

ELASTO-PLASTIC OUT-OF-PLANE BUCKLING STRENGTH OF THROUGH TYPE AND HALF-THROUGH TYPE ARCH BRIDGES

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Elasto-plastic out-of-plane buckling strength of through type and half-through type arch bridges are analyzed by a finite element method which is capable of considering finite displacements and yielding of materials. Examining the numerical results and using an analogy between an arch and a column, we propose a method to determine the slenderness parameter for arch bridges. A designer can use the slenderness parameter to obtain an ultimate stress for arches by substituting it into a column strength formula. The predictions by the proposed procedure are shown to be fairly good correspondence with all of numerical results.

Keywords: arch bridge, out-of-plane buckling, ultimate strength, effective length procedure, slenderness parameter

1. INTRODUCTION

Several studies have been reported on the ultimate load-carrying capacities of arch structures with regard to their out-of-plane behaviour under lateral loads^{1)~3)}. But the ultimate strength of arch bridges which are subjected to uniform in-plane load and fail by lateral instability has not been clarified sufficiently^{4)~9)}.

The ultimate strength of a single arch member subjected to uniformly distributed in-plane load is usually governed by out-of-plane buckling. In this case, the strength of an arch member can be described by the nondimensionalized average normal stress $\bar{\sigma} = N_u / A \sigma_y = \sigma_u / \sigma_y$. In this equation, N_u is the axial force at the springing calculated for the ultimate distributed load p_u by a linear theory; A is the cross-sectional area of the single arch rib and σ_y is the yield point stress of the steel used. When we define a slenderness parameter $\lambda_y = (0.5 L / \pi r_y) \sqrt{\sigma_y / E}$ for the arch member and plot numerical results of elasto-plastic finite displacement analysis as a $\bar{\sigma} - \lambda_y$ relation, we can find that the numerical results coincide fairly well with the standard column strength curve as shown in Fig. 1. In the definition of λ_y , L denotes the curved length of the arch member, r_y is the radius of gyration of the arch rib cross section with respect to out-of-plane bending and E stands for Young's modulus.

This coincidence means that the buckling strength of an arch member can be determined by replacing the arch member with a clamped-clamped column of length L , of which effective length factor K is 0.5. Extensively, even for a more complicated arch bridge if an adequate effective length factor K can be found, there is a great possibility to be able to determine its out-of-plane buckling strength σ_u by substituting the

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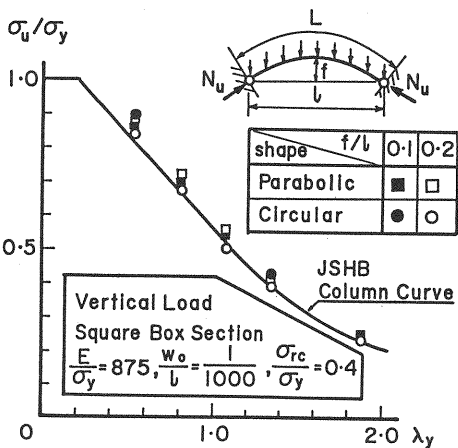


Fig.1 Lateral buckling strength of a single arch rib.

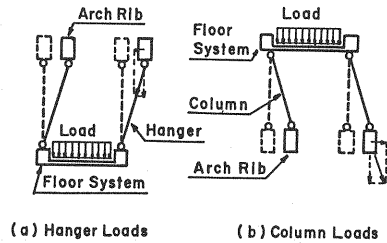


Fig.2 Effect of tilting loads.

slenderness parameter $\lambda_y=(KL/\pi r_y)\sqrt{\sigma_y/E}$ into the column strength formula⁹⁾.
The Japanese Specifications for Highway Bridges (JSHB)¹⁰⁾ recommends a method similar to what described above for evaluating out-of-plane buckling strength of arch bridges. But, in the provisions of JSHB, the effective length factor is determined as $K=\pi/\sqrt{\gamma}$ by using the buckling coefficient γ obtained as the eigen-value for elastic buckling. Further, the buckling coefficient γ , is computed under the assumption that the structure can be considered as a single arch member. As will be discussed later in this paper, it is not acceptable to adopt the same γ value for the determination of the out-of-plane buckling strength of through type or half-through type arch bridges, which have an opening portion with no lateral bracings in order to provide traffic access.
In this paper, the ultimate strength of realistic theoretical models for arch bridges of through or half-through type are first analyzed by a finite element method which is capable of considering finite displacements and material plasticity¹¹⁾. Next, considering the ultimate strength characteristics and behaviour of theoretical models and using an analogy between an arch and a column, we determine adequate effective length factors for arch bridges. The three principal factors governing the strength of arch bridges are the support condition for lateral bending, the effect of tilting loads through hangers or columns (see Fig. 2) and the stiffening effect of lateral bracing members connecting twin ribs. Herein, effective length factors K_e ($=0.5$), K_l and K_β are introduced for the three principal factors respectively and the latter two factors K_l and K_β are determined. The definition of K_l and K_β are modified from those defined in Ref. 9) to describe the behaviour of more realistic theoretical models with stiffening girders. Consequently, the slenderness parameter for arch bridges is defined as follows :

$$\lambda_y=(K_eK_lK_\beta L/\pi r_y)\sqrt{\sigma_y/E} \dots\dots\dots (1)$$

Then it is shown herein that the out-of-plane buckling strength of complicated theoretical models for arch bridges can be determined easily and accurately by substituting the defined slenderness parameter into a standard column strength formula.

2. OUT-OF-PLANE BUCKLING STRENGTH OF THROUGH TYPE ARCH BRIDGES

(1) Theoretical models

Theoretical models studied are 2-hinged parabolic arches shown in Fig. 3. Upper and lower ends of the hanger are assumed to be hinged about the out-of-plane rotation to exclude the effect of their flexural rigidity on the ultimate strength of the model. This is the conservative assumption to avoid complicated secondary effects. An initial out-of-plane deflection of sinusoidal half wave of amplitude $0.001\ l$ is assumed for arch ribs as an imperfection. The distance of twin arch ribs, a , is varied as 20 m, 10 m, 5 m,

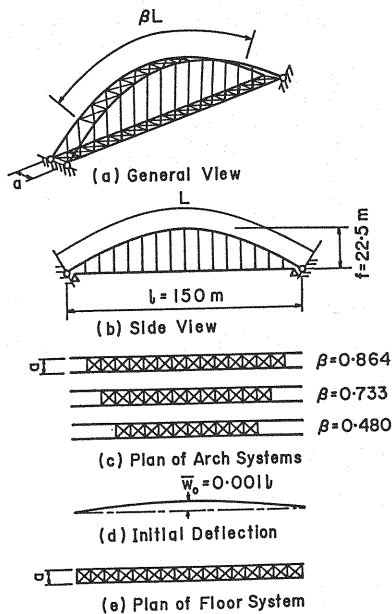


Fig. 3 Theoretical models for through type arch bridges.

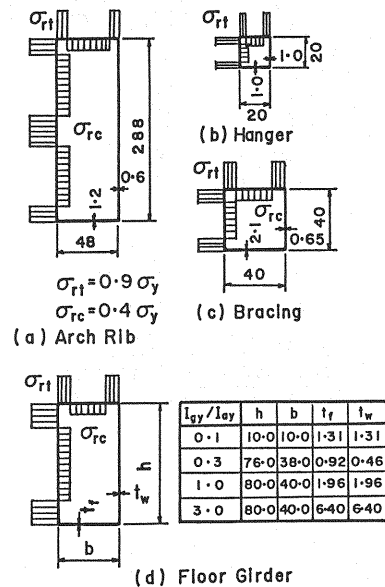


Fig. 4 Cross sections and residual stress distributions (unit in cm).

3 m and 2 m to change the slenderness of the bridge as a whole. Since the radius of gyration of the connected twin arch ribs as a whole is approximately determined by $\sqrt{2 I_y + 2 A(a/2)^2} / 2 A \approx a/2$, the nominal slenderness ratios of the models are determined by $L/(a/2)$ and become 15.6, 31.7, 63.4, 105.7, 158.6 when the value a , is 20 m, 10 m, 5 m, 3 m, 2 m, respectively. Braced length ratio β , which is defined as the ratio of the partial length of arch rib, where lateral bracing members are located, to the total length of the arch rib, and is varied as 0.864, 0.733, 0.48. The ratio of the lateral bending stiffness of the deck system as a whole, EI_{gy} , to the lateral bending stiffness of the connected twin arch ribs as a whole, EI_{ay} , is also varied as 0.1, 0.3, 1.0, 3.0. A uniform load fully distributed on the bridge deck is considered to be the severest one possible and is applied to the floor system as equivalent concentrated nodal forces in the analysis. The material is assumed to be a mild steel of which yield point stress is 235 MN/m^2 (24 kgf/mm^2).

Dimensions of the cross section of each member and the residual stress distribution assumed are shown in Fig. 4. The value of the ratio of the height, h , to width, b , of arch rib cross section is from 1 to 3 in actual bridge arches, but in the analysis the value h/b is assumed to be 6 to avoid in-plane instability of the theoretical model. The cross sectional area of the arch rib is fixed at 448 cm^2 , the smallest value obtained from a survey on actual bridges, because the generality of the numerical results will be maintained by expressing the ultimate strength in terms of stress. The residual stress distribution pattern for the arch rib shown in the figure is determined by assuming a longitudinal stiffener at the midheight of the web plate.

The shape of the cross section of the hanger is square and the cross sectional area is determined to be sufficiently large so as to prevent yielding of the material under the ultimate nodal force.

In order to avoid a reduction in the out-of-plane buckling strength due to the premature buckling of bracing members, the lateral members and struts of bracing system and the floor system are proportioned so that they will not buckled before the overall failure of the arch bridge. The cross section of the floor girder is proportioned as shown in the table of Fig. 4 corresponding to the respective values of I_{gy}/I_{ay} mentioned above.

(2) Estimation of out-of-plane buckling strength by JSBH

Table 1 shows the out-of-plane buckling strength σ_u/σ_y obtained from the ultimate load at which

Table 1 Lateral buckling strength σ_u/σ_y for through type theoretical models.

β	a (m)	I_{gy}/I_{ay}			
		0.1	0.3	1.0	3.0
0.864	20	0.950	0.950	0.950	0.970
	10	0.910	0.950	0.950	0.970
	5	0.850	0.910	0.930	0.965
	3	0.770	—	0.910	—
	2	0.610	—	0.770	0.880
0.733	20	0.690	0.710	0.730	0.730
	10	0.630	0.675	0.730	0.730
	5	0.550	0.620	0.690	0.700
0.480	20	0.530	0.570	0.590	0.590
	10	0.410	0.470	0.560	0.560
	5	0.320	0.357	0.510	0.570

Table 2 Strength σ_{Hu} of theoretical models estimated by JSHB.

a (m)	r (m)	λ_s	σ_{Hu}/σ_y
20	10.00	0.064	1.00
10	5.00	0.128	1.00
5	2.51	0.256	0.97
3	1.52	0.423	0.88
2	1.02	0.627	0.77

out-of-plane deformations increase excessively without further incremental load in the numerical analysis [see Ref.8] . Since the portal frame and lateral bracing members in theoretical models are not proportioned to meet the provisions of JSHB, it might not be appropriate to adopt the provisions of JSHB for out-of-plane buckling to predict the ultimate strength of each theoretical model. But, the following computations

show that the JSHB provisions have serious limitations. According to the definitions of JSHB, the effective buckling length for model arches with $f/l=0.15$ can be determined as follows,

$$l_e = \phi \beta_z l = (1 - 0.35 \chi)(0.54 + 0.65)/2 \times 150 = 59.58 \text{ (m)}$$

where χ denotes a ratio of the load intensity sustained by a hanger to the load intensity applied and can be assumed to be 0.95 approximately. Substituting this effective length into the JSHB column curve, we can determine the ultimate strength of the theoretical model. The ultimate strength determined by JSHB are shown in Table 2. The ultimate strength for each theoretical model obtained by numerical results are shown in Fig.5, where the ultimate strength is defined by the stress σ_{Hu} computed from the ultimate horizontal thrust H_u , ($H_u/A\sigma_y = \sigma_{Hu}/\sigma_y$) as is done in JSHB, and λ_s is also calculated according to JSHB.

In this figure, the ultimate strength determined by JSHB corresponds to the JSHB Column Curve and shown by the solid curve. As is clear from Fig. 5, the estimation of ultimate strength according to JSHB is obviously not conservative for all theoretical models, although the results shown in Fig. 5 do not include the increased safety factor 2.0, which is recommended in JSHB for possible inaccuracy in design analysis. The smaller the value of β , the more the ultimate strength tends to be overestimated. This is believed to be caused by the fact that the effect of the braced length ratio β is not considered in the provisions of JSHB.

(3) Evaluation of K_β

The effective length factor K_β , which accounts for the effects of lateral bracing system on the ultimate strength of through type arch bridges, can be determined as follows. The numerical results for the out-of-plane displacement of the through type arch bridge, which can not have upper lateral bracings at both ends of the bridge to provide traffic access, are shown in Fig.6. The figure shows that very little transverse distortion occurs in the braced portion, βL , of the arch, but large deformation in the portion without bracings. Studies have also shown that twisting deformations of the ribs are found to be insignificant.

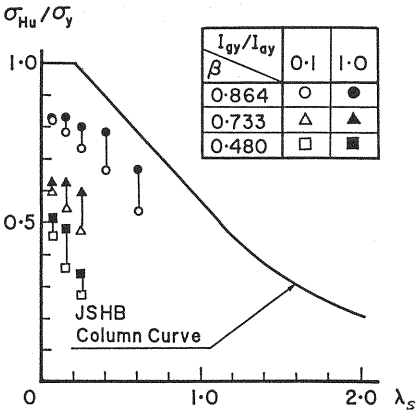


Fig.5 Strength of theoretical models plotted along the slenderness parameter defined in JSHB.

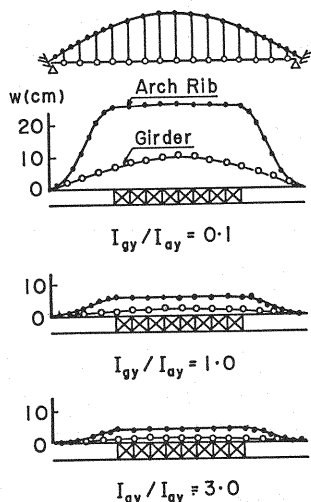


Fig. 6 Lateral displacement of through type arch bridge models (at the ultimate state, $a=10$ m, $\beta=0.48$).

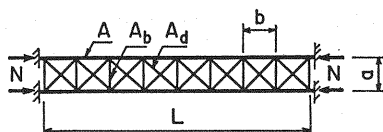


Fig. 8 Laced column with clamped ends.

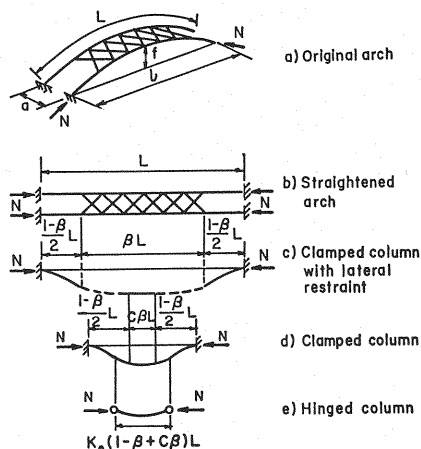


Fig. 7 Effective length for an arch system.

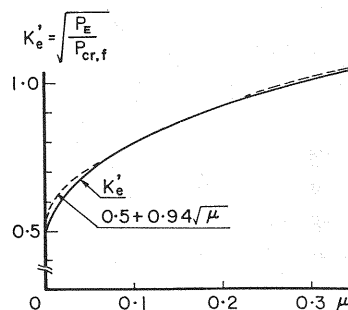


Fig. 9 Shear flexibility parameter μ and effective length factor K'_e .

As a result of these two considerations the decision was made to consider the arch rib in so far as lateral buckling is concerned as a hinged column having a length of $K_e(1 - \beta + C\beta)L$ (see Fig. 7 e) and a radius of gyration equal to r_y . A slenderness ratio, $K_e C\beta L / r_y$, represents the contribution to the entire slenderness ratio of the equivalent column from the braced length, βL . This portion is conceived as a laced column consisting of two rib areas separated by the distance, a , between the ribs as shown in Fig. 8.

When the shear rigidity of lattice members decreases, the buckling strength of the clamped-clamped laced column will be reduced and approach that of the hinged-hinged column. In this case the effective length factor of the laced column, K'_e , can be given by $K'_e = \sqrt{P_E / P_{cr,f}}$, in which P_E is Euler's buckling load for the hinged-hinged column and $P_{cr,f}$ stands for the buckling load for the clamped-clamped laced column. The relation between the value K'_e and the shear flexibility parameter μ can be obtained by using the critical values of clamped-clamped laced column, $P_{cr,f}$, computed by Lin et al.¹²⁾ as shown in Fig. 9. Approximating this relation by $K'_e = 0.5 + 0.94\sqrt{\mu}$ and the radius of gyration of the laced column by $a/2$, we can express the slenderness ratio of the laced column as

$$K'_e L / (a/2) = (0.5 + 0.94\sqrt{\mu}) L / (a/2) \quad (2)$$

where

$$\mu = \frac{1}{2} \left(\frac{a}{L} \right)^2 \left(\frac{a}{b} \right) \frac{A_a}{2A_d} \left[\left[1 + \left(\frac{b}{a} \right)^2 \right]^{3/2} + \frac{2A_d}{A_b} \right] \quad (3)$$

On the other hand, in the case where the arch rib is laterally braced along its total length ($\beta=1$), the slenderness ratio of the equivalent hinged column having a radius of gyration equal to r_y , is written by $K_e CL / r_y$. Then, equating Eq. (2) with $K_e CL / r_y$, we have :

$C=2\ r_y(0.5+0.94\sqrt{\mu})/(aK_e) \dots\dots\dots (4)$

Finally, the effective length factor, K_β , which accounts for the effect of lateral bracings is given as follows;

$K_\beta=1-\beta+|2\ r_y(0.5+0.94\sqrt{\mu})/(aK_e)|\beta \dots\dots\dots (5)$

Using this K_β and $K_e=0.5$, $K_t=0.65$ (explanation for K_t value will appear in the next section), we can obtain Fig.10, where the numerical results given in Table1 are plotted against $\lambda_y=(K_eK_\beta K_t L/\pi r_y)\sqrt{\sigma_y/E}$. As is clear from Fig.10, the ultimate strength of theoretical models closely approximate to the standard column curve.

(4) Evaluation of K_t

The coefficient K_t , which accounts for the effect of tilting loads, relates to the length of hangers and lateral rigidity of the road way. In the case of the through type arch bridge, if the road way to which the loads are applied is laterally stiff, the hangers will tilt as the arch rib moves laterally, as shown in Fig.2. Since this tilt produces a restoring lateral force on the arch, the buckling strength increases. On the other hand if this tilt is ignored or if the road way is laterally flexible, the hanger remains vertical and the load delivered to the deflected rib remains vertical without a restoring component.

Fig. 11 shows the buckling strength of a single arch subjected to the load transmitted through the tilting hangers whose displacements are constrained at their lower ends (shown by —○— in the figure). And also the buckling strength of one subjected to vertical loads are shown by ---△--- in the figure. If the horizontal axis λ_y of the figure is reduced to 65 % for the curve of tilting hanger load, the obtained curve (shown by —●— in the figure) coincides well with the curve for vertical loads. This means that if the road way is laterally stiff enough, $K_t=0.65$ is adequate in determining the value λ_y . This is the reason why $K_t=0.65$ was used in Fig.10 as a temporary measure.

On the other hand, if the road way is not sufficiently stiff, that is, $I_{gy}\leq I_{ay}$, the road way also moves laterally as shown in Fig.6 and the increase in the strength is limited due to a decreasing restoring force. In order to determine the K_t value in consideration of this effect, required values for K_t are plotted against I_{gy}/I_{ay} axis for the case of $\beta=0.733$ in Fig.12. Then, the curve of $K_t=1-0.35(I_{gy}/I_{ay})^{1/4}$ is obtained as a conservative equation to determine the value K_t as the function of I_{gy}/I_{ay} where $I_{gy}/I_{ay}\leq 1.0$. And $K_t=0.65$ can be used when $I_{gy}/I_{ay}>1.0$.

Consequently the coefficient K_t can be given as :

$K_t=\begin{cases} 1-0.35(I_{gy}/I_{ay})^{1/4} & \text{for } I_{gy}/I_{ay}\leq 1.0 \\ 0.65 & \text{for } I_{gy}/I_{ay}>1.0 \end{cases} \dots\dots\dots (6)$

The value I_{gy}/I_{ay} in actual bridges ranges from 0.1 to 2.0 when the stiffness of the concrete deck slab is

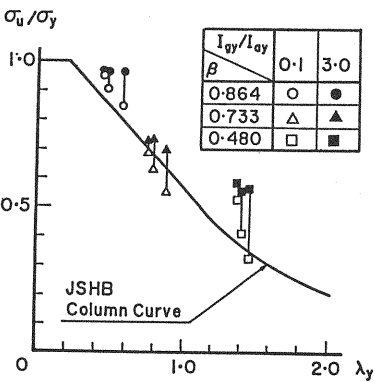


Fig.10 Evaluation of numerical results by considering shear flexibility in K_β ($K_e=0.5$, $K_t=0.65$).

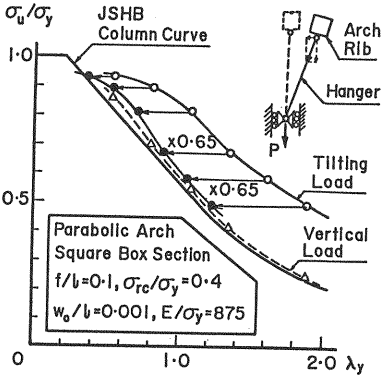


Fig.11 Lateral buckling strength of a single arch rib subjected to hanger loads ($\lambda_y=0.5\ L\sqrt{\sigma_y/E}/\pi r_y$).

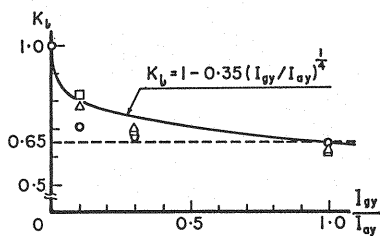


Fig. 12 Relation between K_L and I_{gy}/I_{ay} .

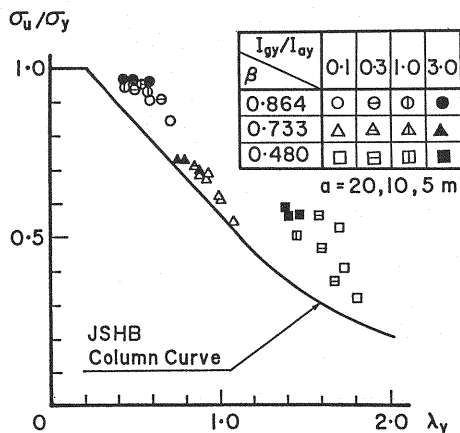


Fig. 13 Evaluation of numerical results for through type arch bridges by the proposed method ($f/l=0.15$).

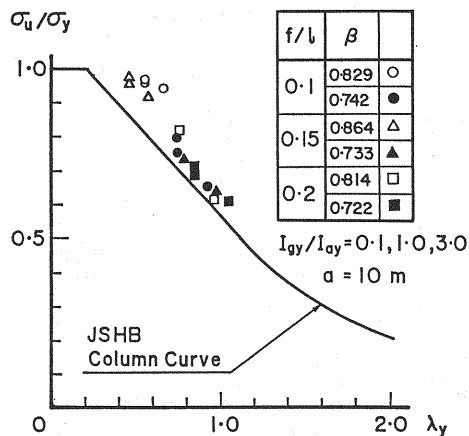


Fig. 14 Evaluation of numerical results for through type arch bridges of different f/l values by the proposed method.

disregarded, and from 1.0 to 3.0 when the stiffness of the concrete deck slab is included.

The slenderness parameters λ_y are determined by using the value K_L , defined in Eq. (6) for all the numerical results shown in Table 1. The results are plotted in Fig. 13. The figure shows that all the numerical results closely correspond

to the standard column strength curve, though those for $\beta=0.48$, which are rare to exist in actual bridges, are somewhat conservative.

(5) Effects of the rise-to-span ratio, f/l

If we use horizontal thrust H and the span length l of arch, as is done in JSHB, to replace an arch to an equivalent column, the effective length of the arch must be a function of the rise-to-span ratio, f/l . But, in the method proposed here, since the tangential force N and the curved length L are used as the representative force and length to describe the replaced equivalent column, the formula is expected to account for the effect of f/l on the ultimate strength without any modification. In order to verify this idea, the ultimate strength of arch bridges with different f/l values are computed and plotted in Fig. 14 along the slenderness parameter λ_y proposed. The numerical results show good correspondence with the column strength curve despite of different f/l values. That is, the procedure can account for the effects of f/l values on the ultimate strength of arch bridges by itself.

3. OUT-OF-PLANE BUCKLING STRENGTH OF ARCH BRIDGE OF HALF-THROUGH TYPE

(1) Theoretical models

Theoretical models studied are 2-hinged parabolic arches as shown in Fig. 15. An initial out-of-plane deflection of sinusoidal half wave of which amplitude is $0.001 l$ is assumed for arch rib as an imperfection. Since the central part of the half-through type arch bridge is similar in configuration to the through type arch bridge, the span l_0 , the rise f_0 , the curved arch length L_0 , and the braced length ratio β_0 , are defined for the supposed through type arch bridge (see also Fig. 18). The distance between twin arch ribs, α , is changed as 20 m, 10 m, 5 m and the number of the braced panels is also changed as 10, 12, 14 within the total of 16 panels, for which $\beta=0.642$, 0.756, 0.875 and $\beta_0=0.381$, 0.578, 0.784, respectively.

Table 3 Lateral buckling strength σ_u/σ_y for half-through type theoretical models.

β [β_o]	a (m)	I_{gy}/I_{ay}			
		0.1	0.3	1.0	3.0
0.875 [0.784]	20	1.005	—	1.010	1.010
	10	1.030 (1.030)	—	1.070 (1.050)	1.030 (1.050)
	5	0.920	1.050	1.070	1.060
0.756 [0.578]	20	0.775	—	0.875	0.875
	10	0.725 (0.855)	—	0.825 (0.875)	0.865 (0.905)
	5	0.630	0.750	0.774	0.825
0.642 [0.381]	20	0.630	—	0.680	0.725
	10	0.582 (0.680)	—	0.694 (0.727)	0.742 (0.760)
	5	0.470	0.590	0.633	0.670

The load condition, the materials and the cross sectional areas of the members are same as those used for the through type model except for the columns. The cross sectional dimensions of the columns are determined as $h=b=40$ cm and $t=2$ cm so as not to buckle as a member.

(2) Estimation of out-of-plane buckling strength by JSHB

Table 3 shows lateral buckling strength σ_u obtained from the ultimate load at which lateral deformation increases excessively without further incremental load in the numerical analysis [see Ref. 8)]. In this table the values given in parentheses indicate the buckling strength of Langer girder or Lohse girder of half-through type arch bridge of which arch rib is rigidly connected to floor girder at their intersection points.

According to the definition of JSHB, the effective buckling length for theoretical models is determined as follows :

$$l_e = \phi \beta_z l = 1.0 \times \frac{0.65 + 0.82}{2} \times 150 = 110.25 \text{ (m)}$$

Using this, the slenderness parameter and buckling strength for each model are computed in terms of the horizontal ultimate stress $\sigma_{Hu}/\sigma_y = H_u/A\sigma_y$. The horizontal ultimate stresses σ_{Hu} , for numerical models are plotted along the slenderness parameter defined in JSHB in Fig. 16.

As is clear from Fig. 16, predictions of the ultimate strength by JSHB (=JSHB Column Curve) is found to be quite unconservative, even when increase in safety factor ($\nu=2.0$) is taken into account. The effect of the unbraced length of the arch rib on the ultimate strength of half-through type arch bridges is the same as that described for through type arch bridges. The smaller the value β , the more the ultimate strength tends to be overestimated. This is caused by the fact that the effect of the braced length ratio β is not accounted for in the provisions of the JSHB. In regard to this point, the provision of JSHB recommends

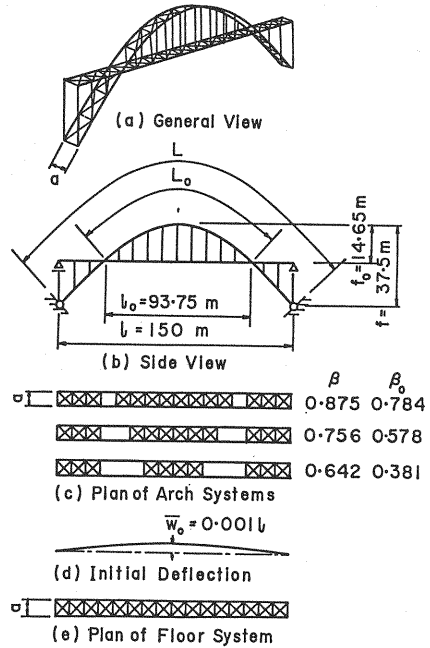


Fig. 15 Theoretical models for half-through type arch bridges.

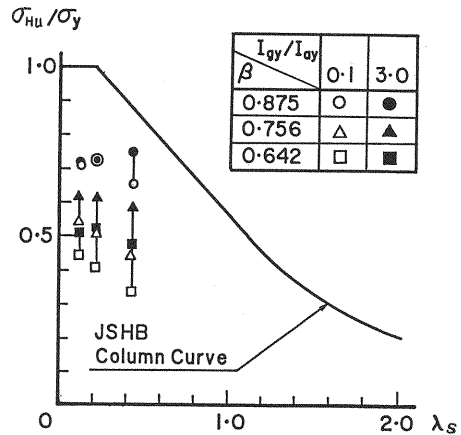


Fig. 16 Strength of theoretical models plotted along the slenderness parameter defined in JSHB.

individual examinations for out-of-plane buckling of the half-through type arch bridge.

(3) Application of the effective length procedure

Fig. 17 shows numerical results for the lateral displacement of half-through type arch bridges at the ultimate state. This figure shows that lateral bending deformation occurs mostly in the portal portion with no bracing and very little transverse distortion occurs in the braced portion of the arch. Also it can be seen from this figure that the deformation modes of the central portion above the road level are very similar to those obtained for the through type arch bridges shown in Fig. 6.

This fact suggests that it is possible to estimate the strength of the half-through type arch bridge as a fictitious through type arch bridge with span l_0 , rise f_0 , curved arch length L_0 and braced length ratio β_0 as shown in Fig. 18.

In this case, the slenderness parameter is redefined as $\lambda'_y = (K_e K_\beta K_L L_0 / \pi r_y) \sqrt{\sigma_y / E}$ by replacing L , β , with L_0 , β_0 in Eq. (1) and in determining K_β , K_L . The ultimate strength is also redefined as $\sigma'_u = N'_u / A$, where N'_u is the axial force of the arch rib at the intersection point between the arch rib and the deck girder. The ultimate strengths σ_u for the numerical models given in Table 3 are re-evaluated and shown in Table 4. Fig. 19, 20 and 21 show the redefined ultimate strength σ'_u of theoretical models plotted along

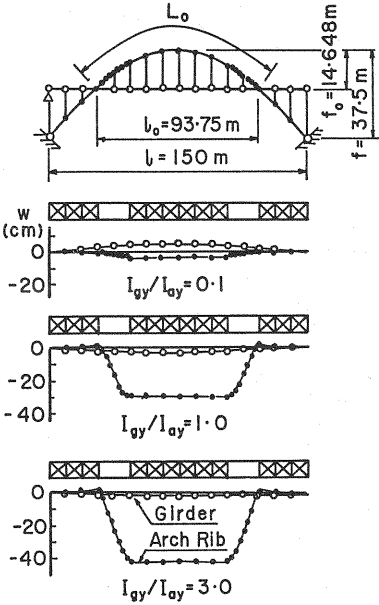


Fig. 17 Lateral displacement of half-through type arch bridges (at the ultimate state, $a=10$ m, $\beta=0.756$).

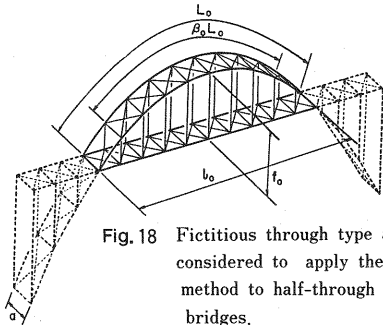


Fig. 18 Fictitious through type arch bridge considered to apply the proposed method to half-through type arch bridges.

Table 4 Lateral buckling strength σ'_u / σ_y for half-through type theoretical models.

β (β_0)	a (m)	I_{gy} / I_{ay}			
		0.1	0.3	1.0	3.0
0.875 (0.784)	20	0.838	—	0.842	0.842
	10	0.859 (0.859)	—	0.892 (0.875)	0.859 (0.875)
	5	0.767	0.875	0.892	0.884
0.756 (0.578)	20	0.646	—	0.730	0.730
	10	0.605 (0.713)	—	0.688 (0.729)	0.721 (0.754)
	5	0.525	0.625	0.645	0.688
0.642 (0.381)	20	0.525	—	0.567	0.604
	10	0.485 (0.567)	—	0.579 (0.606)	0.618 (0.633)
	5	0.392	0.492	0.527	0.558

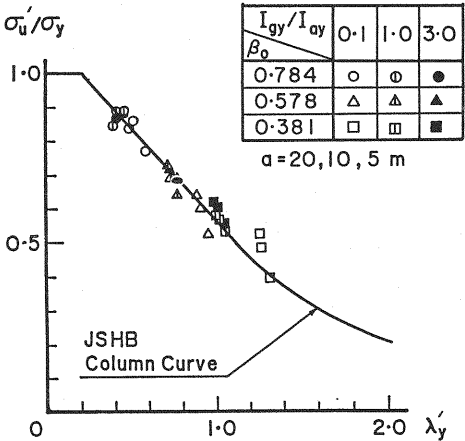


Fig. 19 Evaluation of numerical results for half-through type arch bridges by the proposed method (solid rib arch, $f_0/l_0=0.15$).

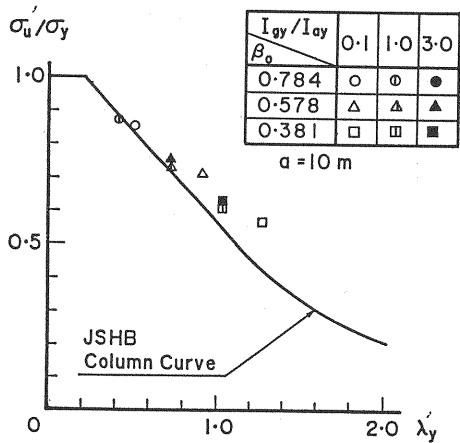


Fig.20 Evaluation of numerical results for half-through type arch bridges by the proposed method (Lohse or Langer girder, $f_0/l_0=0.15$).

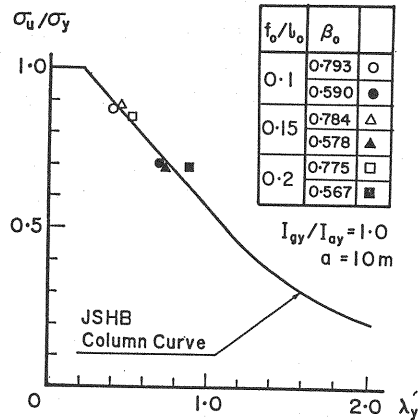


Fig.21 Evaluation of numerical results for half-through type arch bridges by the proposed method (solid rib arch, $f_0/l_0=0.10, 0.15, 0.20$).

re-evaluated slenderness parameter λ'_y . All the numerical results closely correspond to the standard column curve.

4. ULTIMATE STRENGTH FORMULA AND ITS APPLICATION LIMIT

(1) Proposed ultimate strength formula

When λ_y has been determined from Eq. (1), one proceeds next to a column strength curve and determines σ_u . This is the buckling load estimate for the arch. For parabolic arches, the uniformly distributed load per unit length at the ultimate state, p_u , is written in terms of σ_u as follows :

$$\left. \begin{aligned} \text{for through type} \quad & p_u = 2 A \sigma_u / \left(l \sqrt{\frac{1}{16} \left(\frac{l}{f} \right)^2 + 1} \right) \\ \text{for half-through type} \quad & p_u = 2 A \sigma_u / \left(l_0 \sqrt{\frac{1}{16} \left(\frac{l_0}{f_0} \right)^2 + 1} \right) \end{aligned} \right\} \dots\dots\dots (7)$$

Accordingly, the out-of-plane buckling strength of through type and half-through type arch bridges can be checked by the following equation :

$$\nu N_s / A \leq \sigma_u \dots\dots\dots (8)$$

in which

ν : factor of safety or load factor

N_s : axial force computed by an elastic linear theory for uniform load, p , fully distributed along the span, at the springings for through type arch bridges, and at the intersection point of arch rib and deck girder for half-through type arch bridges

$$\text{for example, for parabolic arches of through type} \quad : N_s = \frac{pl}{2} \sqrt{\frac{1}{16} \left(\frac{l}{f} \right)^2 + 1}$$

$$\text{for parabolic arches of half-through type} : N_s = \frac{pl_0}{2} \sqrt{\frac{1}{16} \left(\frac{l_0}{f_0} \right)^2 + 1}$$

A : cross-sectional area of a single arch rib, (for variable cross sections, use the value averaged along the arch length L or L_0 : $A = \sum A_i L_i / L$, $L = \sum L_i$)

σ_u : ultimate stress given by

$$\left. \begin{aligned} \sigma_u / \sigma_y &= 1.0 && \text{for } \lambda_y \leq 0.2 \\ \sigma_u / \sigma_y &= 1.109 - 0.545 \lambda_y && \text{for } 0.2 < \lambda_y \leq 1.0 \\ \sigma_u / \sigma_y &= 1.0 / (0.773 + \lambda_y^2) && \text{for } 1.0 < \lambda_y \end{aligned} \right\} \dots\dots\dots (9)$$

σ_y : yield point stress of steel used, (when several grades of steel are used, use the lowest one)

$\lambda_y: (K_e K_\beta K_t S / \pi r_y) \sqrt{\sigma_y / E}$

$r_y: \sqrt{I_y / A}$ radius of gyration of cross section for a single arch rib

I_y : moment inertia of cross section of a single arch rib about lateral bending, (for a variable cross sections, use a value averaged along the arch length L or L_0 ; $I_y = \sum I_{yi} L_i / L$, $L = \sum L_i$)

K_e : is 0.5 when the rotation about lateral bending is constrained at the support of the arch rib, and is 1.0 when the rotation about lateral bending is free at the support of the arch rib.

$K_\beta: 1 - \beta + [2 r_y (0.5 + 0.94 \sqrt{\mu}) / (a K_e)] \beta$

β : the ratio of the length of the braced portion of the arch rib to the total arch rib length for arch bridges of through type, for arch bridges of half-through type, it should be determined similarly considering the central portion upper than the road level of the bridge as a fictitious arch bridge of through type.

μ : shear flexibility parameter given by Eq. (3), in case of the half-through type, the arch rib length L should be replaced by L_0 .

$K_t: 1 - 0.35 (I_{gy} / I_{ay})^{1/4}$ for $I_{gy} / I_{ay} \leq 1.0$
0.65 for $I_{gy} / I_{ay} > 1.0$

I_{gy} : moment inertia of cross section of the floor system as a whole about lateral bending

I_{ay} : moment inertia of cross section of the twin arch ribs as a whole about lateral bending

S : representative length for determination of effective length, this equals to total length of arch rib, L , for a through type arch bridge, for a half-through type arch bridges, equals to the length of the arch rib of central portion upper than the road way level, L_0 .

Since it has been shown for theoretical models without floor system in Ref. 8), 9) that this type of formula is applicable to braced twin arches with variable cross section, to those made of different grades of steel and to those of circular configuration, the applicability of the proposed formula is believed to be extended without additional computations to the theoretical models with floor system of which twin arches are replaced by those with variable cross section or by those made of different grades of steel or by those of circular configuration. Remarks for extended use of the formula to such arch bridges are given in the above proposal as additional descriptions in brackets.

5. CONCLUSIONS AND REMARKS

The main conclusions of this paper are summarized as follows:

(1) The ultimate strength for realistic theoretical models of through type or half-through type arch bridges which fail by out-of-plane buckling of arch ribs are computed by finite element method considering finite displacements and material plasticity.

(2) A formula for predicting the ultimate strength of through type or half-through type arch bridges is proposed using the results of the computer simulation of this study and other former studies. The formula is based on the effective length procedure using the analogy between an arch and a column.

(3) It is shown that the proposed formula gives fairly good predictions for the out-of-plane buckling strength of all the theoretical models studied.

(4) Though the formula was derived under several limitations, it is useful for preliminary design and in specifying codes for the ultimate state design.

(5) The most important thing is the proportioning of lateral bracing members. In the theoretical models studied, bracing members have been proportioned stiff enough so as not to buckle prior to the overall failure of the arch bridge. When the bracing member is not stiff enough, it is possible that the out-of-plane buckling strength will be governed by the premature buckling of the bracing member and be less than the formula prediction. Therefore, it is worth to point out that the lateral bracing member must never be considered as secondary member as far as the lateral instability of arch bridges is concerned.

Designers must be cautious about this point. Methods for determining the sufficiency of a bracing member are studied in Ref. 2).

(6) The column strength formula which is used for predictions of σ_u need not necessarily be Eq. (9). The curve 2 of the SSRC or the curve c of the ECCS multiple column curves or other equivalent column curves can be used for the prediction of σ_u .

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