Structural Eng. / Earthquake Eng. Vol. 5. No. 1, 213s-216s. April 1988 / Discussion-Closure
Japan Society of Civil Engineers (Proc. of JSCE No. 392/I-9)

SEISMIC RESPONSE ANALYSIS OF NONPROPORTIONAL DAMPING SYSTEM DUE TO RESPONSE SPECTRUM METHOD

(*By Yoshikazu YAMADA and Kenji KAWANO*)

Structural Eng. / Earthquake Eng. Vol. 4, No. 1, 213s~222s, April 1987
(Proc. of JSCE No. 380/I-7)

Discussion——*By Yozo FUJINO and Benito PACHECO* (*University of Tokyo*)

A simple evaluation of peak seismic response of *nonproportionally damped system* is indeed a timely topic and the writers read the authors paper with much interest. We have some questions on certain points, however, that we hope the authors may clarify.

(1) When seismic response spectrum, together with the concept of modal superposition, is used in calculating the peak response of a multi-degree-of-freedom (MDOF) system, certain rules of modal combination (other than simple addition or superposition) become necessary because the respective peaks of the different modes generally do not occur at the same instant.

Examples of modal combination rules are Square-Root-of-Sum-of-Squares (SRSS) (Refs. 14 and 16) and Complete Quadratic Combination (CQC) (Refs. 10 and 15). These have probabilistic background or justification, although they are used even in purely deterministic seismic response analysis. SRSS assumes statistical independence among the modal responses.

CQC (Eq. 9) is originally derived from random vibration theory and it considers statistical correlation between modal responses. The crossmodal coefficient given by Eq. 10 or 11, for example, takes into account that correlation between modal excitations is significant for modes with nearly equal frequencies and nearly equal damping ratios. That is, when the power spectrum of the input ground motion is smooth over a wide range of frequencies including those of the closely spaced modes in question. This type of interaction between modes is induced by the common input excitation and therefore may be called *input-dependent modal correlation* (Also see Ref. 19.).

Meanwhile the modal interaction being dealt with in the paper (called *modal coupling*) depends on structural properties *alone*; unlike the modal correlation discussed above, it does *not* depend upon the power spectrum shape of the input motion. Such coupling occurs when undamped modes are used to transform the equations of motion of nonproportionally damped system (Eq. 1) into "modal" equations (Eq. 2). The resulting equations of motion in new generalized coordinates (q in Eq. 2) are coupled through the damping terms, not like in proportionally damped system where the damping matrix itself is naturally diagonalized.

Equations 10 and 11 are meant to take account of input-dependent modal correlation but not modal coupling as defined above (Ref. 10, p. 189; Ref. 9, p. 420; also Ref. 15, p. 189) (Crossmodal coefficients accounting for both input motion and structural properties are treated in Refs. 13 and 18).

From the comparisons made in the present paper, it appears that the effect of input-independent modal coupling is treated interchangably with effect of input-dependent modal correlation. Since these are conceptually different from each other, the writers could not follow the mathematical and/or physical basis of the method proposed by the authors. We would appreciate greatly if the authors would kindly clarify this point.

(2) For the sake of simplicity, the model in Fig. 1 may be changed to a 2-degree-of-freedom system (Fig. 9) before examining the applicability of the proposed method. Some ratios may again be defined (Eq. 15) for ease of comparison.

$$\left.\begin{array}{l} m\equiv M_p/M_s \\ k\equiv K_p/K_s \\ c\equiv C_p/C_s \\ \omega_i^2\equiv K_i/M_i, \quad i=p,\ s \\ \beta_i\equiv C_2/2M_i\omega_i, \quad i=p,\ s \end{array}\right\} \qquad (15)$$

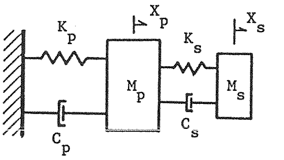

Fig. 9 Simplified version of Fig. 1.

In terms of the quantities defined above, the undamped modes ϕ_j $(j=1,\ 2)$, the associated natural frequencies ω_j, and approximate modal dampings β_j (Eq. 4) can be easily calculated. The important ratios are :

$$\left(\frac{\omega_2}{\omega_1}\right)^2=\frac{(m+k+1)+\sqrt{(m+k+1)^2-4\,mk}}{(m+k+1)-\sqrt{(m+k+1)^2-4\,mk}} \qquad (16)$$

$$\frac{\beta_2}{\beta_1}=\frac{(m+\alpha_1^2)}{(m+\alpha_2^2)}\cdot\frac{(c+1)-2\alpha_2+\alpha_2^2}{(c+1)-2\alpha_1+\alpha_1^2} \qquad (17)$$

$$\alpha_{1,2}=\frac{1}{2}[(-m+k+1)\pm\sqrt{(m+k+1)^2-4\,mk}] \qquad (18)$$

The crossmodal coefficient a_{12}, evaluated using proposed Eq. 12, becomes :

$$a_{12}=\frac{|(c+1)-(\alpha_1+\alpha_2)+\alpha_1\alpha_2|}{\sqrt{[(c+1)-2\alpha_1+\alpha_1^2][(c+1)-2\alpha_2+\alpha_2^2]}} \qquad (19)$$

Fig. 10 plots this crossmodal coefficient versus ratio c, for selected pairs of m and k. Note that large values of c are not unreasonable, because these still mean low damping if the value of β_s is small. When c is close to k, the damping is nearly of the proportional type and therefore the coupling is weak in Eq. 2 (as applied to Fig. 9). On the other hand, when c is either very small or very large compared to k, coupling is very strong.

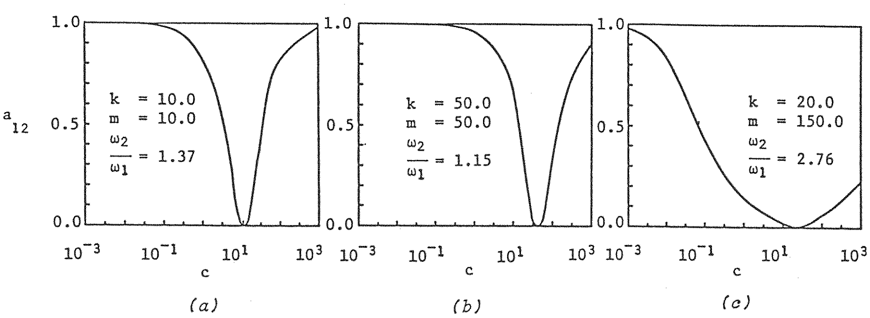

Fig. 10 Crossmodal coefficient a_{12} for selected pairs of k and m.

Fig. 10 clearly shows the sensitivity of a_{12} to the parameter c. In the vicinity of $c=k$, a_{12} approaches zero even when the natural frequencies are close and the corresponding modal damping ratios are nearly equal (Fig. 10 b). That is, when damping is nearly proportional the proposed method reduces to SRSS, not to the more accurate CQC, and becomes unable to take care of correlation between closely spaced modes.

In the other extreme, for values of c that are very far from k, a_{12} approaches 1.0 even when the modal frequencies are well apart (Fig. 10 c). If interpreted as correlation, such a high value for well-separated natural frequencies is seemingly not justified by the results in Refs. 17 and 18 which deal with nonproportionally damped systems.

(3) We are curious how the authors proposed method compares with Refs. 8, 13, and 18, which also propose combination rules for nonproportionally damped systems.

(4) As mentioned above in (1), Eq. 10 by Rosenblueth (Ref. 10) and Eq. 11 by Der Kiureghian (Ref. 9) are derived for proportionally damped system. Moreover they assume relatively small damping. The writers wonder how these and the proposed Eq. 12 would compare when β_p of the system shown in Fig. 1 is smaller than 10 %.

(5) Warburton and Soni's criterion (Eq. 14) is intended as a check whether deletion of the off-diagonal terms of the damping martix (Eq. 4) would be acceptable for practical purposes. The writers are afraid that it may not be fair to conclude from Fig. 3 alone that Eq. 14 "does not always provide good evaluations for the condition of the diagonalization of the nonproportional damping matrix". Looking at Fig. 3, the criterion is safely satisfied—and therefore diagonalization is predicted to be quite acceptable for the range $40 < m < 100$. Nowhere in the paper is this prediction proven wrong.

Despite our questions, we fully appreciate the point that a method like the one proposed would be attractive for being simple and for avoiding complex eigenvalue analysis. We fully agree that no such simple and reliable technique is yet available.

ADDITIONAL REFERENCES

14) Goodman, L., Rosenblueth, E. and Newmark, N. : Aseismic Design of Firmly Founded Elastic Structures, Trans. ASCE, Vol. 120, pp. 782～802, 1955.

15) Wilson, E. L., Der Kiureghian, A. and Bayo, E. P. : A Replacement for the SRSS Method in Seismic Analysis, Earthq. Eng. Struc. Dyn., Vol. 9, pp. 187～194, 1981.

16) Vanmarcke, E. : Structural Response to Earthquakes, *Seismic Risk and Engineering Decisions*, Lomnitz and Rosenblueth (eds.), Elsevier Scientific Publishing Co., Amsterdam, Chapter 8, pp. 287～337, 1976.

17) Igusa, T., Der Kiureghian, A. and Sackman, J. : Modal Decomposition Method for Stationary Response of Non-classically Damped Systems, Earthq. Eng. Struc. Dyn., Vol. 12, pp. 121～136, 1984.

18) Igusa, T. and Der Kiureghian, A. : Response Spectrum Method for Systems with Non-classical Damping, Proc. ASCE-EMD Specialty Conf., West Lafayette, Indiana, May 23-25, 1983, pp. 380～384.

19) Grigoriu, M. : Mean-square Structural Response to Stationary Ground Acceleration, J. Eng. Mech. Div., ASCE, Vol. 107, pp. 969～986, 1981.

(Received June 29 1987)

Closure——*By Yoshikazu YAMADA* (*Kyoto University*) *and Kenji KAWANO* (*Kagoshima University*)

Thank you for your careful consideration of the manuscript. Following further consideration, we would like to comment on your discussion :

(1) The determination and evaluation of natuaral frequencies, critical damping ratios and each vibrational mode play an important role on the seismic response analysis of the response spectrum method. Since the eigenvalue analysis for the undamped system is most convenient and profitable, these results are widely used to analyze the nonproportional damping system. This is why the diagonalization of the nonproportional damping matrix has been studied using the results from the eigenvalue analysis of the undamped system. However, it seems that more exact results could be obtained upon the evaluation of the modal coupling effects on the nonproportional damping system. The modal coupling effects also occur situations of damping and closely spaced natural frequencies. We consider that the effects of the closely spaced natural frequencies are included in the generalized damping term $\tilde{C}_{jk}$ which corresponds to the extent of the natural coupling effects. Thus, it appears that these effects of the response analysis could be evaluated with the weighted coefficient a_{jk} which is represented by the coupling terms of the generalized damping matrix. While there are also the modal coulping effects due to input excitations, it seems that the modal coupling due to the closely spaced natural frequencies and damping characteristics provide significant contributions to the response evaluation of the nonproportional damping system. We think that the evaluation of the modal coupling effects by means of a simplified expression such as the generalized

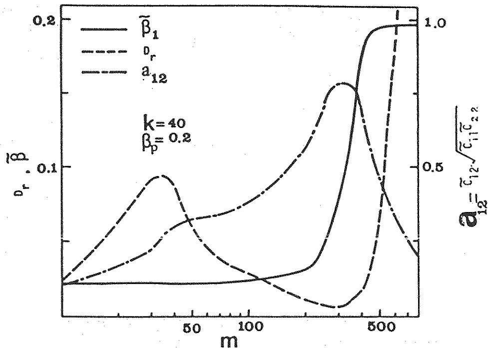

Fig. 12 Evaluations of Diagonalization and Modal Coupling Effects.

damping term $\tilde{C}_{jk}$ meets the requirements of important roles of the seismic response analysis. In this study, it seems that since the seismic response analysis is carried out for the nonproportional damping systems from that point of view, the modal coupling effects are included in contributions which are brought about by input excitations and the dynamic characteristics of the system.

(2) Since there are no modal coupling effects involved in the proportional damping system due to the damping characteristics, as mentioned in your discussion, the proposed method leads to the SRSS method because the generalized damping term becomes zero. Therefore, the modal coupling effects of closely spaced natural frequencies for the proportional damping system cannot be evaluated with this method.

(3) While the studies in references 8, 13, and 18 are applied to the complex eigenvalue analysis approach for the seismic response analysis of the nonproportional damping systems, this study was an attempt to evaluate the modal coupling effects on the response using the classical normal mode approach. Generally, it seems that the more the degrees-of-freedom of the structure increase, the more the complex eigenvalue analysis is difficult to analize. Furthermore, although the approach of reference 8 was examined using the response spectrum method, the appropriate results could not yet be obtained because the modal coupling effects have complex characteristics on the response. Thus, in this study, the evaluation of the modal coupling effects such as Eq. (13) was suggested. Although reference 18 was not considered for the purpose of this study, it appears to provide the more appropriate approach. Since the evaluation of modal coupling effects becomes a somewhat complicated approach owing to the complex eigenvalue analysis, it seems that it would be necessary to examine and to improve for the sake of simplification and application of the response spectrum method.

(4) We think that when the response spectrum method is applied to the seismic response analysis of the nonproportional damping system, the evaluation of the modal coupling effects has a significant role on the response results. The important contributions of the modal coupling effects would be brought about in situations of closely spaced natural frequencies, and it would increase for siuations in which each damping has a very different result. Therefore, in this study the modal coupling effects on the response were not examined because they would provied a slight contribution to the response for situations in which β_p becomes smaller than 0.1.

(5) The coefficient a_{12} represents the modal coupling effects which have important contributions to the responses due to significant vibrational modes. While the coefficient a_{12} increases for the significant modal coupling situation, the coefficient decreases for the slight modal coupling sitation as shown in Fig. 3. and Fig. 12. We think that the coefficient a_{12} provides one of the indicators which gives some estimation of the modal coupling effects on the nonproportional damping system. However, it seems that there is a need for further examination concering a good indicator which provides the exact evaluation of the modal coupling effects on the response using the classical normal mode method.

(Received November 9 1987)