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A SIMPLIFIED SPATIAL ULTIMATE LOAD ANALYSIS OF MEMBERS WITH OPEN CROSS SECTION

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An inelastic finite displacement analysis of arbitrary thin-walled open cross sectional members, using the finite element method, is presented. For the constitutive relation, the tangent modulus approach taking into account the contribution of the St. Venant shear stress to the yielding, is employed. In order to show the efficiency and versatility of this analysis, another analysis based on Prandtl-Reuss flow theory (abbreviated J2F) is developed. It is found that there is no significant difference in the results of the illustrative examples treated by the two analyses. Besides, the J2F based analysis is improved by including the shear stresses caused by non-uiform bending and torsion into the yield condition of von Mises.
Keywords : *thin-walled member, tangent modulus approach, flow theory, shear stress*

1. INTRODUCTION

Ultimate strength of thin-walled open cross sectional members and frames has been in the subject of many experimental and numerical investigations[1)~6)] in recent years. Understanding the behaviour up to collapse and the parameters which have a direct influence on the ultimate load of these structures is rational for safe, economical and simple manipulating design proposals.

Usually this type of structures, before the complete collapse, can undergo large deformations accompanied with plastification of some parts of it. Therefore, in the present analysis, besides the geometrical and structural imperfections, the finite displacement theory as well as material nonlinearity are taken into account.

In keeping track of the large displacement experienced by the member, a moving coordinate system is employed. This selection is motivated by the fact that all rigid body motions will be contained within the motion of the reference system and that all static and kinematic variables are referred to the current local coordinate system. This approach has been successfully used by many investigators in this field[2),6),8),9)].

Among the several constitutive equations which have been proposed in the literature, the Prandlt-Reuss flow theory (J2F)[2),10)] and the tangent modulus approach[1)3)] have been commonly used to describe the nonlinear behaviour of steel. Rajasekran[1)] and Murray[3)] presented analyses based on the tangent modulus approach. These analyses showed satisfactory results in treating problems where bending and compression are dominant. However, their analyses fail when the collapse of the member is caused mainly by the St.

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Venant shear stress, because in their analyses, the contribution of this stress to the yielding has been neglected. In a recent work by Sakimoto et al. [2], a J2F based analysis is presented. Since J2F leads to a coupling between shear and axial deformations in the plastic range, it is necessary to divide each small segment of the wall into a number of layers in order to locate the yielded zones on each cross section. The presented numerical analysis showed more efficiency and wider applicability than the ones mentioned above. However, as far as yielding is assumed to be caused by only the axial and St. Venant shear stresses, it is shown through the present work, that the tangent modulus approach in which the axial and St. Venant shear deformations are kept uncoupled even in the plastic range, practically leads to the same ultimate load that can be obtained by the use of the J2F theory.

It is widely accepted that the shear stresses caused by non-uniform bending and torsion have no significant effect on the behaviour of thin-walled members as long as the material remains elastic. However, when inelastic material is concerned, these stresses play a primary role in the von Mises yield criterion. Despite of this fact, it has been common[1)~3)] to neglect their contribution to the yielding. This is inferred from the fact that their corresponding strains, basically, can not be determined from kinematics developed in the basis of the Bernoulli-Euler assumption for bending and the Vlazov assumption for torsion[11]. Maier[10], using different approach, develops a theory for thin-walled member, where all effects of shear were taken into account. The shear strain on the midline of the wall which is normally taken zero according to Vlazov's assumption, was determined geometrically in this theory, and an incremental procedure based on the J2F theory was presented.

The aim of our work is to show, on one hand, that the use of the tangent modulus approach leads to an efficient, versatile and simplified ultimate load analysis, and on the other hand, that the inclusion of shear stresses due to non-uniform bending and torsion into the yield condition is possible without modifying Vlazov's assumption.

2. BASIC EQUATIONS

(1) Virtual work expression

Because of the nonlinearity imposed by the material law and the finite displacement theory, an incremental equilibrium equation based on the virtual work principle is adopted. In the formulation of the problem, the deformations are assumed to be small while displacements are unrestricted. Under this assumption, the use of a moving coordinate system approach rather than a total Lagrangian one has been found to be more suitable[8)9)]. This approach states that all the static and kinematic unknowns are referred to a known stressed state. As the desired new configuration is supposed to be close enough to that of the reference state, the linearization of the incremental equilibrium equation will be permitted. Without losing generality, these considerations lead to a simplified expression of the virtual work which can be written for one element of volume v and surface s, in the contemporary configuration, as[7]

$$\int_v \bar{\sigma}_{ij}\delta e_{ij}dv+\int_v \sigma_{ij}\delta\eta_{ij}dv=\int_s (T_i+\bar{T}_i)\,\delta u_i ds-\int_v \sigma_{ij}\delta e_{ij}dv \quad \cdots\cdots (1)$$

where $\bar{\sigma}_{ij}$ represents the incremental component of the 2nd Piola-Kirchhoff stress tensor and e_{ij} and η_{ij} are, respectively, the linear and nonlinear part of Green's strain tensor components. $\bar{T}_i$ and u_i are increments in external loads and their corresponding displacements. These quantities are referred to the reference state where σ_{ij} and T_i are acting. In their elastic analysis Hasegawa et al[7] have derived a similar equation. However, the right-hand side of this equation is reduced to $\int_s \bar{T}_i\delta u_i ds$, because they assume that the reference state of one element is locally very close to the contemporary state. The accuracy of their numerical solution is helped by applying very small increments. In the present analysis, since inelastic material is considered, the equilibrium of the reference state is checked at every incremental step aiming to minimize the error caused by the linearization of Eq. (1).

(2) Strain-Displacements relationships

It has been commonly known that the equilibrium equation will have a simple form if some quantities of the unknown displacements are referred to the shear center and others to the centroid. This is true as long as the material does not experience yielding. However, after the initiation of yielding, the equilibrium equation will lose this feature because these reference points are no longer fixed. Therefore, there seems to be no advantage to follow such formulation and the displacement components will be referred to the single centroidal axis of the elastic cross section. Using the usual beam assumptions concerning thin-walled open cross section, the non-zero strains are only the axial strain[6],

$$\bar{\varepsilon}_x = u' - v''(y - z\phi) - w''(z + y\phi) - \omega\phi'' + \frac{1}{2}(v'^2 + w'^2) + \frac{1}{2}(y^2 + z^2)\phi'^2 \quad \cdots\cdots (2\cdot a)$$

and the St. Venant shear strain,

$$\bar{\gamma}_s = 2n\phi' \quad \cdots\cdots (2\cdot b)$$

where u, v and w represent the incremental displacement components of the centroidal point in the respective directions of the local coordinate system (x, y, z). ϕ is the incremental angle of rotation of the cross section with respect to the longitudinal axis x. ω denotes the normalized unit warping coordinate with a pole at the centroid, and y, z and n locate the point of the wall where the strains are evaluated (see **Fig. 1**).

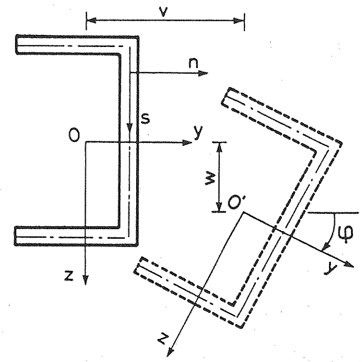

Fig. 1 Local coordinates and rigid motion of the cross section.

(3) Method of analysis

a) Tangent modulus approach

The use of the tangent modulus approach has the advantage of decoupling St. Venant shear and axial deformations leading to a simple constitutive equations[6]. These equations are given in the matrix form

$$\begin{Bmatrix} \bar{\sigma}_x \\ \bar{\tau}_s \end{Bmatrix} = E \begin{bmatrix} c_1 & c_2 \\ c_2 & \frac{G}{E} c_3 \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\gamma}_s \end{Bmatrix} \quad \cdots\cdots (3)$$

where $c_1 = E_t/E$; $c_2 = 0$ and $c_3 = G_t/G$. E_t and G_t being the tangent moduli of tensile and shear tests, respectively. $\bar{\varepsilon}_x$ and $\bar{\gamma}_s$ are the incremental strains given by Eqs. (2・a, b) and $\bar{\sigma}_x$ and $\bar{\tau}_s$ represent the corresponding stresses. In the elastic range, E_t and G_t are identified with the Young modulus E and the shear modulus G, respectively. In the plastic range, however, it was suggested in Ref. 12) that they are approximately related to each other by

$$G_t = E_t / 2\sqrt{3} \quad \cdots\cdots (4)$$

In this case, Eqs. (3) are dependent only on the value of E_t which can be determined provided a yield criterion has been defined. Suppose that the yielding is caused by only axial and St. Venant shear stresses, von Mises yield criterion states that yielding takes place only if

$$\sigma_e^2 \equiv \sigma_x^2 + 3\tau_s^2 = \sigma_y^2 \quad \cdots\cdots (5)$$

where σ_y represents the tensile yield stress of the material. In terms of strains, Eq. (5) can be formally rewritten as

$$\varepsilon_e^2 \equiv \varepsilon_x^2 + 3\left(\frac{G}{E}\right)^2 \gamma_s^2 = (\sigma_y/E)^2 \quad \cdots\cdots (6)$$

Assuming a trilinear stress-strain diagram as shown in **Fig. 2**, the value of E_t is defined as follows,

$$E_t = \begin{cases} E & \text{if} \quad \varepsilon_e < \varepsilon_y \\ E_p & \text{if} \quad \varepsilon_y \leqq \varepsilon_e < \varepsilon_s \\ E_s & \text{if} \quad \varepsilon_e \geqq \varepsilon_s \end{cases} \quad \cdots\cdots (7)$$

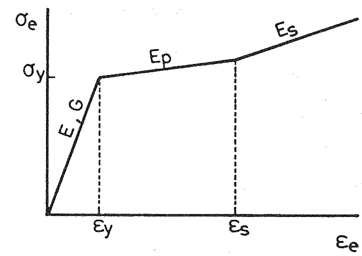

Fig. 2 Stress-strain diagram.

Substituting Eqs. (3) and (2) into Eq. (1), the governing equation becomes

$$\int_0^l \delta \boldsymbol{d}^T \boldsymbol{B} \boldsymbol{d}\, dx + \int_0^l \Big[M_y (v'' \delta\phi + \phi \delta v'') - M_z (w'' \delta\phi + \phi \delta w'') + P (v' \delta v' + w' \delta w') + M_\rho \phi' \delta\phi' \Big] dx = \int_S (T_i + \overline{T}_i)\, \delta u_i ds - \int_0^l \delta \boldsymbol{d}^T \boldsymbol{f} dx \quad (8)$$

where

$$\boldsymbol{B} \equiv E \int_A \begin{bmatrix} c_1 & c_1 y & c_1 z & c_1 \omega & 2nc_2 \\ & c_1 y^2 & c_1 yz & c_1 y\omega & 2nc_2 y \\ & \text{Symm.} & c_1 z^2 & c_1 z\omega & 2nc_2 z \\ & & & c_1 \omega^2 & 2nc_2 \omega \\ & & & & \frac{G}{E} 4n^2 c_3 \end{bmatrix} dA \quad (9)$$

and the vectors $\boldsymbol{d}$ and $\boldsymbol{f}$ are defined as follows,

$$\boldsymbol{d}^T \equiv \langle u' \quad -v'' \quad -w'' \quad -\phi'' \quad \phi' \rangle ; \boldsymbol{f}^T \equiv \langle P \quad M_z \quad M_y \quad M_\omega \quad T_s \rangle$$

with

$$P \equiv \int_A \sigma_x dA ; \quad M_z \equiv \int_A \sigma_x y dA ; \quad M_y \equiv \int_A \sigma_x z dA ;$$

$$M_\omega \equiv \int_A \sigma_x \omega dA ; \quad T_s \equiv \int_A 2n\tau_s dA \quad (10\cdot a)$$

and the remaining stress resultant M_ρ which appears in Eq. (8) is given by

$$M_\rho \equiv \int_A \sigma_x (y^2 + z^2)\, dA \quad (10\cdot b)$$

In the evaluation of the stiffness equation given by Eq. (8), a numerical integration over A, the area of the cross section, is required to determine the stress resultants (Eqs. (10)) and the cross sectional properties contained in matrix $\boldsymbol{B}$. These properties are dependent not only on the shape of the section and its dimensions but also on the value of E_t which is path dependent. To this end, consider an arbitrary thin-walled open cross section whose profile has been divided into a convenient number of small segments such that the linear distribution of the axial strain ε_x inside each segment can be neglected. Fig. 3 shows a small segment of the wall and the distribution of the strains inside it. As the St. Venant shear strain has its largest value γ_s^* on the surfaces and according to Eq. (6), yielding commences simultaneously there and propagates toward the midline of the wall as γ_s increases. Consequently, at any reference state where the total strains ε_x and γ_s^* are known, it is always possible to detect the elastic, plastic and strain-hardening zones for each small segment. These zones are located by the following expressions (see Fig. 3)

Elastic zone $\quad t_e = t \dfrac{E}{G} \left[\dfrac{\varepsilon_y^2 - \varepsilon_x^2}{3\gamma_s^{*2}} \right]^{1/2}$

Plastic zone $\quad t_p = t \dfrac{E}{G} \left[\dfrac{\varepsilon_s^2 - \varepsilon_x^2}{3\gamma_s^{*2}} \right]^{1/2} - t_e$

Strain-hard. zone $\quad t_s = t - t_e - t_p \quad (11\cdot a, b, c)$

In a segment where γ_s^* is zero, the stress state is uniform and there exists only one of the three zones, and thus the value of E_t will be constant through its thickness. Now it becomes possible that a partially yielded cross section is governed by only the elastic modulus E provided that the original thickness t of each segment is replaced by t_m which is defined by

$$t_m = t_e + \frac{E_p}{E} t_p + \frac{E_s}{E} t_s \quad (12)$$

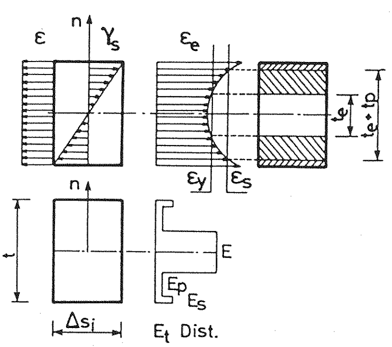

Fig. 3 Location of the yielded zones and distribution of the tangent modulus in each segment.

where E_p and E_s are shown in Fig. 2 and t_e, t_p and t_s are given by Eqs. (11). By making use of the distribution of E_t inside each segment, the stress resultants given by Eqs. (10) will be also evaluated by numerical integration except St. Venant torque T_s which will be obtained by accumulation of its increments ΔT_s calculated from displacements as follows

$$\Delta T_s = GJ_m\phi' \cdots\cdots (13)$$

here J_m represents the St. Venant constant of the modified cross section which can be obtained easily since the distribution of the tangent modulus is known. This can be computed by the following expression

$$J_m = \sum \Delta b \left\{ \frac{t_e^3}{3} + [(t_e + t_p)^3 (E_p - E_s) + E_s t^3 - E_p t_e^3]/6\sqrt{3}\,G \right\} \cdots\cdots (14)$$

where the summation is taken over the total number of the small segments. The width of each one is Δb.

b) Prandlt-Reuss flow theory (J2F)

This theory, successfully employed by Sakimoto et al.[2], utilizes Hooke's law for the elastic range and Prandlt-Reuss incremental equations for the inelastic range in conjunction with the von Mises yield criterion. This theory formally results in the same incremental stress-strain relationships as Eqs. (3), but with different expressions for c_1, c_2 and c_3. In this case, c_1, c_2 and c_3 must be replaced by $(1-D_1)$, $-D_2$ and $(1-D_3)$ of Ref. 2), respectively. The von Mises yield criterion which will be used in this analysis, includes the effects of the axial stress σ_x, St. Venant shear stress τ_s and the shear stress τ_w due to non-uniform bending and torsion and is defined as follows

$$\sigma_e^2 = \sigma_x^2 + 3(\tau_s + \tau_w)^2 = \sigma_y^2 \cdots\cdots (15)$$

Unlike σ_x and τ_s, τ_w can not be determined from kinematics because the corresponding strain is originally assumed to be zero. Therefore, it will be determined from equilibrium consideration[6]. Referring to Fig. 4 and using the technique of the finite element method, we can approximately determine the shear stress τ_w for an arbitrary segment ij of the cross section as follows

$$\tau_w^j = \tau_w^i - \frac{\sigma_x^q - \sigma_x^p}{l}\Delta s_i \cdots\cdots (16)$$

where σ_x^p and σ_x^q denote the average of the axial stress acting on the considered segment at the end points p and q of one element of length l, respectively. This equation can be easily solved if an appropriate contour coordinate s with origin located at an open end point free from shear is selected. The variation of these shear stresses inside the segment will not be taken into account, and only their average will be considered.

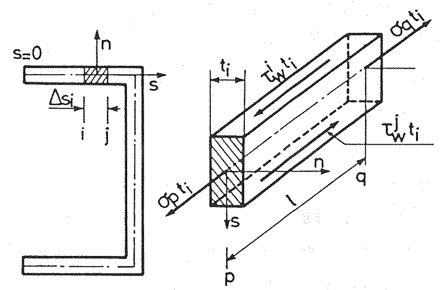

Fig. 4 Determination of the shear stress τ_w.

In comparison with those of the previous section, the expressions of c_1, c_2 and c_3 become more complicated, and their distribution inside the segment can not be known in advance. Therefore, the thickness of each segment is divided into many layers, and at the center of each one of the layers, the stresses, the plastic strain and then c_1, c_2 and c_3 are evaluated in order to carry out the numerical integration required to determine $\boldsymbol{B}$ and $\boldsymbol{f}$.

(4) Tangent stiffness equation

The governing equation Eq. (8) with four unknown incremental displacements; i.e. u, v, w and ϕ, will be discretized by the F.E.M. technique and solved by an iterative procedure. The usual Hermitian shape functions $\langle \boldsymbol{n}_1 \rangle = \left\langle 1-\frac{x}{l} ; \frac{x}{l} \right\rangle$ and $\langle \boldsymbol{n}_3 \rangle = \left\langle 1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3} ; -x+\frac{2x^2}{l}-\frac{x^3}{l^2} ; \frac{3x^2}{l^2}-\frac{2x^3}{l^3} ; \frac{x^2}{l}-\frac{x^3}{l^2} \right\rangle$ are employed here to describe, respectively, the variation of u and v, w and ϕ along the length of the element. Upon substituting these shape functions into Eq. (8) and making some usual manipulations, the incremental stiffness expression for one element in the local coordinate system (x, y, z) can be derived and written in the familiar form of F. E. M. as

$$\delta \boldsymbol{r}^T[\boldsymbol{K}_{ep}+\boldsymbol{K}_g]\,\boldsymbol{r}=\delta \boldsymbol{r}^T(\boldsymbol{R}-\boldsymbol{F}) \quad \cdots\cdots (17)$$

where $\boldsymbol{K}_{ep}$ and $\boldsymbol{K}_g$ represent the elasto-plastic small displacement stiffness matrix and the geometrical stiffness matrix, respectively. These two matrices resulted from the first and second terms of the left-hand side of Eq. (8), respectively. While $\boldsymbol{R}$ and $\boldsymbol{F}$ which denote the total applied external force and internal force vectors, are obtained from the first and second terms of the right-hand side of Eq. (8), respectively. The incremental nodal displacement vector $\boldsymbol{r}$ is given in the following order

$$\boldsymbol{r}^T=\langle u_p \;\; v_p \;\; w_p \;\; \phi_p \;\; -w'_p \;\; v'_p \;\; -\phi'_p;\, u_q \;\; v_q \;\; w_q \;\; \phi_q \;\; -w'_q \;\; v'_q \;\; -\phi'_q\rangle \quad \cdots\cdots (18\cdot a)$$

and the explicit form of $\boldsymbol{F}$ is

$$\boldsymbol{F}^T=\langle -(P_p+P_q)/2 \;\; -(M_z^q-M_z^p)/l \;\; -(M_y^q-M_y^p)/l \;\; -(M_\omega^q-M_w^p)/l \;\; -(T_s^p+T_s^q)/2 \;\; -M_y^p \;\; M_z^p$$
$$M_\omega^p-(T_s^q-T_s^p)\,l/12;\,(P_p+P_q)/2 \;\; (M_z^q-M_z^p)/l \;\; (M_y^q-M_y^p)/l \;\; (M_\omega^q-M_\omega^p)/l+(T_s^p+T_s^q)/2$$
$$M_y^q \;\; -M_z^q \;\; -M_\omega^q+(T_s^q-T_s^p)\,l/12\rangle \quad \cdots\cdots (18\cdot b)$$

It should be noted that, in deriving Eq. (17) and (18・b), except for the stress resultant P, all the components of $\boldsymbol{B}$ and $\boldsymbol{f}$ were assumed to vary linearly through the length of one element in the following form

$$B_{ij}=(B_{ij})_p+[(B_{ij})_q-(B_{ij})_p]\frac{x}{l} \quad \cdots\cdots (19\cdot a)$$

$$f_i=(f_i)_p+[(f_j)_q-(f_i)_p]\frac{x}{l} \quad \cdots\cdots (19\cdot b)$$

(5) Transformation and Assembly[8)]

Before proceeding with the derivation of the incremental equations of equilibrium of the structure as whole, it is necessary that the nodal displacement vector real as well as virtual of Eq. (17) be transformed into a single global Cartesian system (x, y, z) of the structure. This requires that the element local unit vectors (x, y, z) be determined in terms of the global ones. For this purpose, two assumptions are taken into account : (1) rotations are assumed to be reasonably small and hence they can be, approximately, transformed like vectors and (2) warping parameters (moments and deformations) have no equivalents in the global system and hence they are not transformed. When the element deforms, its end cross sections will rotate successively through their corresponding angles $(\theta_y, \theta_z, \theta_x)$ or in terms of the generalized displacements $(-w', v', \phi)$ into their new directions. Since in most cases the local z-axes at the two end cross sections do not lie in the same direction, the corresponding local axis of the element will be selected to be along the average direction. The longitudinal x-axis of the element will be determined from the global coordinates of the reference points at the element's end cross sections, while the y-axis which must be normal to the z-x plane, will be obtained using this property. The local z-axis will be ascertained by making use of the orthogonality condition with the x-y plane. Finally, the transformation matrix can be readily set up and by making use of it, Eq. (17) can be written in the global coordinate system. The tangent stiffness for the whole structure under consideration will be deduced from this equation by usual assembly procedure. Recognizing the arbitrary nature of the virtual displacements, leads to a set of non-linear equations to be solved iteratively in the subsequent chapters.

3. SOLUTION PROCEDURE

The solution procedure of Eq. (17) for the two presented analyses is almost the same. The major difference is in how to judge the elastic, plastic and strain-hardening ranges. The method employed will be briefly reviewed below.

(1) Tangent modulus based analysis

In this analysis, the thickness of each segment of the wall needs not to be layered since its elastic, plastic and strain-hardening parts are located from the known distribution of the strains as was explained in the previous chapter. This will enable the evaluation of the components of $\boldsymbol{B}$ and $\boldsymbol{f}$ required for the formation of the assembled tangent stiffness matrix and the internal force vector. Then the right-hand side of Eq. (17) is checked whether it is zero or not. If it is satisfied, the new configuration is in equilibrium state and thus we can move to the next incremental step. Otherwise, the incremental displacement vector r_t is evaluated, and updated are the coordinates and rotations of the end sections of each element required for the formation of the transformation matrix. Then these displacements are transformed to the local coordinate system to find the strain increments by using shape functions and subsequently stress increments at each segment. Then the total strains and stresses are calculated by simple accumulation of increments and the entire process is repeated. The ultimate load is defined as the load after which iterative convergence could not be achieved although the incremental load has been reduced to 0.1 % of it ; and the computation is stopped.

(2) J2F based analysis

The solution procedure for this analysis and the previous one differs on the manner of the discretization of the cross section and on how to evaluate c_1, c_2 and c_3 of Eqs. (3). The numerical evaluation of these expressions depends on whether the shear stress of warping and bending does contribute to yielding or not. In the case of no such contribution, their evaluation follows exactly the process given in Ref. 2). However, in the other case, St. Venant shear stress τ_s in Ref. 2) will be replaced by $(\tau_s+\tau_w)$ in the evaluation of the equivalent stress σ_e and parameters c_1, c_2 and c_3. Here τ_w represents the shear stress due to warping and bending found from the equilibrium condition with the axial stress σ_x. The other steps of the solution procedure remain unaltered.

4. RESULTS AND DISCUSSIONS

In order to show the efficacity and versatility of the tangent modulus based analysis, a variety of problems that include torsion, bending and compression are solved, and the results are compared with experiments and the more theoretically sophisticated (J2F) analyses.

(1) Eccentrically compressed members

A Wide-Flange cross section (5×6H) column with residual stresses ($\sigma_{rc}=-0.145\,\sigma_y$) subjected to the axial load P applied eccentrically with respect to both y and z axes, is examined up to failure. For the tangent modulus based analysis, the cross section was divided into 44 segments while 192 were required for the J2F analysis. Using 8 elements for both, the F. E. solution in Fig. 5 of these two analyses shows good agreement with the experimental results of Birnstiel[4] and the numerical ones by Sakimoto et al.[2]. In this problem, since the shear stresses of bending and warping are playing a secondary role, their effect on the

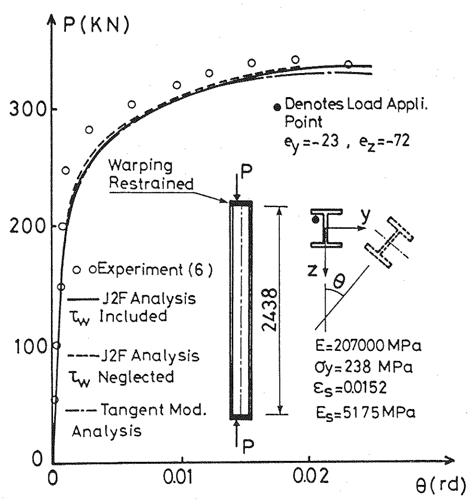

Fig. 5 Eccentrically compressed member.

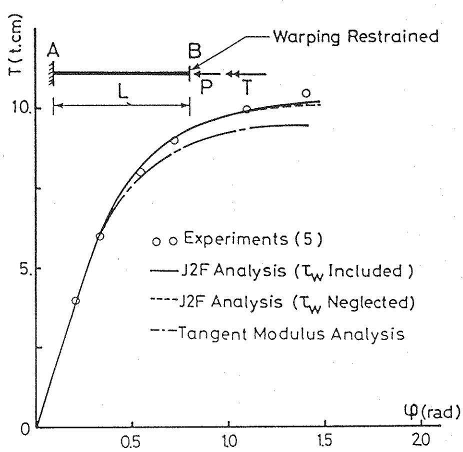

Fig. 6 Member subjected mainly to torsion.

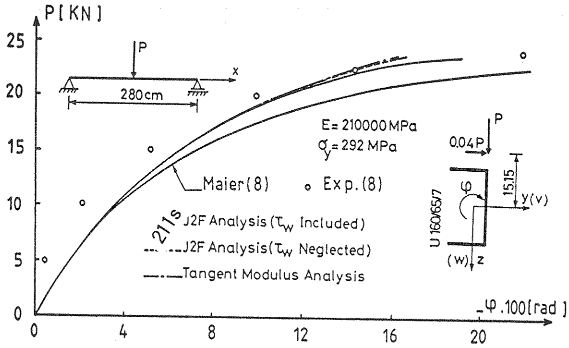

Fig. 7 Channel under flexure and torsion.

ultimate load is not observed.

(2) Members subjected mainly to torsion

To demonstrate the ability of the tangent modulus formulation in analyzing torsion, a wide flange cross section member that correspond to a specimen HT-2 in Ref. 5) is analyzed. The geometrical and mechanical properties as well as the distribution of the residual stresses and their magnitudes are given in Ref. 5). Fig. 6 indicates that the proposed tangent modulus formulation can predict with acceptable accuracy the ultimate strength of members where St. Venant torsion plays a leading role.

(3) Beam under the combined action of flexure and torsion

Treated next is the ultimate strength of beams with unsymmetrical cross section under a simultaneous action of bending and torsion. A theoretical model to be solved is shown in Fig. 7. The effect of load application point is taken into account, and an additional term "$-0.1515P+0.04Pd$", for this particular example, must be introduced in the diagonal element of the assembled geometric stiffness matrix. Here P is the total applied load at the reference state and d is the distance between the centroidal point and the web center of the cross section. The cross section profile is divided into 32 segments for the tangent modulus solution but for the other analyses, each segment is divided into 8 layers. As can be seen from the figure, there is a large discrepancy between the experimental and numerical results from the very beginning of the deformation. According to Ref. 10), this error is mainly due to the setting-up of the experiments and the difficulty of keeping the load in the plane of the web. The solution based on the tangent modulus approach seems to be, again, in agreement with the J2F one when the shear stress τ_w is not included. However, when this stress is taken into consideration, the solution seems to be closer to experiments and that of Maier [10]. The formulation of Maier was not based on shape functions and it was necessary to divide the member into 35 elements (here only 8 elements) for accuracy requirements.

5. CONCLUSION

An inelastic F. E. M. analysis of arbitrary thin-walled member with open cross section, is presented. Two different constitutive equations have been employed and a variety of problems for which experimental results exist, have been examined numerically.

Compared with the more theoretically sophisticated analysis (J2F analysis), the presented tangent modulus analysis has adopted a more simplified constitutive equation that keeps St. Venant torsion uncoupled with other deformations. Thus the plastic flow is prevented and the thickness of the wall does not have to be layered. In spite of this, satisfactory results for a large range of applicability, have been obtained. Moreover, the computation time has been reduced as a consequence of the way of the discretization of the cross section profile and minimization of non-zero cross sectional stiffness parameters.

In the case of J2F analysis, without modifying Vlazov assumption concerning the shearing deformation in the mid-surface of the wall, the warping and bending shear stress resulting from equilibrium with the axial stress has been incorporated into the yield condition. The numerical results confirm the validity of this approach and show that the effect of these shear stresses on the ultimate state may be negligible.

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