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EFFECTS OF SUPPORT FLEXIBILITY ON MODAL DAMPING OF CABLES

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Effects of support flexibility on the modal damping of flexural oscillation of cables are examined in relation to the dynamic characteristics of cables. The modal damping is experimentally measured using elastically-supported cable model. Finite element method is employed in order to calculate natural frequencies, normal modes and additional dynamic strains. It is found that the flexibility of support has significant effects upon the modal damping of only symmetric modes in the region of modal transition. The experiment indicates that the energy dissipation from support is one of the major sources of damping and that this must be further studied.

Keywords : *cable, dynamic strain, experiment, finite element method, flexible support, modal damping*

1. INTRODUCTION

Cables have been widely used with recent development of high strength cable. Oscillations of cables can occur easily due to wind because of their light weight and low flexibility. Indeed remarkable oscillations, such as galloping and buffeting of cables in transmission lines, telecommunication lines, cable-stayed bridges, and suspension bridges, have been reported frequently and recently the relatively new problem of rain and wind-induced vibration of cable[1)~3)] becomes a serious engineering issue in cable-stayed bridges. Occurrence of this kind of wind-induced oscillation is dependent on the damping of cable itself. The damping mechanism of cable, therefore, is very important to consider suppression of such wind-induced oscillation.

Several studies[4)~10)] have been made on the damping characteristics of cables and wire ropes. Most of them deal either with the first flexural modal damping of taut cables or with the hysteresis damping of wire ropes during axial oscillation. This means that researches have been conducted mainly on the material damping of ropes, while there are very few investigations[9),10)] on the structural damping of sagged cable, that is, the modal damping of flexural oscillation of cables.

The authors have studied, in a previous paper[10)], the characteristics of modal damping in suspended cables based on the free oscillation experiments. Their findings can be summarized as follows ; (1) the in-plane symmetric modal damping is very large when the sag-to-span ratio is around the modal crossover point ; (2) the modal damping of cable is closely related to the additional dynamic strains of normal mode

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and the primary cause of modal damping is the internal damping due to the hysteresis energy of cable ; and (3) the damping of cable is also affected by the initial tension. Only the rigidly-supported cables, however, were treated in Ref. 10) and the damping of elastically-supported cable, which is common in real structures, still remains to be solved. The condition of support seems to be actually very important in cable damping. An example can be seen in the wind tunnel study on the galloping of figure-8 overhead telecommunication cables[11] ; the mode during galloping was different when the different flexibility of end support of telecommunication cable was used.

The primary objective of this paper is to investigate the effects of support flexibility on the modal damping of flexural oscillation in suspended cables. Natural frequencies, normal modes and additional dynamic strains of elastically-supported cables are firstly discussed by means of numerical analysis based on the finite element method. The modal damping characteristics of suspended cables with elastic supports are next measured through free-oscillation experiments under various support conditions and the effects of support flexibility are clarified. Relations between modal damping and normal oscillation are also compared with each other.

2. DYNAMIC CHARACTERISTICS OF ELASTICALLY-SUPPORTED CABLES

Dynamic analysis of elastically-supported cables has been made by Luongo et al.[12] using the finite difference method. They assume that cable is inextensible and only effect of support flexibility on modes and natural frequencies is discussed (additional dynamic strain of each normal mode is not discussed quantitatively). In these respects, their study is not sufficient for clarifying the damping characteristics of elastically-supported cables. In this study, therefore, the free oscillation analysis using the finite element method is made to calculate natural frequencies, normal modes and additional dynamic strains for extensible cables, and the effects of support flexibility are studied on the dynamic characteristics of elastically-supported cables.

(1) Finite element analysis of elastically-supported cables

Several numerical methods[13]~[15] have been proposed to calculate natural frequencies, normal modes and additional dynamic strains of suspended cables. The finite element method[15] is employed in this study since it can easily accommodate the elastic-support condition. The 3-node quadratic element is used for the cable with the shape function of quadratic polynomial. The support of cable is modeled as a spring-mass system which consists of three equivalent springs in the directions of Cartesian coordinates and one equivalent mass (Fig. 1), while the damping at supports is not included in the present analysis.

The static equilibrium configuration due to the self weight was analyzed first, and next the eigenvalue problem was solved by evaluating the tangential stiffness matrix at the static equilibrium state. Using the mode vectors obtained, the additional dynamic strain at the internal node of each element was calculated for a reference amplitude which will be mentioned later. The dynamic strain has a different value for each internal node of element and the root mean square of additional dynamic strain is taken as a representative value for each normal mode. The detailed analytical procedure such as the calculation of dynamic strain is discussed in Ref. 10).

The elastically-supported cables used in the experiment (see 3.) are analyzed. The specifications of cable models and elastic supports are shown in Tables 1 and 2, respectively. The horizontal in-plane spring with spring constant k_1 (Fig. 1) is only considered as the stiffness of support in the analysis and the characteristics of in-plane oscillation are discussed. The numbers of elements and nodes used in the numerical analysis were taken to be 21 and 43, respectively.

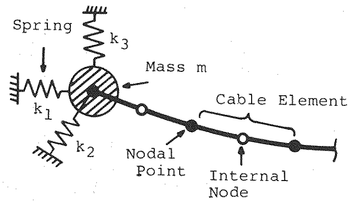

Fig. 1 Analytical models of cable and support.

(2) Natural frequencies and normal modes

Fig. 2 shows the natural frequencies for three support conditions ; Support 1 (more flexible), Support 2 (flexible) and Support 3 (rigid) in Table 2. Four analytical curves for the first symmetric mode (Sym. 1), the first asymmetric mode (Asym. 1), the second symmetric mode (Sym. 2) and the second asymmetric mode (Asym. 2) are presented for each support condition. As can be seen from Fig. 2, there exist modal crossovers[13] in a certain region of sag ratio, where the natural frequency of symmetric mode increases with the transition of symmetric mode into higher symmetric mode. On the other hand the natural frequencies of the asymmetric modes are not influenced by the support condition and decreases monotonically with increasing sag ratio. The modal transition sag-ratio can be recognized but the critical sag-ratio shifts to larger sag-to-span ratios as the support becomes more flexible. The natural frequency of symmetric mode for more flexible support, therefore, decreases for sag-to-span ratios in the modal transition region. As for cables with sag ratios larger than the modal crossover point, there is not so much difference in the natural frequency of symmetric mode. This is because the elastic supports hardly move for the symmetric mode with sag ratios outside the region of the modal transition and also for the asymmetric mode in the linear theory, while the movement of support is relatively large for the symmetric mode in the modal crossover region. Fig. 4 shows the comparison between Support 2 (flexible) and Support 3 (rigid) in the first symmetric and asymmetric mode shapes at the crossover point where the movement of support becomes maximum.

(3) Additional dynamic strain

Fig. 3 shows a plot of calculated dynamic strain $\Delta\varepsilon$ versus sag-to-span ratio γ, comparing three cases of support condition. As can be seen from Fig. 3, the additional dynamic strain of symmetric mode takes large value in the modal transition region and has a maximum at the modal crossover point for each support condition. The maximum dynamic strain of the flexibly-supported cable (Support 1 or Support 2) is much smaller than that of the rigidly-supported cable (Support 3) and the maximum value decreases as the flexibility of support increases. On the other hand, the additional dynamic strains of asymmetric modes for each support condition, which are smaller in comparison with the symmetric modes, lie on the same curves in Fig. 3.

The span-wise distributions of additional dynamic strain for the first two modes are indicated in Fig. 5 as examples corresponding to the normal mode shapes in Fig. 4. The differences of dynamic strain modes are not remarkable between the flexibly-supported cable and the rigidly-supported cable. In Fig. 5, the dynamic strain of the first symmetric mode is distributed almost constantly and that of the first asymmetric

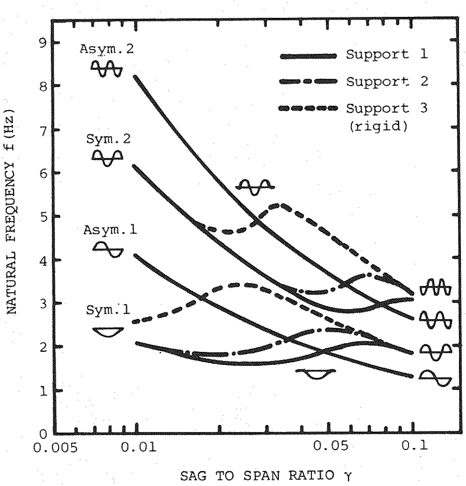

Fig. 2 Natural frequency versus sag-to-span ratio.

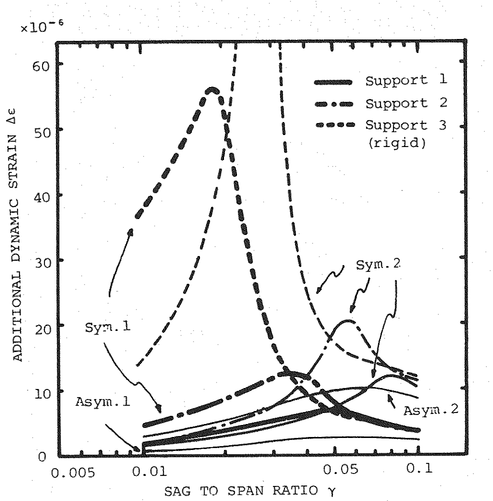

Fig. 3 Additional dynamic strain versus sag-to-span ratio.

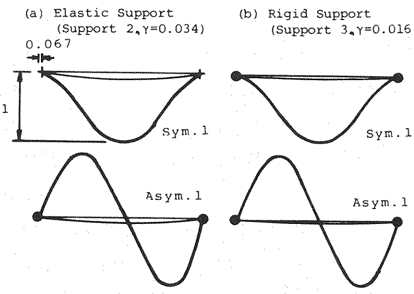

Fig. 4 Comparison of normal mode shapes.

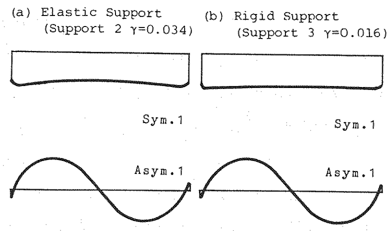

Fig. 5 Modes of additional dynamic strain.

mode has a shape similar to the normal mode shape, while the dynamic strain modes change complicatedly along the span for larger sag ratios and higher modes.

3. FREE OSCILLATION EXPERIMENT OF MODEL CABLE

(1) Models of cable and elastic support

The cable model employed in the experiments is a 7-wire strand rope to which lead weights (15.0 g/weight), available as sinker in the market, were attached at interval of about 9.5 cm distances in order to adjust the cable weight. The details of cable model with lead weight are shown in Fig. 6 and the model specifications are listed in Table 1.

As is shown in Fig. 7, the cable was wound with several turns around a horizontal bar-steel fixed to a thin steel plate, which was connected rigidly to a steel rigid support, and then mounted in the support through a turnbuckle. Each supporting point was set in a same level and the sag of cable was adjusted by the turnbuckle.

Fig. 8 shows the details of elastic support. The elastic support was modeled by a cantilever of 15 cm× 54 cm rectangular plate which was embedded 25 cm from the upper end. At the lower end of plate, the bar-steel was attached horizontally in order to wind the model cable. The plate thickness was changed to achieve a different support condition and three kinds of support flexibility were prepared in this experiment.

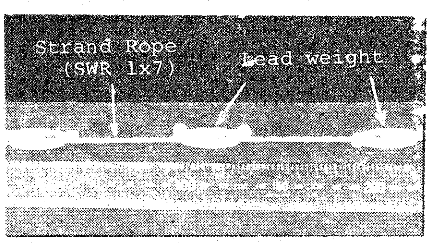

Fig. 6 Details of model cable with lead weight.

Table 1 Cable model specifications.

Type (Dia. of Wire)	Tensile Rigidity EA(kN)	Break Tension T_B(kN)	Mass m(kg/m)	Span Length ℓ(m)	Sag-to-Span Ratio γ
SWR1×7 (0.5mm)	2.40×10^2	2.74	0.17	7.28	0.017 ∼0.1

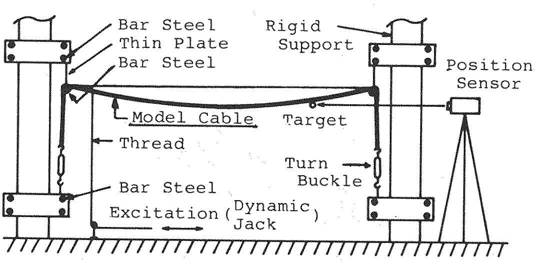

Fig. 7 Schematic diagram for experimental set-up.

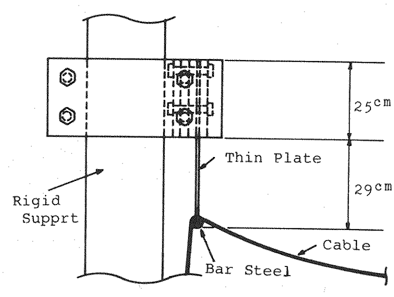

Fig. 8 Details of elastic support.

Table 2 Specifications of elastic supports.

	Plate Thickness (mm)	Equivalent Spring const. k_1 (kN/m)	Generalized Mass for 1st Mode(kg)	Natural Freq. (Hz)	Damping (Log-Decrement)
Support 1	2.0	2.38	0.481	11.2	0.01~0.02
Support 2	2.5	5.99	0.550	16.6	0.02~0.04
Support 3	10.0	1.90×10^2	1.21	63.0	0.06~0.09

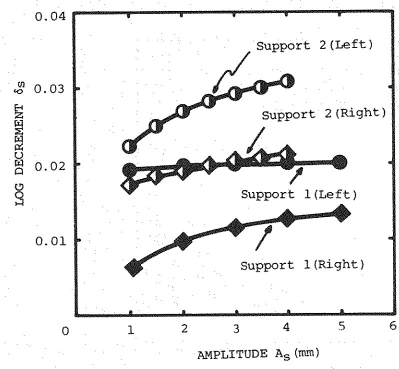

Fig. 9 Damping of support versus amplitude.

The dynamic characteristics of supports were measured directly by performing static and dynamic tests before suspending the cable model. The equivalent spring constant at the bottom of cantilever was first measured by static loading test. The free oscillation test was then conducted to measure natural frequency and modal damping. The results are shown in Table 2, where the generalized mass is estimated from the first natural frequency of the cantilever and the horizontal spring constant k_1. The damping of support, which is the first modal damping of the cantilever, depends upon the amplitude of support as is indicated in Fig. 9. The support damping at each end differed slightly from each other in the experiment (Right and Left in Fig. 9).

The support conditions in the present study are not intended to model real structures and the condition of 'Support 3' is quite close to a fixed condition, that is, a rigid support.

(2) Experimental procedure

The test cable set up with various sag-to-span ratios was forced vertically to oscillate at each natural frequency by using a dynamic jack which was connected to a point of the cable through a thread (Fig. 7). The thread was cut after stationary oscillation was attained, and the subsequent decay of free vibration was recorded. The excitation point was changed such that the mode concerned was purely excited. The dynamic displacement was measured by means of an electro-optical displacement follower (position sensor), a target of which was attached to the cable at the point of the largest amplitude of the mode shape. The signal from the position sensor was digitized and recorded on a personal computer through using a digital dynamic strain meter. The digitized free oscillation records were then processed by the personal computer in order to compute the natural frequency and modal damping (logarithmic decrement). The method of direct reading of analog data recorded on a strip chart pen-recorder was also sometimes used.

The logarithmic decrement depends on the amplitude[10], so the largest amplitude of the mode concerned nearly equal to 0.1% of span length, i.e., 7.5 mm, was adopted as a reference amplitude at which the damping was calculated. This reference amplitude in the present paper is smaller than that in the author's previous experiment[10], because various modal couplings, such as the coupling of in-plane and out-of-plane motions or the superimposition of higher modes in the mode of concern[16], occurred in this experiment due to the geometrical nonlinearity of cable at larger amplitudes than 7.5 mm.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

(1) Natural frequency

Fig. 10 shows a plot of the natural frequency versus sag-to-span ratio for each support condition, comparing the experimental results with the analytical results. The static deformation of support was too large to conduct free oscillation experiment in case of smaller sag ratios of elastically-supported cables; Support 1 and Support 2. The experimental values of the second symmetric mode for large sag

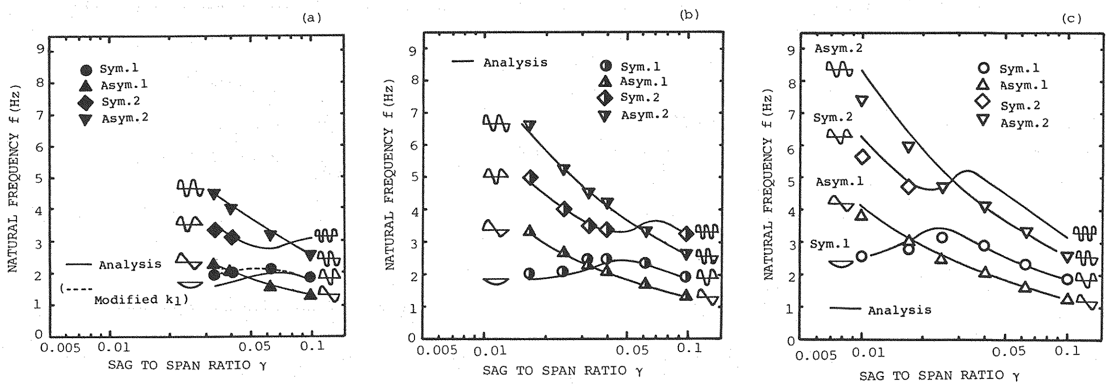

Fig. 10 Natural frequency versus sag-to-span ratio : (a) Support 1; (b) Support 2 ; and (c) Support 3.

ratios is not shown in Fig. 10 because the second symmetric mode was not purely excited in the experiment.

As can be seen from Fig. 10, the experimental values agree fairly well with the theoretical results except for sag ratio of 0.01 in the rigidly-supported cable ; Support 3 (Fig. 10(c)) and for the first symmetric mode of elastically-supported cable ; Support 1 (Fig. 10(a)) in the region of smaller sag ratio. The difference between experimental and theoretical results in Support 3 probably can be attributed to the setting error of such a small sag ratio of 0.01. As for Support 1, the cause for the error is the increase of spring constant of support after the suspension of cable with small sag ratio, which is due to the addition of geometric stiffness of cable to the stiffness of largely deformed support. The dotted line in Fig. 10(a) indicates the analytical curve obtained by using the increased spring constant of support which was measured by means of static experiment of support with cable. The experimental values agree better with this theoretical result.

(2) Modal damping and effects of support flexibility

The experimentally measured values of modal damping for each support condition are compared in Fig. 11 for (a) the first symmetric mode, (b) the second symmetric mode, (c) the first asymmetric mode, and (d) the second asymmetric mode. Our previous study[10] dealing with rigidly-supported cables reveals that the change of modal damping with respect to the sag-to-span ratio have nearly one-to-one correspondence to the change of additional dynamic strain. As for flexibly-supported cables, there is no fundamental difference in this point from rigidly-supported cables, as is indicated in Figs. 3 and 11.

In case of the symmetric modes (Fig. 11 (a) and (b)), the modal damping is large in the modal transition region, and takes a peak for the sag-to-span ratio around the modal crossover point. The damping of second

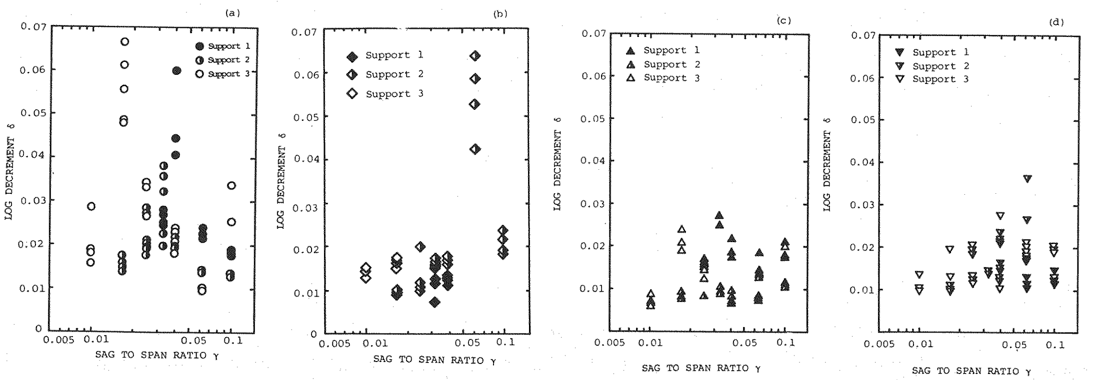

Fig. 11 Effects of support flexibility on modal damping : (a) 1st symmetric mode ; (b) 2nd symmetric mode ; (c) 1st asymmetric mode ; and (d) 2nd asymmetric mode.

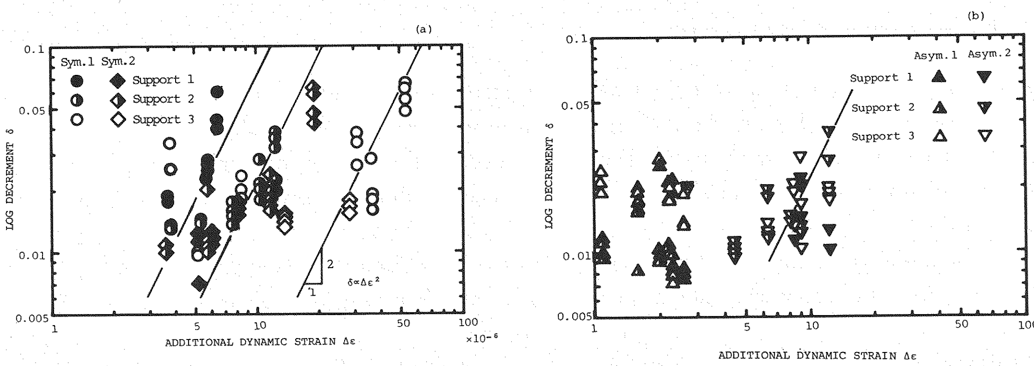

Fig. 12 Log decrement versus additional dynamic strain : (a) Symmetric modes ; and (b) Asymmetric modes.

symmetric mode (Fig. 11 (b)) could not be measured for large sag ratios as stated above but is estimated to be very large by analogy because the sag ratios are just in the modal transition region. The mutual relation of symmetric-mode damping in magnitude for different support condition depends on the sag-to-span ratio, because the modal transition region for more flexible support shifts to the region of larger sag ratio as mentioned in 2. (2). That is, the damping of the symmetric mode in more flexibly-supported cable is smaller for small sag ratios but is larger for large sag ratios. It should be noted on the first symmetric-mode damping in Fig. 11 (a) that the mutual relation of maximum damping value at the modal crossover point for each support condition does not correspond to the mutual relation of maximum dynamic strain in Fig. 3, while the change of modal damping with respect to the sag ratio is very similar to that of dynamic strain.

In the case of asymmetric mode (Fig. 11 (c) and (d)), modal damping is relatively small as compared with the damping of symmetric mode and is not significantly influenced by the flexibility of support. This is because the asymmetric mode of elastically supported cable does not induce movement of support as mentioned in 2. (2). The damping of asymmetric mode increases slightly as the sag-to-span ratio increases and this tendency coincides with the tendency of additional dynamic strain of the asymmetric modes in Fig. 3.

The relation between modal damping and dynamic strain is shown more directly in Fig. 12 where the abscissa is the calculated additional-dynamic-strain and the ordinate is the measured modal-damping both on log scales. In Fig. 12(a), the data points of symmetric mode plotted for each support condition lie roughly in a straight line of slope 2 although there is a some scatterness. This means that the modal damping is in proportion to the square of dynamic strain, i. e., the hysteresis energy and that the internal damping due to the hysteresis energy is one of the primary causes of modal damping of cable. The straight line of slope 2 for different support conditions, however, differs from each other, nevertheless the same cable was used in all the cases in the experiment : the straight line shifts to the left for more flexible support. This results suggests the existence of another cause of modal damping which might be the result of energy loss at the support. That is, the damping at support, which becomes larger for larger movement of support, i. e., for more flexible support, might have direct effects upon the total damping of cable. This effect of energy dissipation from flexible support still remains to be solved quantitatively.

The data points of asymmetric mode plotted in Fig. 12(b) are scattered for all the support conditions, especially for low dynamic-strain, and the value of modal damping is relatively small. This suggests that there might be other sources of modal damping of cable such as the fluid dynamic damping caused by viscous and pressure drag from the air[17].

5. CONCLUDING REMARKS

The dynamic characteristics were analyzed numerically by the finite element method and the modal damping were measured through the free oscillation experiments for elastically-supported cables. The effects of support flexibility on the modal damping of flexural oscillation were then discussed in relation to the dynamic characteristics of cables. The major conclusions obtained through the present investigation are as follows :

(1) The modal transition region of symmetric mode exists regardless of support condition, while the region for more flexible support shifts to the region of larger sag-to-span ratio.

(2) The dynamic characteristics of symmetric mode for sag ratios out of the modal transition region and the dynamic characteristics of asymmetric mode are independent of the support condition, because the normal modes are uncoupled with the movement of support.

(3) The maximum value of additional dynamic strain at the modal crossover point decreases as the flexibility of support increases.

(4) The symmetric-mode damping of a flexibly-supported cable becomes large in its modal transition region, especially for the sag-to-span ratio around the modal crossover point.

(5) The damping of symmetric mode in more flexibly-supported cable is smaller for small sag ratios but is larger for large sag ratios.

(6) There is no significant dependency of modal damping on the flexibility of support in the case of the asymmetric modes.

(7) The modal damping of cable is closely related to the additional dynamic strain of normal mode.

(8) The internal damping due to the hysteresis energy of cable is one of the primary cause of modal damping even in elastically supported cables, while the energy dissipation from support might play an important role on the total damping of the normal mode coupled with the motion of support. The further study on the effects of energy dissipation from elastic support must be conducted next.

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