is 0.1 in these examples. Forecasting the annual rate of interest is very difficult. Because, it depends on the economic conditions of the world or nations from now on, greatly. It is, however, assumed in this paper that the annual interest rate varies from 0.05 to 0.06. The assumption is based on the moderate rate of public investment in the world, and the anticipation that there will be no extreme rise of interest rates in the near future. The value of $C_f^*=C_f/ab$ is assumed to vary from 10 to 50. Also, the design life of structures N_0 varies from 30 to 50 years.



(a) (b) (c)

Fig. 3 Optimum Safety Index.

It is stated from Fig. 3(a) to 3(c) that if long term serviceability is to be expected to N, giving higher annual interest rate and the short term design life of N_0 for the same N, the more economical design is obtained when relatively high safety level is specified to the structure. For example, for $\alpha=1.0$, $V_S=0.1$, $N_0=50$ in Fig. 3(c) (right top), the optimum safety index value for $N=40$ can be specified greater than the optimum value for $N=20$. But the difference of C_f^* between the annual rate is not so significant.

For the fixed V_R, as stated by Sugiyama et al. [10], more economical design can be obtained by setting the safety level relatively high, for small V_S. Also, the safety reduction function's parameter α has a significant effect in design of structures or structural systems. For example, other factors being equal, if $\alpha=0.5$ then the optimum safety index is around about 4.0 for $V_S=0.1$, and 3.0 for $V_S=0.3$, while if $\alpha=1.0$ then they are 5.0 and 4.0, respectively. From this, it can be stated that the more economical design may be obtained when the safety level is set relatively high for the structures that have a relatively small safety reduction with time.

The dimensionless expected total loss cost $E_{CT}^*=(C_T-a)/ab$ is shown in Fig. 4(a) to 4(d). In Fig. 4(a) to 4(d), the c. o. v. of load effect varies from 0.1 to 0.3, and C_f^* is 10 to 30. The minimum expected total loss cost corresponding to the optimum safety index is also drawn in the figures as a white circle. From the figures, it is seen that when $V_S=0.1$, the difference between $C_F^*=10$ and $C_F^*=30$ is little when compared to that with $V_S=0.3$. On the other hand, the difference of E_{CT}^* between $\alpha=0.5$ and $\alpha=1.0$, through Fig. 4 (a), (b) and (c), is significant if other conditions are equal. Also, the difference between $i=0.05$ and $i=$

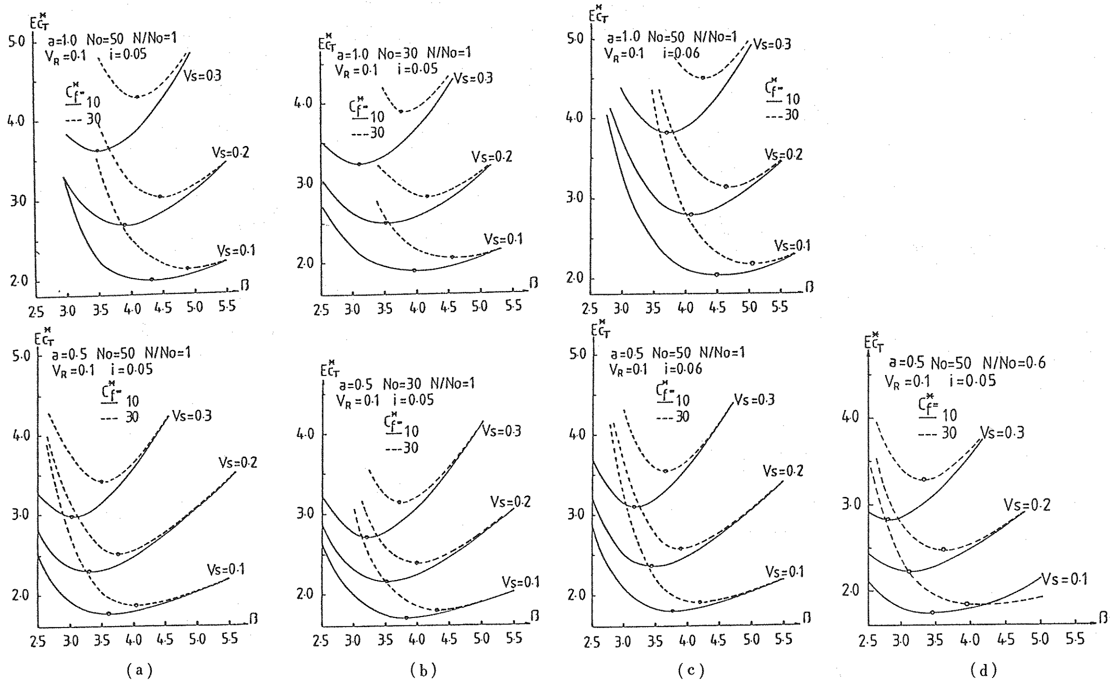


Fig. 4 Expected Total Loss Cost E_{CT}^{*}.

0.06 is not very significant when $V_S=0.1$ and $V_S=0.2$. Furthermore, the optimum safety level lies between about 3.0 and 4.5 for $\alpha=0.5$, and between about 3.5 and 5.0 for $\alpha=1.0$, for all designs considered in this paper, regardless of the values for N_0, i or V_S. The optimum E_{CT}^{*} for the N for $i=0.05$ and $\alpha=1.0$ is also shown in **Fig. 5**. From **Fig. 5**, it is easily seen that the E_{CT}^{*} is affected greatly by the variation of V_S. To obtain an optimal design, the estimation of V_S is considered to be important.

(2) An application

The structures designed by the failure criteria $Z=R-S=0$ should usually be designed for maximum load or load effect that may be expected to occur during the design life. The load has an expected return period T. The relation between the design life term N_0 of structures and T can be shown as[7] :

$$T=\frac{1}{1-q^{1/N_0}} \quad \cdots\cdots (20)$$

in which q=the probability that the annual maximum load does not exceed the design load or load effect. According to Eq. (20), if $N_0=50$ then $T=480\simeq 10N_0$ for $q=0.9$, and $T=220\simeq 4N_0$ for $q=0.8$, respectively. But, in the actual design, T is considered approximately to be equal to from N_0 to $2N_0$ at most. In this case, therefore, q must be reduced to about 0.5. If it is considered to be appropriate for $q=0.5$[7,11], then the maximum return period for T is taken to be from $T=N_0$ to $T=2N_0$, here.

The failure criterion for limit state subject to repeated loading during the return period T can be shown as[12] :

$$Z=R-S_{M,T}=0 \quad \cdots\cdots (21)$$

in which $S_{M,T}$ is the maximum load observed during the $T=n\cdot\Delta$, ($n=1, 2, \cdots$), Δ=unit period (1 year). The distribution function of $S_{M,T}$ is shown to be :

$$F_{M,T}(s)=P(\max_{1\leq i\leq n}.\{S_i\}<s)=\{F(s)\}^T \quad \cdots\cdots (22)$$

in which $F(\ )$=distribution function of S, $\{S_i\}$=the realization of load S_i.

$F(r)$ and $F(s)$ are assumed to be normally distributed, though $\{F(s)\}^T$ is not. A further assumption,

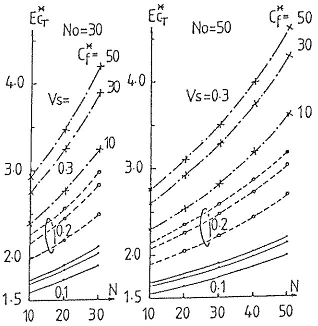


Fig. 5 Optimum Total Loss Cost.

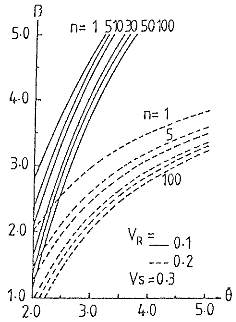


Fig. 6 Safety Index $\beta^{(n)}$.

Table 1 a_1 and b_1 for n.

n	1	5	10	30	50	100
a_1	0	1.163	1.539	2.043	2.249	2.507
b_1	1.000	0.448	0.344	0.246	0.216	0.185

Table 2 E^*_{CT} for $\beta^{(n)}$ (V_R=0.1, V_S=0.3).

	T=50		T=100	
C^*_F \ i	0.05	0.06	0.05	0.06
10	3.80	3.90	5.52	8.08
30	4.29	4.59	9.86	17.55
50	4.79	5.28	14.20	27.02

Table 3 E^*_{CT} for β_0 (α=1.0, V_R=0.1, V_S=0.3).

	N_o=30		N_o=50	
C^*_F \ i	0.05	0.06	0.05	0.06
10	3.27	3.34	3.60	3.81
30	3.88	3.99	4.28	4.50
50	4.20	4.32	4.61	4.83

then, is introduced that $\{F(s)\}^T$ is also normally distributed. In this case, approximate mean values and standard deviations of exact distribution $\{F(s)\}^T$ are given as[13),14)]:

$$\mu_S^{(n)}=\mu_S^{(1)}+a_1\sigma_S^{(1)} \quad \cdots\cdots (23)$$

$$\sigma_S^{(n)}=\sqrt{b_1}\,\sigma_S^{(1)} \quad \cdots\cdots (24)$$

in which $\mu_S^{(1)}$, $\mu_S^{(n)}$, $\sigma_S^{(1)}$, $\sigma_S^{(n)}$ are mean values and standard deviations for $T=1$ and $T=n$ of $S_{M,T}$, respectively. The constants a_1 and b_1 are given, for various n, in Table 1[13)].

Using these values a_1, b_1 from Table 1, the safety index $\beta^{(n)}$ of Eq. (21) can be obtained by the method proposed by Hasofer and Lind[15)], as shown in Fig. 6. The central safety factor θ in Fig. 6 is defined as:

$$\theta=\mu_R/\mu_S^{(n)}, \quad (n=1, 5, \cdots, 100) \quad \cdots\cdots (25)$$

in which μ_R and $\mu_S^{(n)}$ are mean value of R and $S_{M,T}$, respectively.

From Fig. 6, if T is assumed to be from 50 to 100 year, then the design life term of structures are considered to be from 30 to 50 year, approximately. The c. o. v. s of strength and load may not always be constant but vary through N_0, but c. o. v. for R and $S_{M,T}$ are estimated here as V_R=0.1-0.2 and V_S=0.3 as the mean value during N_0.

The expected total loss cost calculated from Eq. (14) for V_R=0.1 and θ=3.0, is $\beta^{(n)}$=3.8-4.0 for n=50-100 in Fig. 6, is shown in Table 2. The results for V_R=0.2 are excluded because the safety index for V_R=0.2 seem to be too small for actual design.

The optimum expected total loss cost for N_0=30 to 50 that corresponds to a T (=n) =50 to 100 are also shown in Table 3. It is seen from Tables 2 and 3 that the design using β_0 which can be obtained from the method proposed in this paper is more economical than the design using $\beta^{(n)}$ (n=1, $\cdots$), if the life term or serviceability term of structures is taken into consideration in design. Furthermore, it is shown by calculation that the more economical design can be obtained by using the β_0 proposed here, even if V_R=0.2.

5. CONCLUSIONS

The optimization of the cost of structures or structural systems is usually approached using the principle of minimum total cost. It is expressed as the expected total loss cost, $E[C_T]=C_I+p_fC_F$. The life term or serviceability term is very important to this analysis. Thus, the need arises to consider the interest rate during the expected life of the project when estimating the failure cost (to compare with equivalent to initial cost). Furthermore, the safety reduction function is proposed to estimate the safety level of structures or structural systems. This function can be used to calculate the mean failure probability.

The optimum safety indices, considering serviceability term or design life term of structures or structural systems, are obtained by minimizing $E[C_T]$. It is concluded that the minimum expected total loss cost is obtained by setting the safety level relatively high when c. o. v. for load or load effect is small and when the design life term or serviceability term of structures is long, provided the c. o. v. of strength is fixed. For the design of higher interest rate through the life or serviceability term of structures, it is also concluded that if c. o. v. s of both strength and load or load effect are fixed, then setting the safety level relatively high will yield more economical structures.

REFERENCES

1) Ang, A. H-S. and Amin, M. : Safety factors and probability in structural design, J. Struct. Div., ASCE, Vol. 95, No. ST7, pp. 1389–1405, 1969.
2) Ang, A. H-S. and Cornell, C. A. : Reliability bases of structural safety and design, J. Struct. Div., ASCE, Vol. 100, No. ST9, pp. 1755–1769, 1974.
3) Cornell, C. A. : Structural safety specifications based second moment reliability analysis. Final Report, IABSE Symposium on concepts of safety method on design, pp. 235–246, 1969.
4) Mau, S. and Sexsmith, R. : Minimum expected cost optimization, J. Struct. Div., ASCE, Vol. 100, No. ST9, pp. 2043–2058, 1972.
5) Moses, F. and Kinser, D. E. : Optimum structural design with failure probability constraints, J. AIAA, pp. 1152–1158, 1967.
6) Switzky, H. : Minimum weight design with structural reliability, AIAA, 5th Annual Structures and Materials Conference, pp. 316–322, 1964.
7) JSCE : Safety and reliability of structures, published by Japan Society of Civil Engineer, 1976 (in Japanese).
8) Fujino, Y. : Structural design and safety checking method based on probability theory (in Japanese), J. of JSCE, pp. 33–39, No. 2, 1982.
9) Lind, N. C. : Approximate analysis and economics of structures, J. Struct. Div., ASCE, Vol. 102, No. ST6, pp. 1177–1195, 1976.
10) Sugiyama, T. et al. : Decisions on Reliability Level and Safety Factor for Structural Design (in Japanese), Proc. of JSCE, pp. 21–28, 1982.
11) Fujino, Y. and Hasegawa, A. : What are safety factors-An approach to uncertainty-Topics. (in Japanese), JSCE, pp. 39–44, No. 9, 1980.
12) Grigoriu, M. D. and Turkastra, C. : Structural safety indeces for repeated loads, J. Engrg. Mech. Div., ASCE, Vol. 104, No. EM4, pp. 829–843, 1976.
13) Ruben, H. : On the moments of order statistics in samples from normal populations, Biometrika, Vol. 41, pp. 200–227, 1954.
14) Tippet, L. H. C. : On the extreme individuals and the range of samples taken from a normal population, Biometrika, Vol. 17, pp. 364–387, 1925.
15) Hasofer, A. M. and Lind, N. C. : Exact and invariant second-moment code format, J. Engrg. Mech. Div., ASCE, Vol. 100, No. EM1, pp. 111–121, 1974.

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