

PARAMETRIC STUDIES ON TUNED LIQUID DAMPER (TLD) USING CIRCULAR CONTAINERS BY FREE-OSCILLATION EXPERIMENTS

By Yozo FUJINO, Benito M. PACHECO**, Piyawat CHAISERI***
 and Li Min SUN*****

A new kind of damper, Tuned Liquid Damper (TLD) relying on motion of shallow liquid in a rigid cylinder, is experimentally studied. Prototype-sized circular containers with diameters 40 cm and 60 cm and partially filled with water, are attached to a single-degree-of-freedom structural model with natural period of 2 sec. The damper effect is measured in terms of the increase in the logarithmic rate of decrement of free oscillation of the main structure. The structural displacements range from 8 cm down to 0.25 cm. It is seen that, for large damping effect at small amplitude of structural vibration, it is necessary to tune the fundamental sloshing period of the liquid to the natural period of structure; hence the name Tuned Liquid Damper. Breaking of surface waves, which is dependent on structural vibration amplitude, appears to be a major mechanism of energy dissipation in the range of displacements considered. Also investigated are the effects of: ratio of liquid frequency to structure frequency; liquid viscosity; container bottom roughness; container roof height; ratio of liquid mass to structure mass; and container diameter.

Keywords: amplitude, circular container, damper, experiment, free oscillation, liquid, logarithmic decrement, prototype, tuning, wave, wave breaking

1. INTRODUCTION

Relatively light, flexible, and weakly damped bridges, towers, building and other structures are increasing in number because of growing use of high-strength materials and welded joints. Structural vibrations then can create possible problems from the viewpoint of serviceability or even safety. Base isolation is one way to control such vibrations in case of ground motions¹⁾.

When vibration suppression is required, another option is to use mechanical dampers²⁾. Tuned Mass Dampers (TMD) are a class of such devices for absorbing and dissipating vibration energy. TMD now finds practical application in bridges and buildings but not without problem areas of its own, such as sensitivity of device to external conditions, maintenance requirements, or unreliability of automatic activation.

This paper discusses a new type of so-called passive mechanical damper, named Tuned Liquid Damper (TLD) [Fig. 1], which relies on motion of shallow liquid inside a rigid container, for absorbing and dissipating structural vibration energy. Dampers using liquid motion have been in use in space satellites and marine vessels [Fig. 2]^{3)~6)}. Modi and Sato^{7), 8)} were among the first to suggest their application to ground structures including buildings and towers. Some other recent studies may also be cited^{10)~23)}. Growing interest in TLD is attributable to several potential advantages, including: low cost; easy installation especially in already existing structures which may otherwise have some space problems; adaptability to temporary use; and few maintenance requirements.

* Member of JSCE, Ph. D., Assoc. Prof., Eng. Res. Inst., Univ. of Tokyo, Yayoi 2-11-16, Bunkyo-ku, Tokyo 113

** Member of JSCE, Dr. Eng., Asst. Prof., Dept. of Civil Eng., Univ. of Tokyo, Hongo 7-3-1, Bunkyo-ku

*** Member of JSCE, M. Eng., Graduate Student, Dept. of Civil Eng., Univ. of Tokyo

**** Member of JSCE, Graduate Student, Dept. of Civil Eng., Univ. of Tokyo

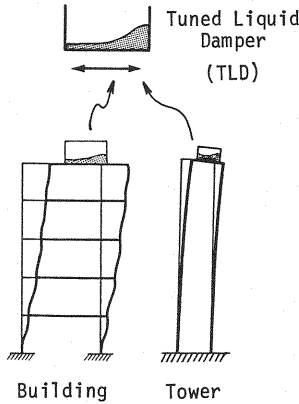


Fig.1 TLD installed on a building and a tower.

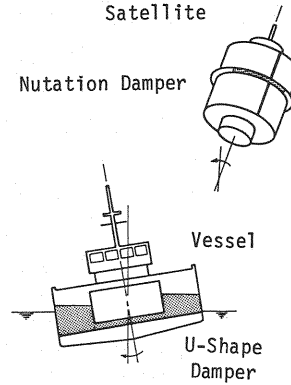


Fig.2 Other liquid dampers.

Studies on liquid dampers so far indicate their effectiveness ; but adequate mathematical modelling and clear explanation of mechanism are lacking. This paper presents data and insights from an extensive experiment designed to demonstrate the TLD's fundamental mechanism, with emphasis on parameters that the practicing engineer may have immediate interest in, e. g. ratio of liquid mass to mass of structure.

2. EXPERIMENT SET-UP AND PROCEDURE

A steel platform as shown schematically in Fig.3 was designed to simulate a tall flexible tower or building, with a natural period of $T_s=1.98$ sec (or frequency $f_s=0.505$ Hz), and [modal] mass m_s ranging from 116 to 440 kg. The structural mass was large enough to keep the ratio of liquid mass to structural mass in the range usually acceptable for dampers, i.e. about 1 %.

Unlike in past experiments that employed small models, large prototype-size cylinders with diameters of 40 and 60 cm, were used here as liquid containers, with a view to using multiple identical TLDs in massive actual structures. The inside diameters were 38.9 and 58.9 cm, respectively. With mass ratio, dimension, and vibration frequency thus made equal to those of prototype, it was possible to sidestep some dynamic fluid similarity requirements that would be difficult, if not impossible, to satisfy simultaneously. One such similarity requirement would be the similarity of ratio of liquid surface tension force to liquid inertia force.

The structure, i.e. platform, was essentially a single-degree-of-freedom system allowing only horizontal motion. It was given initial displacement while taking care that the liquid in the TLD was quiescent before the structure was released into free vibration. Both structural displacement and liquid surface height at one side of the TLD were pen-recorded and simultaneously stored as digitized data.

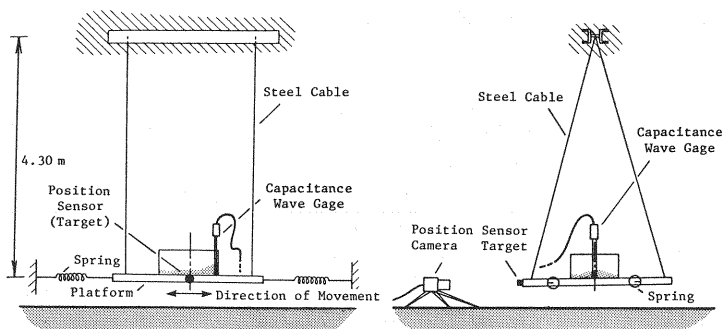


Fig.3 Single-degree-of-freedom structural model with TLD.

Damping in the structure (either with or without TLD attached) was calculated as amplitude-dependent locally-averaged logarithmic rate of decrement of oscillation amplitude, $\delta = \delta(A)$:

$$\delta(A_n) = (1/4) \ln (A_{n-2}/A_{n+2}) \dots \dots \dots (1)$$

where A_n is structural vibration amplitude for n th cycle; A_{n-2} is amplitude two cycles before; and A_{n+2} is amplitude two cycles after. Computerized processing was done on the digital displacement data; and continuous δ -versus- A curve was fitted to the discrete points obtained by Eq. (1). Additional damping due to TLD was then calculated as

$$\Delta\delta = \delta_T - \delta_S \dots \dots \dots (2)$$

where δ_S is logarithmic decrement of structure alone (Fig. 4); and δ_T is logarithmic decrement of structure with TLD attached. Thus additional damping $\Delta\delta$ could be experimentally determined as function of structural vibration amplitude A . Fig.5 (a) shows sample time histories of structural displacement without and with damper. In some cases with TLD attached, beating phenomenon was observed in the end-portion of the displacement record, i.e. at small amplitude of remaining vibration, as in the example shown in Fig.5 (b). Additional damping $\Delta\delta$ could not meaningfully be measured from such records. Beating or amplitude modulation during free-vibration indicates that a large fraction of absorbed energy is transferred back from liquid to structure.

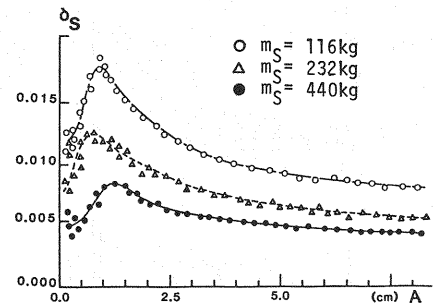


Fig.4 Logarithmic decrement of structure without TLD.

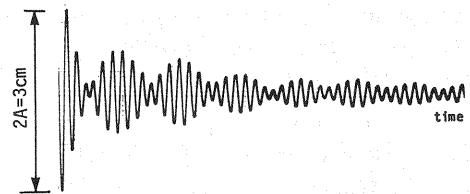
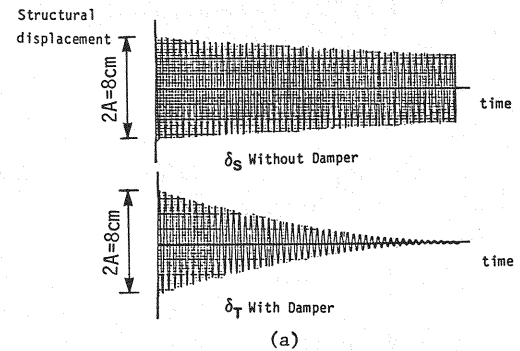


Fig.5 Structural displacement histories.

3. PARAMETERS UNDER EXPERIMENTAL STUDY

In the experiment described above, the following parameters were varied, in order to demonstrate their effects on TLD mechanism and on additional damping, $\Delta\delta$:

- Amplitude of structural vibration, A
- Ratio of liquid sloshing frequency to structural frequency, $\gamma = f_L/f_s$
- Kinematic viscosity of liquid, ν
- Roughness of bottom of cylinder [Fig.6]
- Gap between liquid surface and roof of container, G [Fig.7]
- Ratio of liquid mass to structural mass, $\mu = m_L/m_s$
- Diameter of cylinder, $\phi = 2a$

The undamped natural frequency of fundamental mode of sloshing of liquid, f_L , was calculated as:

$$f_L^2 = [1.84 g \tanh(1.84 h/a)] / [a(2\pi)^2] \dots \dots \dots (3)$$

from linear potential theory [Fig.8]. g is gravitational acceleration; h is depth of liquid; and $a = \phi/2$ is cylinder radius.

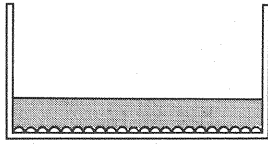


Fig. 6 Roughness of container bottom.

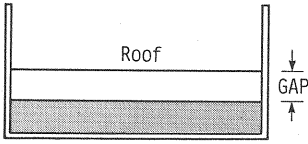


Fig. 7 Gap with container roof.

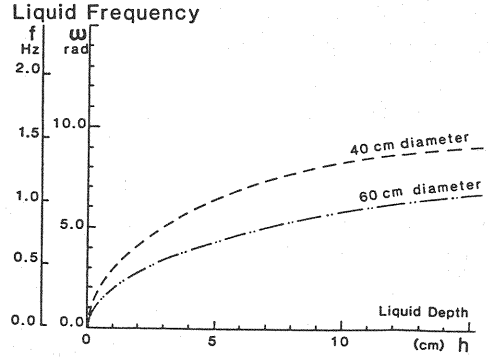


Fig. 8 Natural frequency vs. liquid depth.

4. OBSERVED EFFECTS OF DIFFERENT PARAMETERS ON ADDITIONAL DAMPING $\Delta\delta$

Additional damping $\Delta\delta$ is shown in Fig. 9 for selected cases.

(1) Effect of frequency tuning

Each curve of $\Delta\delta$ versus A in Fig. 9 corresponds to a different liquid depth h and, accordingly, different frequency ratio γ . On the other hand, each curve of $\Delta\delta$ versus γ in Fig. 10 corresponds to a different amplitude A . One thing to be noted from Fig. 10 is that in most of the cases where additional damping is significant, the liquid in TLD is generally shallow, e. g. the ratio h/a is below 0.1. Considering the liquid wave length, L , in an oscillating container to be twice the container diameter (or $L=4a$), the present ratios $h/L < 0.025$ indicate that shallow water wave or long wave theory should be applied in wave analysis⁽¹⁹⁾ [Sect. 4.1].

As these figures corresponding to $\phi=40$ cm show, high additional damping $\Delta\delta$ is obtainable when the frequency ratio is about $\gamma=1.0$, which is the nominal tuning condition for the present system. The additional damping $\Delta\delta$ at tuned condition is particularly high when the amplitude A is small. In fact when $\gamma=1.0$, $\Delta\delta$ exhibits strong dependence on A . In the case of 60-cm diameter TLD, it was observed that the highest $\Delta\delta$ was obtained when the nominal frequency ratio γ was about 0.9 instead of 1.0.

On the other hand, additional damping $\Delta\delta$ is rather low at frequency ratios that are very different from $\gamma=1.0$. Indeed, for very high frequency ratio, say $\gamma > 1.5$, and accordingly deep liquid, little $\Delta\delta$ is attainable even as the mass ratio μ is large. It should be pointed out that the high $\Delta\delta$ mentioned above ($\gamma=1.0$) corresponds to a liquid mass that is only about 1 % of structure.

The condition of tuning seems unnecessary, however, for large amplitude A , where the same moderate

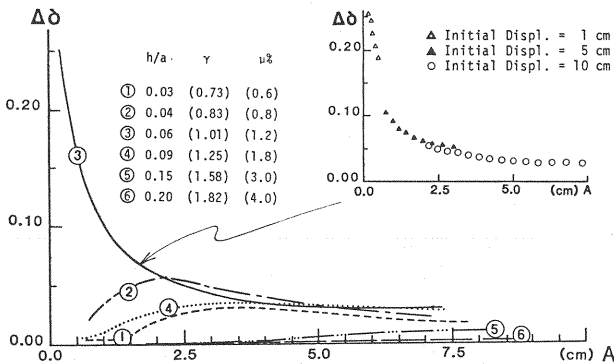


Fig. 9 Additional damping due to TLD ($\phi=40$ cm, $m_s=116$ kg).

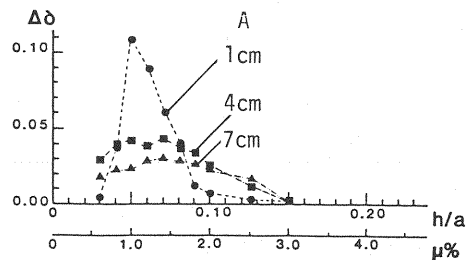


Fig. 10 Additional damping vs. frequency ratio ($\phi=40$ cm, $m_s=116$ kg).

additional damping $\Delta\delta$ is obtained almost regardless of frequency ratio γ . This may suggest the use of γ that is much less than 1.0, to make the liquid mass ratio μ accordingly lower. Still, γ should be close to 1.0 if high $\Delta\delta$ is to be obtained specially at small A ; the name *Tuned Liquid Damper* is on account of this latter requirement.

As for the sensitivity of γ or f_L to errors or deviations in the liquid depth h , Fig. 8 shows that for given $f_L=f_s$ the sensitivity to h is less when the cylinder is bigger, although the required h itself is deeper. This suggests that tuning may be more reliably accomplished with a relatively big cylinder.

(2) Effect of liquid viscosity

In usual oil or viscous dampers it is preferable to use high-viscosity liquids. For TLD, Fig. 11 compares two liquids with the same density but different kinematic viscosities. It turns out that somewhat higher $\Delta\delta$ is obtained from the less viscous liquid when the frequency ratio γ is about 1.0 or less; and there is no significant difference in $\Delta\delta$ when the amplitude A is very large. It is noteworthy that lower-viscosity liquid yields higher additional damping around $\gamma=1$ even though the viscous boundary layer is predicted to be thinner than that of high-viscosity liquid^[9] [Sect. 4.6].

(3) Effect of roughness of bottom of container

The liquid in TLD being generally shallow, the

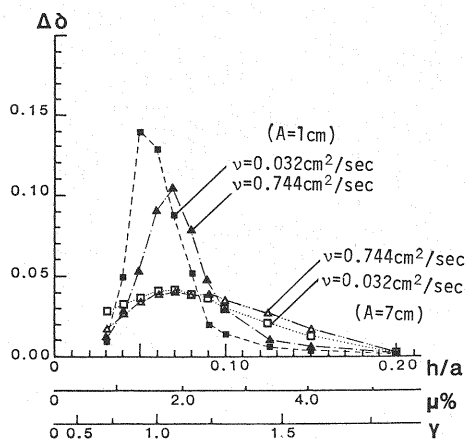


Fig. 11 Effect of liquid kinematic viscosity ($\phi=40$ cm, $m_s=116$ kg).

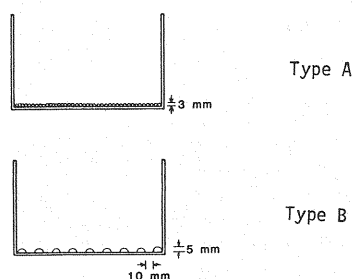


Fig. 12 Semispheres as roughness elements at container bottom.

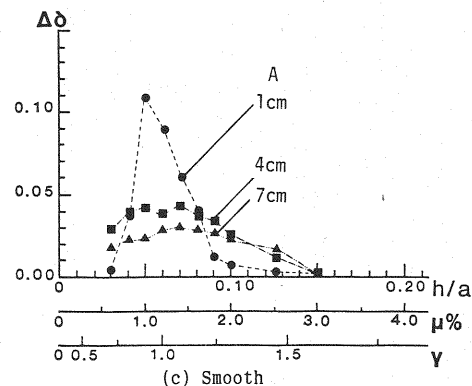
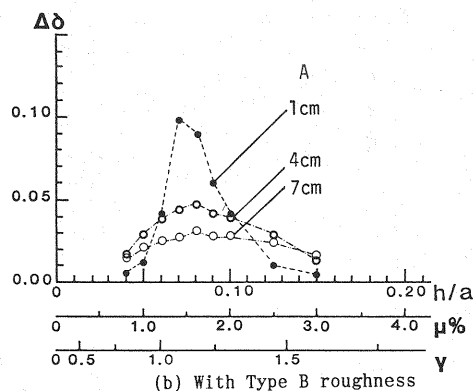
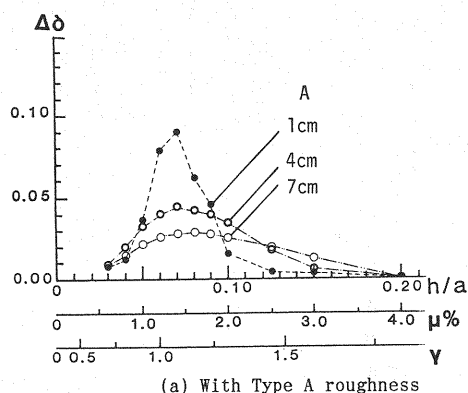


Fig. 13 Effect of bottom roughness on additional damping ($\phi=40$ cm, $m_s=116$ kg).

bottom is the major contact surface and roughness here may affect the TLD's effectiveness, too. The rough bottom was made of semisphere as shown in Fig. 12. The comparison in Fig. 13 shows that the value of peak $\Delta\delta$ is only slightly affected by the roughness elements. Upon introduction of bottom roughness elements, the h (and accordingly the γ) where peak $\Delta\delta$ is attained is shifted up, indicating that a certain layer of liquid at the bottom is restrained and the effective depth is less than the actual h .

(4) Effect of gap between liquid and roof of container

With a closed roof, it is possible for the liquid to hit it and dissipate energy in the process. At the same time, if the gap is small, liquid motion may be restrained and the original TLD mechanism may be suppressed. Fig. 14 demonstrates the effect of roof in a tuned case, showing that $\Delta\delta$ has a peak at gap ratio

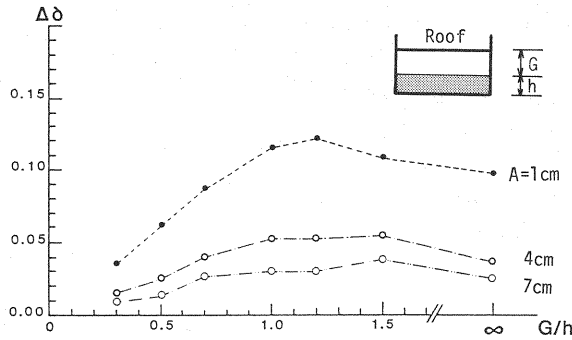


Fig. 14 Effect of roof height (gap G) on additional damping ($\gamma=1.0$, $\phi=40\text{ cm}$, $m_s=116\text{ kg}$).

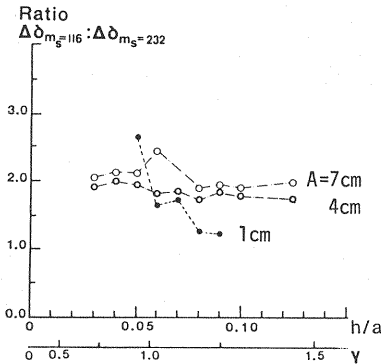


Fig. 15 Effect of mass ratio on additional damping ($\phi=40\text{ cm}$).

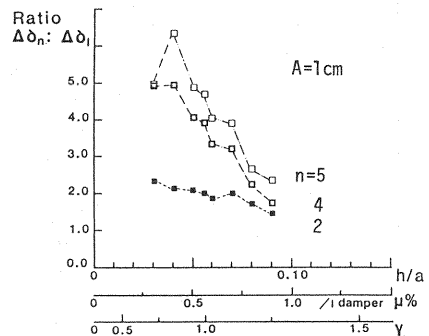


Fig. 16 Effect of number n of TLDs on additional damping ($\phi=40\text{ cm}$, $m_s=232\text{ kg}$).

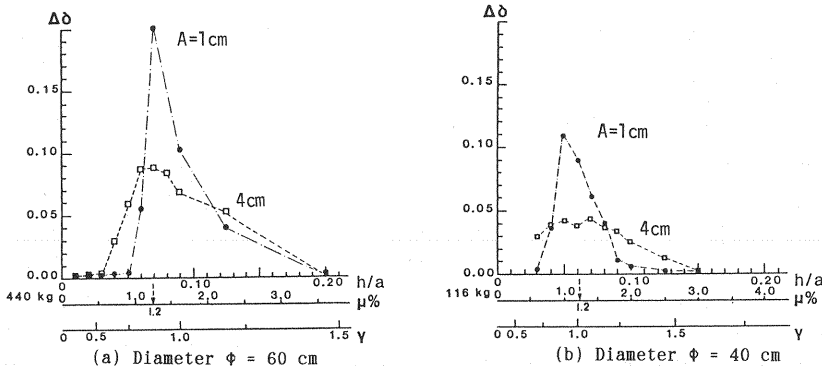


Fig. 17 Comparison of single large TLD and single small TLD.

$1.0 < G/h < 1.5$ although the increase over the infinite-gap or no-roof case is not much. Instead, the indication is that it is wiser to provide a high roof and avoid restraining the liquid motion.

(5) Effect of liquid mass ratio and number of dampers

Fig. 15 shows the effect on $\Delta\delta$ when, for the same TLD, the structural mass is halved from 232 kg to 116 kg and hence the liquid mass ratio μ is doubled. For doubled mass ratio μ , the additional damping $\Delta\delta$, too, is increased by a factor of approximately 2 when the frequency ratio is about $\gamma=1.0$ (It is uncertain whether the calculated ratios for $\gamma>1.5$ are in fact smaller than 2; in this range of γ , the individual $\Delta\delta$ values themselves are very small and prone to errors).

Mass ratio could also be increased, for the same TLD configuration and same structural mass, by using multiple identical TLDs as done for Fig. 16. This in fact may be the way of applying 40 or 60 cm diameter TLDs in prototype structures. Fig. 16 shows trends similar to Fig. 15. These figures indicate that for as long as the frequency ratio is about $\gamma=1.0$, additional damping $\Delta\delta$ is practically linearly proportional to the number of identical dampers or to liquid mass ratio, at least within the present experimental range.

(6) Effect of container size

As Eq. (3) and Fig. 8 show, in tuning the natural frequency of liquid to the frequency of structure, it is possible to choose from among different sizes and their corresponding required liquid depths. The question in practice is whether to use one large TLD or multiple smaller ones.

Firstly, Fig. 17 compares single 40 cm and 60 cm TLDs. On comparing the respective peaks of $\Delta\delta$ at about $\gamma=1.0$ and $\mu=1.2\%$, it is seen that the larger TLD gives higher peak $\Delta\delta$. Fig. 18 compares one large TLD with five small ones, again showing that the peak $\Delta\delta$ is higher for larger TLD, even as almost equal mass ratios correspond to these peaks. Thus for the present case it is indicated that, for the same mass ratio, the bigger TLD is more effective in producing large $\Delta\delta$. It is not conclusive, however, that still bigger TLDs are better. For given structural period (or frequency) and amplitude, there may be a size that gives an optimal effectiveness. This is further discussed below.

5. DISCUSSIONS

(1) Liquid motion and additional damping

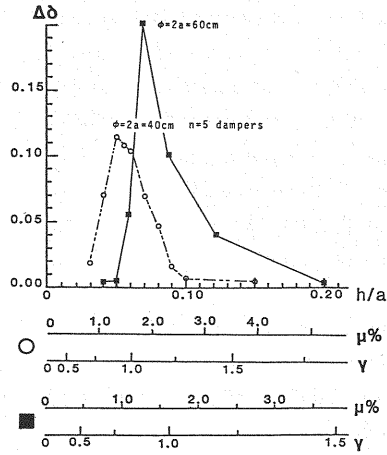
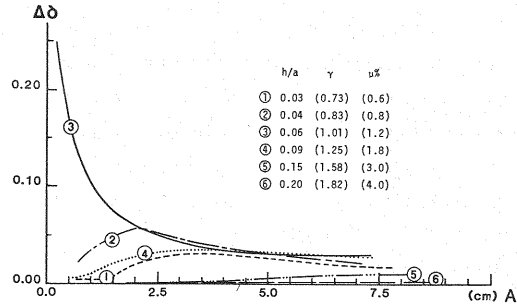


Fig. 18 Comparison of one large TLD and five small TLDs ($m_s=440$ kg).



CURVE	TYPE OF LIQUID MOTION		
1	VI	II	III
2	VI	II	III
3	VI	II	III
4	VI	V	IV
5	VI	V	IV
6	VI	V	IV

0.0 2.5 5.0 7.5 (cm) A

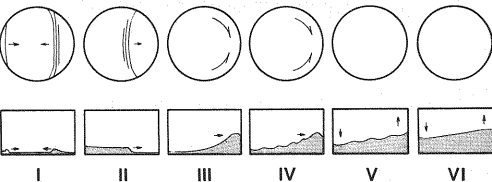


Fig. 19 Visual classification of type of liquid motion ($\phi=40$ cm, $m_s=232$ kg).

As Fig.9 illustrates, the additional damping $\Delta\delta$ due to the TLD depended very much on structural vibration amplitude A . It may be recalled that on account of the procedure adopted for the test, the effective motion of the cylinder was forced oscillation that is approximately sinusoidal with gradually decreasing amplitude. According to frequency and amplitude of such cylinder motion, the liquid motion was expected to vary^(7,9). In fact the type of liquid motion changed as the amplitude A changed during the free-vibration test, and liquid motion type could be correlated with $\Delta\delta$.

The $\Delta\delta$ -vs- A curves of Fig. 9 were replotted in Fig. 19, together with descriptions of liquid motion. The transition between types of motion could not be identified exactly, as the classification was done visually; but it was clear that Type III motion consistently corresponded to rather large $\Delta\delta$. Six types of liquid motion were visually identified :

- Type I : two smooth waves travelling back and forth (ϕ 60 cm only)
- Type II : one smooth wave travelling back and forth
- Type III : two waves travelling on half-circles along the wall
- Type IV : similar to III but not as smooth
- Type V : wavy liquid surface moving only vertically
- Type VI : similar to V but smooth

Wave height was also measured at a point near the container wall, on a diameter along the direction of structural motion. Fig.20 shows two example wave records and their corresponding structural displacement records, corresponding to curves (3) and (4) in Fig. 9 or 19. These wave records provided a good estimate, even if the recorded positive peaks included some effect of splashing during type III motion, and even if very low wave troughs could not be recorded at all.

For the case shown in Fig. 20(a) and curve (3) in Fig. 19, inspection of the wave record showed that for $A>0.25$ cm, the wave height H was greater than 1.2 cm which was the depth of liquid h in that TLD. Applying approximately a criterion commonly used in coastal hydrodynamics, wave breaking, with the associated turbulence at the wave crest, was to be expected when wave height H reached a value about equal to liquid depth h . Turbulence at the wave crest was indeed observed during the experiment, and it seems to indicate that wave breaking may have contributed to $\Delta\delta$ for $A>0.25$ cm.

Wave breaking seems to have occurred not only in the above tuned case but even when frequency ratio was *not* $\gamma=1.0$, e.g. curves (1), (2) and (4) in Fig. 19, during vibration at very large amplitudes. The threshold wave height in the case of curve (4) and Fig. 20(b) was about $H=1.8$ cm, which matched the corresponding liquid depth h . For deep liquid corresponding to $\gamma\gg 1.0$, however, liquid motion tended to be smooth sloshing (Type VI) even for large amplitude A ; and no significant $\Delta\delta$ could be attained.

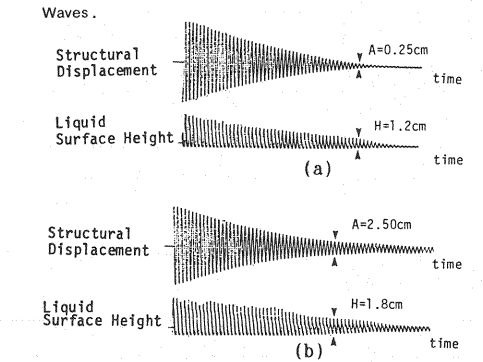


Fig.20 Histories of structural displacement and liquid surface height at one side of TLD.
(a) Case 3 in Fig.19($\gamma=1.0$; $h=1.2$ cm)
(b) Case 4 in Fig.19($\gamma=1.25$; $h=1.8$ cm)

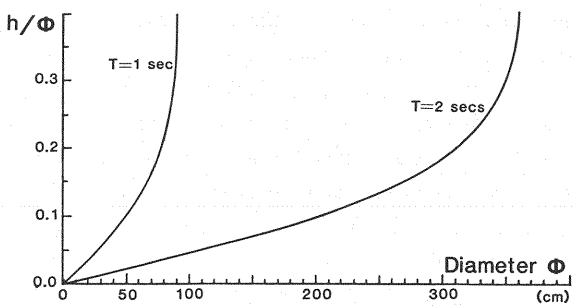


Fig.21 Required depth ratio h/ϕ vs. container diameter ϕ for specified natural period.

For enhancing surface wave development and subsequent wave breaking, it may well be recalled from Sect. 4. (2) and Fig. 11 that, for given liquid density, low viscosity liquid was slightly more effective. From Sect. 4. (4) and Fig. 14 it was also learned that sufficient gap between liquid surface and container roof was favorable to the natural development and possible breaking of high waves.

(2) Choosing the appropriate size of cylinder

Section 4. (6) and Figs. 17 and 18 showed that, for the same liquid mass ratio, the bigger (60 cm) TLD was better since higher peak $\Delta\delta$ was produced.

The advantage of the bigger diameter, for given period of TLD oscillation, may be associated with the manner of liquid motion when it is Type III : two waves develop along half circles and hit together twice per cycle [Fig. 19]. Each time that the waves meet, the energy dissipated is proportional to the velocity of the converging waves. For given frequency of oscillation, this velocity is in turn proportional to the length of the half-circle arc that they traverse every half-cycle.

Other considerations may rule out the use of very large TLD, however. Fig. 21 illustrates that for tuning to given period (or given frequency), there is a wide range of possible cylinder sizes ; each size ϕ has its own corresponding required liquid depth h . The necessary relative liquid depth h/ϕ is not constant, however ; instead it increases rapidly in the range of large ϕ . Very high h/ϕ in very large TLD would not be very efficient, as some liquid near the bottom would tend to be immobile and ineffective. Relative advantage of having shallow liquid was also observed in the study by Welt⁽²¹⁾. As for wave breaking, the deeper liquid would require a higher wave height H , hence a larger structural amplitude A , before energy dissipation by wave breaking could possibly start. For example, the 40 cm TLD at $\gamma=1.0$ required about $A=0.25$ cm [Fig. 20] ; the 60 cm TLD required about $A=0.5$ cm. There may be an optimal size of TLD for given structural period (or frequency) and amplitude of concern.

6. CONCLUSIONS AND RECOMMENDATIONS

An extensive free-oscillation experiment was performed in order to investigate the fundamental properties of Tuned Liquid Damper (TLD). The model was essentially a single-degree-of-freedom structure with attached TLD. The structural period was about 2 sec. The main findings are briefly summarized as follows.

- (1) For the range of structural displacements considered in the experiment ($A=0.25$ cm~8 cm), it appears that wave breaking is a prevalent mechanism of energy dissipation in TLD. Some TLD properties that were exhibited may be correlated with wave breaking.
- (2) Additional damping $\Delta\delta$ is highly dependent on amplitude of vibration A .
- (3) To attain large additional damping specially at small to moderate vibration amplitude, it is necessary to tune the liquid frequency to the natural frequency of the structure. This enhances energy absorption by the TLD, prior to possible energy dissipation by wave breaking. For very large amplitude, TLD is rather insensitive to tuning ; but the additional damping itself is not so large.
- (4) High-viscosity liquid is not necessarily preferable as it may inhibit the development and breaking of high waves.
- (5) Making the cylinder bottom rough by attaching semispheres does not improve TLD effectiveness, when the energy dissipation occurs primarily on the free surface of the liquid as in wave breaking.
- (6) Closed roof has two counteracting effects : allow energy dissipation through impact of liquid, and restrain the development of high waves. The net effect differs only slightly from no-roof condition. A more reliable TLD design is to make the roof sufficiently high.
- (7) When liquid mass ratio is small and tuning parameter is approximately unity, the additional damping is practically linearly proportional to mass ratio or to number of identical TLDs.
- (8) For the given structural period of 2 sec, and within the range of parameters considered in the experiment, the larger TLD is more effective. However, because of changing liquid condition from shallow

to deep as TLD size is increased, for given frequency and amplitude of structure there may be a cylinder diameter that produces optimal amount of additional damping. The subject of optimal TLD size requires further investigation.

Forced harmonic tests at different displacement amplitudes will be useful in verifying the amplitude-dependent additional damping $\Delta\delta$ that has been defined in this study. As for developing a simple mechanical model for the TLD itself, which may then be interfaced and used in the analysis of complicated structure, a more straightforward experiment would be to measure the interface force between TLD and structure^{(14), (15)}. The mass, stiffness, and damping elements of the mechanical model would then be easier to identify although these are expected to depend on displacement amplitude.

Further study is also needed for comparing the present TLD with other shapes, e. g. doughnut-shape⁽⁷⁾, rectangular⁽¹⁰⁻¹³⁾, and spherical⁽⁷⁾, or with dampers completely filled with two immiscible liquids⁽¹⁸⁾. Further experiments in a set-up similar to the one described here, but using other sizes of circular cylinder and also other shapes of container, have shown that simple empirical formulas may be obtained for amplitude-dependent energy dissipation per cycle in TLD regardless of container shape and size⁽²²⁾. Such formulas may be conveniently used in designing practical TLDs for structures with period of approximately 2 sec and with low structural damping.

ACKNOWLEDGMENTS

The authors would like to thank Prof. M. Ito of University of Tokyo, and M. K. Fujii and Mr. T. Sato of Shimizu Corp. for their very helpful comments. Discussions with Dr. M. Isobe of University of Tokyo, on wave theory and hydrodynamics were extremely informative. Thanks are expressed also to Mr. I. Shino and Mr. P. Warnitchai of University of Tokyo for their help in the experiment. This research was supported partially by a Grant-in-Aid from the Japan Ministry of Education, Science, and Culture.

REFERENCES

- 1) Fujita, T. : Earthquake isolation technology for industrial facilities-research, development and applications in Japan, *Bul. New Zealand Natl. Soc. Earthq. Eng.*, Vol. 18, No. 3, pp. 224~249, 1985.
- 2) Tsuji, M. : Review of various mechanical means for suppressing wind-induced vibration of structures, *Japan J. Wind Eng.*, Vol. 20, pp. 81~102, 1984 (In Japanese).
- 3) Alfriend, K. : Partially filled viscous ring nutation damper, *J. Spacecraft*, Vol. 11, No. 7, pp. 456~462, 1974.
- 4) Bhuta, P. G. and Koval, L. R. : A viscous ring damper for a freely precessing satellite, *Intl. J. Mech. Sci.*, Vol. 8, pp. 383~395, 1966.
- 5) Amieux, J. C. and Dureigne, M. : Analytical design of optimal nutation damper, *J. Spacecraft*, Vol. 9, No. 12, pp. 934~936, 1972.
- 6) Watanabe, S. : Methods of vibration reduction, *Proc. Japan Naval. Arch. Soc. Symp.*, pp. 156~179, 1969 (In Japanese).
- 7) Modi, V. J. and Welt, F. : Vibration control using nutation dampers, *Proc. Intl. Conf. Flow Induced Vib.*, England, pp. 369~376, 1987.
- 8) Sato, T. : Tuned sloshing damper, *Japan J. Wind Eng.*, Vol. 32, pp. 67~68, 1987.
- 9) Hashimoto, H. : Future trend and topics of liquid sloshing, *J. Japan Soc. Mech. Engr.*, Vol. 89, No. 810, pp. 512~517, 1986.
- 10) Miyata, T., et al. : Feasibility study on damping of wind-induced vibrations of structures by breaking of sloshing water, *Japan J. Wind Eng.*, Vol. 32, pp. 65~66, 1987 (In Japanese).
- 11) Miyata, T., et al. : Suppression of tower-like structures vibration by damping effect of sloshing water contained, *J. Struct. Eng.*, JSCE, Vol. 34 A, pp. 617~625, March 1988 (In Japanese).
- 12) Noji, T., et al. : Study on a vibration damper system using hydrodynamic force of sloshing (Parts 1, 2, 3), *Proc. Conf. Arch. Inst. Japan*, pp. 867~872, 1987 (In Japanese).
- 13) Matsuura, Y., et al. : On a mean to reduce excited-vibration with the sloshing in a tank, *Proc. Japan Naval. Arch. Soc.*, No. 160, pp. 424~432, 1986 (In Japanese).
- 14) Fujii, K., Sato, T. and Otsuki, T. : Damper using liquid sloshing motion - tuned sloshing damper, *Proc. Conf. Arch. Inst. Japan*, pp. 1483~1484, 1987 (In Japanese).
- 15) Sun, L. M., et al. : Measurement of wave motion and wave-induced force in TLD - An experiment using shaking table, *Proc. 43rd JSCE Annual Meeting*, 2 pp., October 1988.

- 16) Chaiseri, P., Pacheco, B., Fujino, Y. and Sun, L. M. : Damper for bridge tower - utilizing water motion as energy dissipator, Proc. Intl. Conf. Cable-stayed Bridges, Vol.1, pp.545~556, 1987.
- 17) Sayar, B. and Baumgarten, J. : Linear and nonlinear analysis of fluid slosh dampers, Amer. Inst. Aero Astro J., Vol.20, No.11, pp.1534~1538, 1982.
- 18) Bauer, H. : Oscillations of immiscible liquids in a rectangular container : a new damper for excited structures, J. Sound Vib., Vol.93, No.1, pp.117~133, 1984.
- 19) Chaiseri, P. : Damper for tower-like structure utilizing liquid motion as energy dissipator (Masteral thesis), University of Tokyo, 120 pp., Sept. 1987.
- 20) Fujino, Y., Pacheco, B.M., Chaiseri, P. and Fujii, K. : An experimental study on TLD using circular containers, J. Struc. Eng., JSCE, Vol.34 A, pp.603~616, March 1988 (In Japanese).
- 21) Welt, F. : A study of nutation dampers with application to wind-induced oscillations. (Doctoral thesis), University of British Columbia, 204 pp., January 1988.
- 22) Chaiseri, P., et al. : Study of size and shape effects on TLD, Proc. 43rd JSCE Annual Meeting, 2 pp., October 1988.

(Received April 20 1988)