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DYNAMIC STABILITY OF AN ANNULAR SECTOR PLATE SUBJECTED TO IN-PLANE DYNAMIC MOMENTS

By Kazuo TAKAHASHI, Yoshihiro NATSUAKI**, Yasunori KONISHI*** and Michiaki HIRAKAWA*****

Dynamic stability problem of an annular sector plate subjected to in-plane dynamic moments at the radial edges is examined. The basic equation is reduced to a set of ordinary differential equations by applying a Galerkin method, and transformed into an eigen-value problem by using the harmonic balance method. The stability of the system can be directly determined from the sign of the real parts of the eigen-values.

The dynamic unstable regions composed of both simple parametric and combination resonances, which contain the secondary as well as the primary unstable regions, are obtained for the annular sector plates with various boundary conditions and geometrical parameters. The effect of a static moment on the unstable regions is also examined.

Keywords: dynamic stability, annular sector plate, plate vibration

1. INTRODUCTION

Out-of-plane vibrations of a thin plate may by observed under in-plane periodic forces by reason of parametric excitation. Since the parametric resonance may induce fatigue cracks or acoustic radiations, it is important to clarify the conditions under which the unstable motions occur.

The dynamic stability problem of plates has been studied by a number of researchers. Bolotin[1], Hutt et al.[2] and Yamaki et al.[3] investigated the dynamic stability of a rectangular plate subjected to in-plane uniformly distributed periodic forces. Duffield et al.[4] treated stiffened rectangular plates. Takahashi et al.[5] analyzed the rectangular plate under linearly distributed dynamic loads such as the in-plane moments and the triangularly distributed load.

From these results, the dynamic instability of a rectangular plate subjected to various in-plane periodic forces is essentially understood. In practical structures, however, various shaped plates have been incorporated as structural members. For instance, such curved plates as web plates of the arch rib or corner members of the rigid-frame may be regarded as annular sector plates subjected primarily to in-plane moments. It seems that no investigation of the dynamic stability of those plates has ever been initiated.

In this paper, the dynamic stability problem of an annular sector plate simply supported along radial edges is analyzed. The equation of motion with respect to out-of-plane vibrations of a plate subjected to in-plane dynamic moments is reduced to a set of ordinary differential equations by applying a Galerkin

* Member of JSCE, Dr. Eng., Associate Professor, Department of Civil Engineering, Nagasaki University (Bunkyo-machi 1-14, Nagasaki City)
** Member of JSCE, Manager of Research and Development Sect., Katayama Iron Works, Co., Ltd. (2-21, Minamiokajima 6-chome, Taisho-ku, Osaka)
*** Member of JSCE, Dr. Eng., Professor, Department of Civil Engineering, Nagasaki University
**** Member of JSCE, Graduate Student of Nagasaki University

procedure, and transformed into an eigen-value problem by using the harmonic balance method described first by Bolotin[1] and lately extended by the first author[6]. Then the stability of the system can be directly determined from the sign of the real parts of the eigen-values.

As numerical examples, unstable regions of an annular sector plate subjected to in-plane dynamic moments at the radial edges are obtained under various boundary conditions along the circumferential edges, geometrical parameters and the static moment. Their influences on the unstable regions are also examined.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS

Fig. 1 shows an annular sector plate with the opening (subtended) angle α, outer radius a and inner radius b. The polar co-ordinates (r, θ) are taken in the neutral surface of the plate.

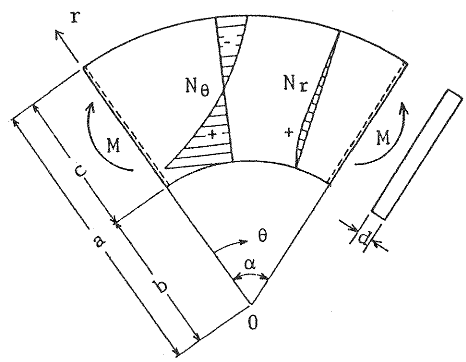

Fig. 1 Geometry and co-ordinates.

Equal and opposite moments M, which consist of the static moment M_0 and periodic dynamic moment $M_t \cos \Omega t$, act at the radial edges. In-plane forces N_r, N_θ and $N_{r\theta}$ are given by[7]

$$N_r=-\frac{4M}{N}\left(\frac{a^2b^2}{r^2}\ln\frac{a}{b}+a^2\ln\frac{r}{a}+b^2\ln\frac{b}{r}\right) \cdots\cdots (1)$$

$$N_\theta=-\frac{4M}{N}\left(-\frac{a^2b^2}{r^2}\ln\frac{a}{b}+a^2\ln\frac{r}{a}+b^2\ln\frac{b}{r}+a^2-b^2\right) \cdots\cdots\cdots\cdots (2)$$

$$N_{r\theta}=0 \cdots\cdots (3)$$

where $N=(a^2-b^2)^2-4a^2b^2\{\ln(a/b)\}^2$ and $M=M_0+M_t\cos\Omega t$. In-plane forces N_r and N_θ are functions of the independent variable r. The basic equation for linear free vibrations of a plate subjected to these in-plane forces then, can be written by adding an inertia force term to the governing equation of the corresponding buckling problem[8] as follows :

$$D\nabla^4 w+\rho d\frac{\partial^2 w}{\partial t^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(rN_r\frac{\partial w}{\partial r}\right)-\frac{N_\theta}{r^2}\frac{\partial^2 w}{\partial\theta^2}=0 \cdots\cdots (4)$$

where w denotes the plate deflection, ρ is the mass density, t is the time, $D=Ed^3/12(1-\nu^2)$ is the flexural rigidity, E is Young's modulus, ν is Poisson's ratio, d is the plate thickness and $\nabla^2=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial^2\theta}\right)$ is a Laplacian operator in the polar co-ordinates.

Considering the web plate divided by adjacent radial stiffeners of a vertically curved I-girder to be an annular sector plate, the boundary condition for radial edges of the plate can be assumed to be simply supported. The following three boundary conditions along the circumferential edges are considered in the present analysis, case Ⅰ : simply supported, case Ⅱ : clamped and case Ⅲ : free.

3. METHOD OF SOLUTION

Taking the boundary conditions for radial edges into account, the defelection along the angular direction is given by the sine wave. The solution of Eq. (4) then can be assumed to be of the form

$$w=\sum_s T_{sn}(t)W_{sn}(r,\theta) \cdots\cdots (5)$$

in which T_{sn} is an unknown function of the time variable, n is an arbitrary integer which denotes the half-wave number in the θ direction, and W_{sn} is an eigen-function associated with the free vibration of the corresponding annular sector plate loading with no in-plane forces and satisfying geometrical boundary conditions of the plate, defined as[9]

$$W_{sn}(r,\theta)=R_{sn}(r)\sin(\alpha_n\theta) \cdots\cdots (6)$$

where $R_{sn}=A_{sn}J_{\alpha n}(k_{sn}\xi)+B_{sn}Y_{\alpha n}(k_{sn}\xi)+C_{sn}I_{\alpha n}(k_{sn}\xi)+D_{sn}K_{\alpha n}(k_{sn}\xi)$, in which A_{sn}, B_{sn}, C_{sn} and D_{sn} are constants of integration dependent on each boundary condition, $J_{\alpha n}$ and $Y_{\alpha n}$ are Bessel functions of order α_n, $I_{\alpha n}$ and $K_{\alpha n}$ are modified Bessel functions of order α_n, k_{sn} is the eigen-value of free vibrations, $\alpha_n=n\pi/\alpha$, and $\xi=r/a$.

Substituting Eq. (5) into Eq. (4) and applying a Galerkin method, one has

$$[I]\{\ddot{T}_n\}+[A]\{T_n\}+(\overline{M}_0+\overline{M}_t\cos\overline{\omega}\tau)[B]\{T_n\}=\{0\} \quad (7)$$

in which $[I]$ is the unit matrix, $[A]$ and $[B]$ are coefficient matrices (see Appendix), and $\{T_n\}$ is a column vector consisting of the dependent variable T_{sn}. The following non-dimensional qualities have been introduced in the above equation :

$$\overline{M}_0=\frac{M_0}{M_{cr}},\quad \overline{M}_t=\frac{M_t}{M_{cr}},\quad \overline{\omega}=\frac{\Omega}{\Omega_1^1}\quad \text{and}\quad \tau=\Omega_1^1 t.$$

Here $\Omega_1^1=k_{11}^2\sqrt{(D/\rho d a^4)}$ is the lowest natural circular frequency[9], $M_{cr}=\lambda_{cr}D$ is the buckling moment due to static in-plane moment[8] and λ_{cr} is the eigen-value of buckling.

The solution of Eq. (7) is now sought in the form[6]

$$\{T_n\}=e^{\lambda\tau}\left\{\frac{1}{2}\boldsymbol{b}_0+\sum_m(\boldsymbol{a}_m\sin m\overline{\omega}\tau+\boldsymbol{b}_m\cos m\overline{\omega}\tau)\right\} \quad (8)$$

where $\boldsymbol{b}_0$, $\boldsymbol{a}_m$ and $\boldsymbol{b}_m$ are vectors that are independent of time.

Substituting Eq. (8) into Eq. (7) and applying the harmonic balance method yield a set of homogeneous algebraic equations as

$$([M_0]-\lambda[M_1]-\lambda^2[M_2])\{X\}=\{0\} \quad (9)$$

in which $[M_0]$, $[M_1]$ and $[M_2]$ are the coefficient matrices of the zeroth (constant), first and second powers of λ, respectively, and $\{X\}$ is the column vector consisting of $\boldsymbol{b}_0$, $\boldsymbol{b}_m$ and $\boldsymbol{a}_m$.

The eigen-value λ can be obtained by solving a double sized matrix as an eigen-value problem in the form[6]

$$\begin{bmatrix}[0] & [I]\\ [M_2]^{-1}[M_0] & -[M_2]^{-1}[M_1]\end{bmatrix}\begin{Bmatrix}X\\ Y\end{Bmatrix}=\lambda\begin{Bmatrix}X\\ Y\end{Bmatrix} \quad (10)$$

where $\{Y\}=\lambda\{X\}$. Then, the stability of the system can be directly determined from the sign of the real parts of the eigen-values.

The convergency and accuracy of the space and time functions assumed in the Galerkin and harmonic balance methods, respectively, have been confirmed in the previous papers[10)~12)].

The geometrical parameters in the present analysis are the opening angle α and the radius ratio β $(=b/a)$. For the comparison with the corresponding rectangular plate, the aspect ratio of an annular sector plate may be defined by the rectangular plate analogy as $\mu=l/c$, in which $l=(a+b)\alpha/2$ is the mean arc length and c is the radial edge length.

4. NUMERICAL RESULTS

Based upon the vibration analysis[9] and the preceding theoretical analysis, numerical solutions have been obtained for an annular sector plate under various boundary conditions along the surcumferential edges, geometrical parameters and the static moment. First, the natural frequencies are presented. Then, the dynamic unstable regions are determined and compared with those of a rectangular plate.

(1) The property of natural frequencies

The natural frequencies of an annular sector plate are examined under various geometrical parameters in order to estimate the effects of the opening angle α and the radius ratio β on natural frequencies. Figs. 2 and 3 show the variation of natural frequencies with the opening angle α and the radius ratio β, respectively. The frequency curves are obtained by calculating with parameters at suitable intervals. In these figures, the ordinate κ_s^n $(=k_{sn}^2/\pi^2=\Omega_s^n a^2\sqrt{\rho d/D}/\pi^2)$ denotes the non-dimensional natural frequency and the notation (n, s) represents the half-wave number of the vibration mode in the θ and r directions, respectively. The natural frequencies rapidly increase with a decrease of the opening angle α

and with an increase of the radius ratio β. Since the natural frequencies are strongly influenced by the opening angle α and the radius ratio β, it follows that the dynamic unstable regions are also influenced by these parameters.

(2) Effect of static moment $\overline{M}_0$ on natural frequencies

The natural frequencies of an annular sector plate subjected to a static moment $\overline{M}_0$ can be obtained by solving Eq. (7) at $\overline{M}_t=0.0$ as an eigen-value problem, in which the eigen-function W_{sn} is taken up to $s=10$. The moment $\overline{M}_0$ vs. the natural frequency $\overline{n}$ for case I, $\alpha=60^\circ$ and $\mu=1.0$ is shown in Fig. 4. In this figure, the ordinate $\overline{M}_0=M_0/M_{cr}$ shows the static moment M_0 normalized to the buckling moment M_{cr}, while the abscissa $\overline{n}=\Omega_s^n/\Omega_1^1$ denotes the natural circular frequency Ω_s^n normalized to the lowest relevant natural circular frequency Ω_1^1.

The frequencies change with an increase of the static moment $\overline{M}_0$. Natural frequencies decrease with an increase of the static moment except for the modes (2, 1), (3, 1) and (4, 1), in which the frequencies remarkably increase. The effect of the static moment $\overline{M}_0$ on natural frequencies of an annular sector plate is different from that of the uniformly distributed in-plane force on natural frequencies of a rectangular plate, which never increase with an increase of the in-plane force[3]. Moreover, in the case of a square plate subjected to an in-plane moment, natural frequencies corresponding to the mode $(m, 1)$ decrease and the other modes' frequencies are almost constant or slightly increase with an increase of the static moment $\overline{M}_0$[5].

Next, the effect of the static moment $\overline{M}_0$ on the modes of vibration can be examined. The changes in the first modal shape (1, 1) with moments are illustrated in Fig. 5. The maximum amplitude moves towards the compressive force side. When the normalized static moment $\overline{M}_0$ is equal to unity, the mode of vibration corresponds to that of buckling. Since the buckling mode ($\overline{M}_0=1.0$) is different from the free vibration mode ($\overline{M}_0=0.0$), the out-of-plane responses of an annular sector plate subjected to in-plane moments should be composed of several free vibration modes without an in-plane moment. Therefore, it is predictable that the combination resonances will appear in the dynamic instability of the present problem.

(3) Dynamic unstable regions for various boundary conditions

The kinds and widths of the unstable regions depend on the contents of the coefficient matrix $[B]$ in Eq. (7). Since the matrix $[B]$ is symmetric ($b_{ij}=b_{ji}$) and the diagonal element b_{ii} is non-zero as shown in APPENDIX, the sum type combination resonances in the neighborhood of $\overline{\omega}=(\omega_i^n+\omega_j^n)/k$ and simple resonances in the neighborhood of $\overline{\omega}=2\,\omega_i^n/k$ will be simultaneously obtained in the present analysis[13]. Here, $k=1, 2, \cdots$ is an integer and the unstable region corresponding to $k=1$ is called the primary unstable region, while that of $k\geqq 2$ is called the secondary unstable region. This is quite different from the

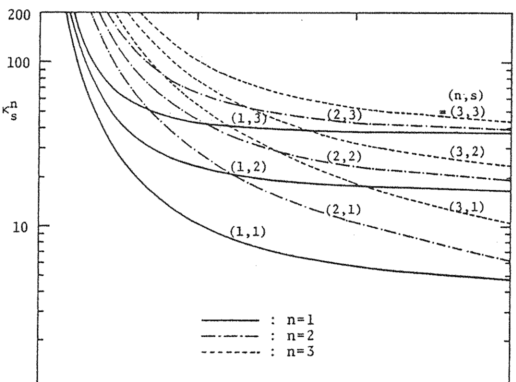

Fig. 2 The variation of natural frequency with the opening angle α : case I and $\beta=0.5$.

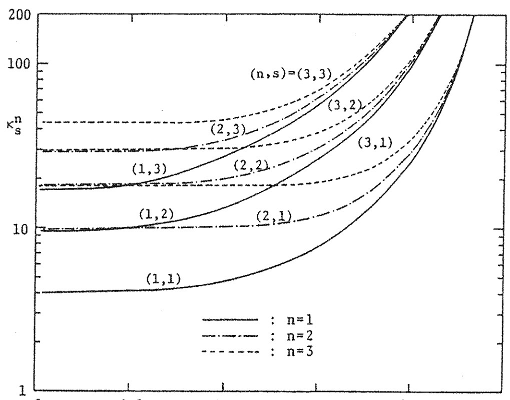

Fig. 3 The variation of natural frequency with the radius ratio β : case I and $\alpha=60^\circ$.

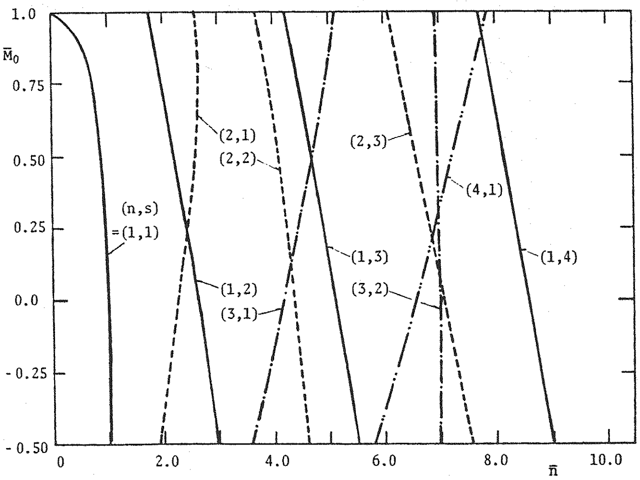

Fig. 4 Moment $\overline{M}_0$ vs. the natural frequency $\overline{n}$: case I, $\alpha=60^\circ$ and $\mu=1.0$.

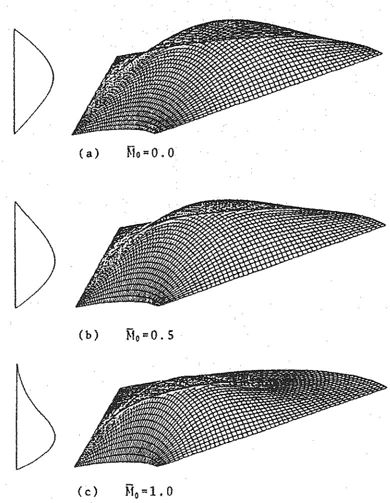

Fig. 5 Effect of static moment $\overline{M}_0$ on the mode (1, 1).

case of a rectangular plate subjected to in-plane moments, in which parametric resonances occur only through the coupling terms because the diagonal elements of the parametric coefficient matrix are zero[5].

The dynamic unstable regions of the annular sector plate for different boundary conditions with no static moment ($\overline{M}_0=0.0$) are shown in Fig. 6 through Fig. 8. The unstable regions for case II are so numerous that they are shown in two figures as Figs. 7(a) (for $n=1$ and 2) and 7(b) (for $n=3$, 4 and 5). In these figures, the ordinate $\overline{M}_t=M_t/M_{cr}$ denotes the amplitude M_t of the periodic moment normalized to the corresponding buckling moment M_{cr}, while the abscissa $\overline{\omega}=\Omega/\Omega_1^1$ is the exciting frequency Ω normalized to the lowest natural circular frequency Ω_1^1. Further, the cross-hatched portions represent the regions of various types of instability such as both simple parametric resonances $(2\,\omega_i^n/k)$ and combination resonances of the sum type $((\omega_i^n+\omega_j^n)/k)$, which contain the secondary unstable region $(k\geqq 2)$ as well as the primary unstable region $(k=1)$. The narrower unstable regions of $\overline{\omega}$ less than 0.1 at $\overline{M}_t=0.5$ are omitted in the figures.

Non-dimensional natural frequencies ω_s^n $(=\Omega_s^n/\Omega_1^1)$, the lowest eigen-value of vibration k_{11} and the eigen-value of buckling λ_{cr} of annular sector plates with $\alpha=60^\circ$ and $\mu=1.0$ are shown in Table 1 for three boundary conditions.

The widths of primary unstable regions of the simple resonance are broader than those of the combination resonance. This property of unstable regions of an annular sector plate is quite different from that of a rectangular plate, for which the combination resonances are predominant[5]. As for the combination resonances in the present problem, the unstable regions which have adjacent half-wave numbers in the radial direction and the same half-wave number in the angular direction, such as $\omega_i^n+\omega_{i+1}^n$, are predominant. For an annular sector plate subjected to in-plane dynamic moments, the number of unstable regions is much affected by the boundary conditions, from comparisons of Figs. 6, 7 and 8. Since natural frequencies corresponding to unstable motions are close to each other in case II and apart from each other in case III as shown in Table 1, case II has more unstable regions than case III.

(4) Effects of opening angle α and radius ratio β on unstable regions

The opening angle α and the radius ratio β influence the vibration property (see Figs. 2 and 3) and the buckling property[8]. Their effects on unstable regions were also examined. The variations in unstable regions with the opening angle α and the radius ratio β are shown in Figs. 9 and 10, respectively. In these figures, unstable regions were obtained for $n=1$ of case I at $\overline{M}_t=0.5$.

When the opening angle α or the radius ratio β is relatively small, the natural frequencies for $n=1$ are

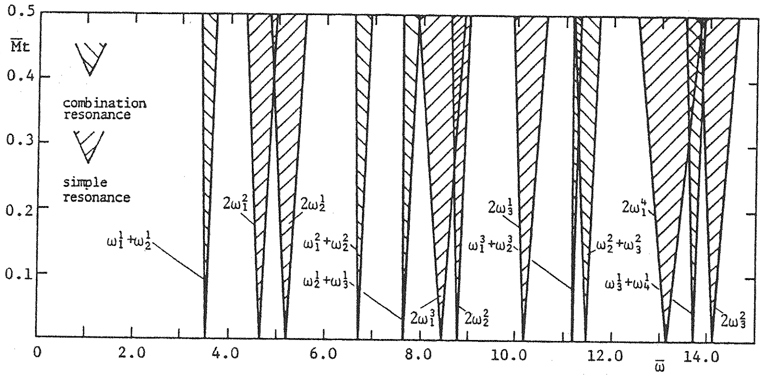

Fig. 6 Unstable regions for an annular sector plate subjected to moment $\overline{M}_t$: case I, $\alpha=60°$ and $\mu=1.0$.

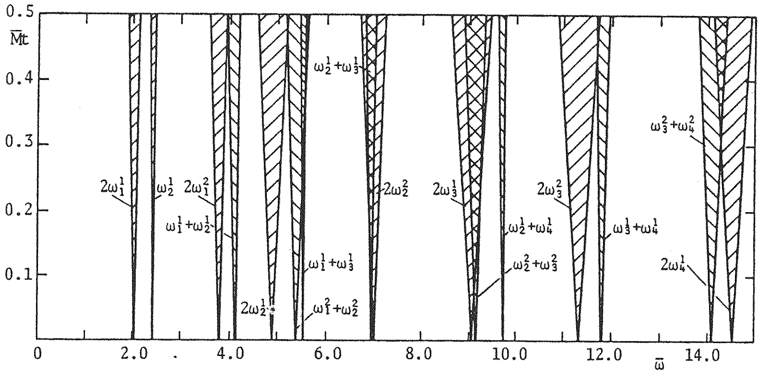

Fig. 7(a) Unstable regions for an annular sector plate subjected to moment $\overline{M}_t$: case II, $\alpha=60°$ and $\mu=1.0$ (for $n=1$ and 2).

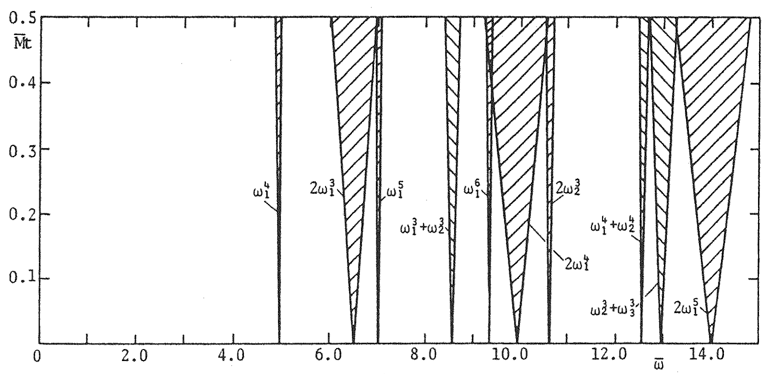

Fig. 7(b) Unstable regions for an annular sector plate subjected to moment $\overline{M}_t$: case II, $\alpha=60°$ and $\mu=1.0$ (for $n=3$, 4 and 5).

so close to each other (see Figs. 2 and 3) that the unstable regions corresponding to them are crowded as shown in Figs. 9 and 10. The widths of primary unstable regions of the simple resonances widen slightly with an increase of the opening angle α, while the others remain almost unchanged as shown in Fig. 9. On the other hand, the radius ratio β influences the widths of unstable regions, which are generally narrower with an increase of radius ratio β as shown in Fig. 10.

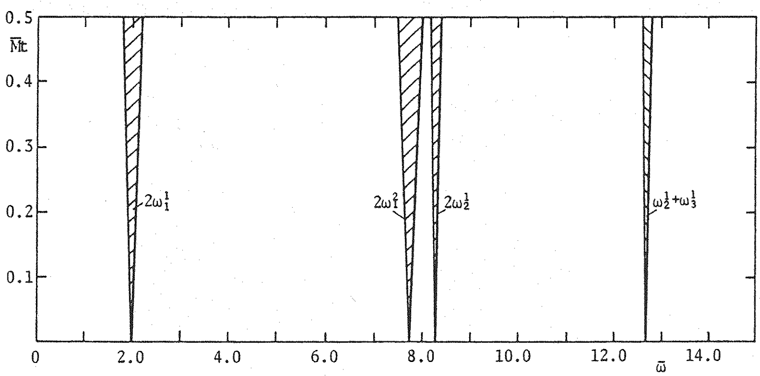

Fig. 8 Unstable regions for an annular sector plate subjected to moment $\overline{M}_t$: case Ⅲ, $\alpha=60^\circ$ and $\mu=1.0$.

Table 1 ω_s^n, k_{11} and λ_{cr} for different boundary conditions : $\mu=1.0$ and $\alpha=60^\circ$.

case		Ⅰ				Ⅱ						Ⅲ		
	s \ n	1	2	3	4	1	2	3	4	5	6	1	2	3
ω^n_s	1	1.000	2.320	4.201	6.579	1.000	1.877	3.242	4.959	7.009	9.387	1.000	3.871	8.344
	2	2.610	4.397	7.017	10.207	2.429	3.494	5.296	7.571	10.210	13.197	4.147	9.993	17.577
	3	5.122	7.099	10.255	14.178	4.510	5.648	7.682	10.412	13.586	17.127	8.557	17.065	24.431
	4	8.598	10.630	14.058	18.583	7.259	8.435	10.553	13.579	17.230	21.298	15.170	25.224	38.625
k_{11}		6.5019				7.8300						3.4982		
λ_{cr}		-28.1428				-44.7665						2.4200		

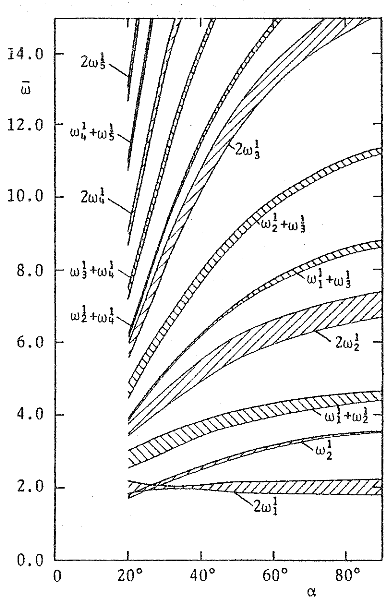

Fig. 9 The variations of unstable regions with the opening angle α : case Ⅰ, $\beta=0.5$, $n=1$ and $\overline{M}_t=0.5$.

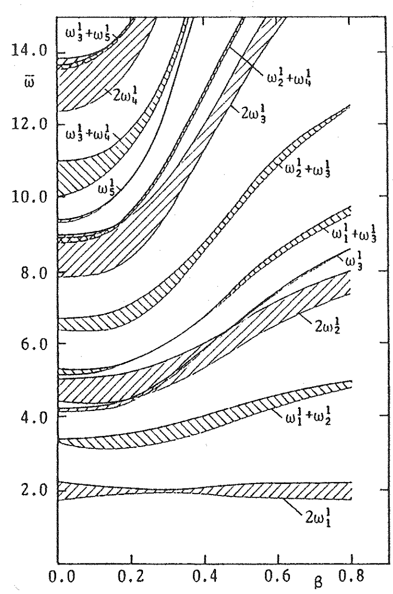

Fig. 10 The variations of unstable regions with the radius ratio β : case Ⅰ, $\alpha=60^\circ$, $n=1$ and $\overline{M}_t=0.5$.

(5) Effect of static moment $\overline{M}_0$ on unstable regions

Fig. 11 shows the variations of unstable regions with the static moment $\overline{M}_0$. In this figure, the unstable regions were obtained for case Ⅰ with $\alpha=60^\circ$ and $\mu=1.0$ at $\overline{M}_t=0.5$. The narrower unstable regions of $\overline{\omega}$

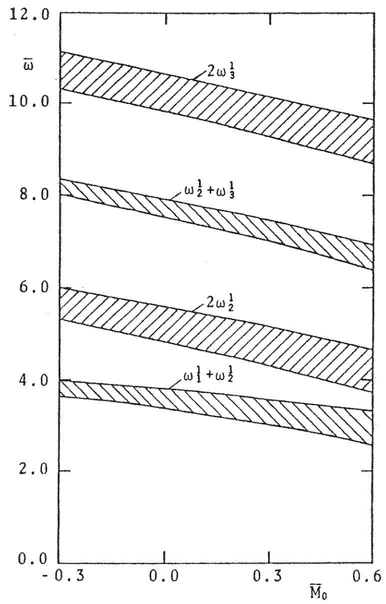

Fig. 11 The variations of unstable regions with the static moment $\overline{M}_0$: case I, $\alpha=60°$, $\mu=1.0$, $n=1$ and $\overline{M}_t=0.5$.

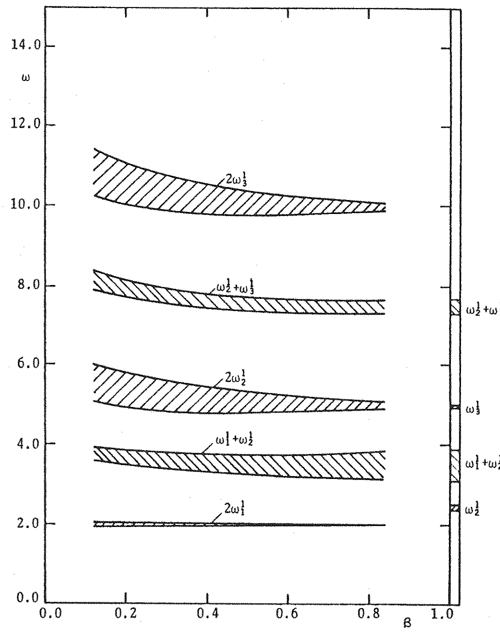

Fig. 12 Comparison of case I with a square plate : $\mu=1.0$, $n=1$ and $\overline{M}_t=0.5$.

less than 0.1 at $\overline{M}_0=0.0$ are omitted in the figure. The widths of unstable regions become slightly broader with an increase of the static moment, but the static moment has little affect on the unstable regions as shown in Fig. 11. The effects of the static moment on unstable regions for an annular sector plate are quite different from the case of a rectangular plate, in which the widths of simple resonances remarkably widen under the influence of static moments[5].

(6) Comparison of case I with a square plate

The kind of unstable regions which appear on the annular sector plate with $\alpha=60°$ and $\mu=1.0$ is quite different from that of a rectangular plate as mentioned above. That is, the simple resonances predominate over the combination resonances, in contrast to a rectangular plate. In order to compare the dynamic stability of an annular sector plate to that of a square plate, the unstable regions of an annular sector plate can be examined by keeping the aspect ratio μ at unity.

Fig. 12 shows the variation of the unstable regions with the radius ratio β for case I, $\mu=1.0$, $n=1$ and $\overline{M}_t=0.5$. When β approaches 1.0, the annular sector plate with $\mu=1.0$ resembles a square plate in shape. In this figure, the unstable regions of a square plate are also shown on the right side ordinate. For an annular sector plate with $\mu=1.0$, the widths of unstable regions of the simple resonances become narrower and those of the combination resonances become wider, resembling a square plate in distribution of in-plane forces, as β approaches 1.0. The combination resonances predominate over the simple resonances when the radius ratio β is larger than 0.8, in which the in-plane forces can be presumed to form a linear distribution[8].

It is announced that the property of free vibrations of an annular sector plate for cases I and II can be estimated by the rectangular plate analogy[9]. On the other hand, the dynamic unstable regions of an annular sector plate are influenced by the distribution of in-plane forces like the buckling property[8]. Therefore, the dynamic stability of an annular sector plate with $\mu=1.0$ can not be estimated by that of a square plate when the in-plane forces of the annular sector plate are not so close to the linear distribution, that is, when the radius ratio β is less than 0.8.

5. CONCLUSIONS

The dynamic stability of an annular sector plate subjected to in-plane time-varying moments has been investigated by using the linear theory. The conclusions are as follows :

(1) The natural frequencies of an annular sector plate subjected to in-plane moments change with an increase of the static in-plane moment. Some of them increase and others decrease.

(2) The dynamic unstable regions of the present problem consist of the simple parametric and sum type combination resonances.

(3) Simple resonances predominate over combination resonances for an annular sector plate with the large opening angle or the small radius ratio subjected to in-plane dynamic moments. This behavior is quite different from the case of a rectangular plate, in which the combination resonances predominate.

(4) The widths and numbers of unstable regions depend primarily on the boundary conditions. The unstable regions become more numerous with an increase in the degree of restraint along the circumferential edges.

(5) The unstable regions are more influenced by the radius ratio than the opening angle, while the effect of static moments on unstable regions is not so remarkable.

(6) The dynamic stability of an annular sector plate under the influence of the nonlinearity of in-plane forces can not be estimated by that of a rectangular plate.

ADDITIONAL REMARK

It has been shown that the in-plane force distributions of an annular sector web plate are transfigured by the size of flange plates[14), 15)]. Therefore, it is necessary to take the effect of flange plates into account for the determination of unstable regions of the web plate of a vertically curved I-girder.

The motions on unstable regions grow indefinitely with time under the assumption of the small deflection. Although the linear theory is useful in determining the initial growth or decay, the amplitudes of motions are bounded by the stretching of a middle plane of the plate. It is necessary to take the nonlinearity into account for the determination of the amplitudes of unstable motions[16)]. The effects of damping and initial deflection on the unstable motions also remain to be considered.

The dynamic stability analysis considering these effects will be reported in the subsequent papers.

In the present study, numerical examples are calculated by means of FACOM M-180 II AD of Information Processing Center, Nagasaki University.

APPENDIX : Coefficient matrices $[A]$ and $[B]$

$[A]=\mathrm{diag}\,(a_{in}^2)$

where $a_{in}=k_{in}^2/k_{11}^2$,

$$[B]=\begin{bmatrix} b_{11n} & b_{21n} & \cdots b_{N1n} \\ b_{12n} & b_{22n} & \cdots b_{N2n} \\ \cdots & \cdots & \cdots \\ b_{1Nn} & b_{2Nn} & \cdots b_{NNn} \end{bmatrix}$$

where $b_{ijn}=I_{ijn}/I_{in}$, $I_{in}=\int_\beta^1 R_{in}^2\xi d\xi$, $I_{ijn}=-\dfrac{4}{\overline{N}}\dfrac{\lambda_{cr}}{k_{11}^4}\displaystyle\int_\beta^1\left\{\xi f_1(\xi)\frac{dR_{in}}{d\xi}\frac{dR_{jn}}{d\xi}+\frac{\alpha_n^2}{\xi^2}f_2(\xi)R_{in}R_{jn}\right\}d\xi$,

$$\overline{N}=(1-\beta^2)^2-4\beta^2\left(\ln\frac{1}{\beta}\right)^2,\quad f_1(\xi)=\frac{\beta^2}{\xi^2}\ln\frac{1}{\beta}+\ln\xi+\beta^2\ln\frac{\beta}{\xi}$$

and $f_2(\xi)=-\dfrac{\beta^2}{\xi^2}\ln\dfrac{1}{\beta}+\ln\xi+\beta^2\ln\dfrac{\beta}{\xi}+1-\beta^2$.

REFERENCES

1) Bolotin, V. V. : The Dynamic Stability of Elastic Systems, Holden-Day Inc., San Francisco, 1964.
2) Hutt, J. M. and Salam, A. E. : Dynamic Stability of Plates by Finite Elements, Proc. of ASCE, Vol. 97-EM3, pp. 879-899, 1971.
3) Yamaki, N. and Nagai, K. : Dynamic Stability of Rectangular Plates under Periodic Compressive Forces, Report of the Institute of High Speed Mechanics, Touhoku University, Vol. 32, No. 288, pp. 103-127, 1975.
4) Duffield, R. C. and Willems, N. : Parametric Resonance of Stiffened Rectangular Plates, Journal of Applied Mechanics, Vol. 39, pp. 217-226, 1972.
5) Takahashi, K. and Konishi, Y. : Dynamic Stability of a Rectangular Plate Subjected to Distributed In-plane Dynamic Force, Journal of Sound and Vibration, Vol. 123, pp. 115-127, 1988.
6) Takahashi, K. : Instability of Parametric Dynamic Systems with Non-Uniform Damping, Journal of Sound and Vibration, Vol. 85, pp. 257-262, 1982.
7) Timoshenko, S. P. and Gere, J. M. : Theory of Elastic Stability, 2nd ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1961.
8) Natsuaki, Y., Takahashi, K., Konishi, Y. and Hirakawa, M. : Buckling of an Annular Sector Plate Subjected to Inplane Moment, Journal of Structural Engineering, Vol. 34 A, pp. 181-190, 1988.
9) Yamasaki, T., Chishaki, T. and Kaneko, T. : Free Transverse Vibration of Circular Ring Sector Plates, Technology Reports of the Kyushu University, Vol. 42, No. 4, pp. 576-583, 1969.
10) Takahashi, K., Natsuaki, Y., Konishi, Y. and Hirakawa, M. : Buckling of an Annular Sector Plate Subjected to In-plane Moments, Proc. of JSCE, No. 404／I-11, pp. 91-99, 1989.
11) Takahashi, K., Tagawa, M., Ikeda, T. and Matsukawa, T. : Dynamic Stability of a Rectangular Plate Subjected to Inplane Forcing, Proc. of JSCE, Vol. 341, No. 1, pp. 179-186, 1984.
12) Takahashi, K. : An Approach to Investigate the Instability of the Multiple-degree-of-freedom Parametric Dynamic Systems, Journal of Sound and Vibration, Vol. 78, pp. 519-529, 1981.
13) Hsu, C. S. : On the Parametric Excitation of a Dynamic System Having Multiple Degrees of Freedom, Journal of Applied Mechanics, Vol. 30, pp. 367-372, 1963.
14) Chu, K. Y. : Beuluntersuchung von ebenen Stegblechen Kreisförmig gekrümmter Träger mit I-Quershnitt, Stahlbau, 35 Jg., H. 5, s. 129-142, 1966.
15) Mikami, I., Akamatsu, Y. and Takeda, H. : Elastic Local and Coupled Buckling of Vertically Curved I-Girders under Pure Bending, Proc. of JSCE, Vol. 230, pp. 45-54, 1974.
16) Takahashi, K., Konishi, Y., Ikeda, T. and Kawano, R. : Nonlinear Response of a Rectangular Plate Subjected to Inplane Dynamic Moment, Proc. of JSCE, No. 374／I-6, pp. 79-87, 1986.

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