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AN INTERPOLATION FUNCTION METHOD FOR STOCHASTIC FEM ANALYSIS UNDER DYNAMIC LOADS USING FREQUENCY RESPONSE ANALYSIS

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This paper proposes a new method of frequency response analysis of probabilistic structures using FEM to evaluate the stochastic dynamic behavior of structures with uncertain parameters. It presents some numerical examples including verification by Monte-Carlo Simulation (MCS). The frequency transfer function $H(\omega)$ of displacement of a system is expanded into the first-order Taylor series at the mean values of input parameters. Then a function similar to the frequency transfer function of a SDOF system is specified to interpolate each input parameter axis. Using these specific interpolated frequency transfer functions, the mean and standard deviation time history of response can be obtained.

Keywords: *probabilistic structures, finite element method, frequency response analysis*

1. INTRODUCTION

With the remarkable progress of computers and advances in such techniques of numerical analysis as the Finite Element Method (FEM), the Stochastic Finite Element Method (SFEM) has become the focus of attention. As a result, the estimation of stochastic deformation or stress has become possible.

As for stochastic dynamic response analysis of structures with uncertain parameters, there have been several basic investigations[1),2)]. Hisada and Nakagiri[3),4)] proposed stochastic dynamic response analysis by SFEM using the perturbation method. The method, however, may not be viable when the degree of uncertainty of input parameters is large because the perturbation method is valid only when the degree of uncertainty is small and/or simple enough to be approximated, in general, in the first-order (linear). Yamazaki and Shinozuka[5)] evaluated the response variability of a viscoelastic finite element system using a modified Monte-Carlo simulation method which employs the Neumann expansion. This method has a possibility of practicability supported by the advancement of super-computers, although it requires a great amount of computing efforts.

The authors proposed a method of frequency respose analysis of probabilistic structures by SFEM using the perturbation method to evaluate the stochastic dynamic behavior of structures with uncertain parameters[6)]. Results similar to those in the reference 3) and 4) were obtained. That is, the method is valid so far as the degree of uncertainty is small and/or simple enough just as when the viscous damping factor is an only random variable.

This paper proposes a new method of frequency response analysis of probabilistic structures using FEM

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to evaluate the stochastic response of structures with uncertain Young's modulus, unit weight and Poisson's ratio including uncertain viscous damping factor, and presents some numerical examples together with verification by Monte-Carlo Simulation (MCS). The main concept of the proposed method is to obtain the overall mean and variance using the contribution (mean and variance) of each random variable to an imaginary SDOF system (interpolation function), which is computed by frequency response analysis using the frequency transfer functions of a given MDOF system and their derivatives with each random variable. The method is concretely outlined in the following. The frequency transfer function $H(\omega)$ of displacement of a system is expanded into the first-order Taylor series at the mean values of the input parameters. Then a function similar to the frequency transfer function of a SDOF system is specified to interpolate each input parameter axis (a_i) from the values of $H(\omega;0)$ and $\partial H(\omega;0)/\partial a_i$. Using these specific interpolated frequency transfer functions, the mean and standard deviation time history of response can be obtained.

In the following, the summary of analysis is presented in the second chapter (2.). and some numerical examples, including verification by MCS, are introduced in the third chapter (3.), Finally, some concluding remarks obtained in this study are summarized in the last chapter (4.).

2. SUMMARY OF ANALYSIS

(1) Frequency transfer function[7)]

The equation of motion of a system subjected to support point motions is given as follows;

$$\begin{bmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{Y}_A \\ \ddot{Y}_B \end{Bmatrix} + \begin{bmatrix} K^*_{AA} & K^*_{AB} \\ K^*_{BA} & K^*_{BB} \end{bmatrix} \begin{Bmatrix} Y_A \\ Y_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_B \end{Bmatrix} \quad \cdots (1)$$

where $[M]$: the mass matrix,
$[K^*]$: the complex stiffness matrix,
$\{Y_A\}$: the absolute displacement vector of moving points,
$\{Y_B\}$: the absolute displacement vector of fixed points,
$\{P_B\}$: the reaction force vector at fixed points.

From the upper part of eq. (1), the frequency transfer function $\{H(\omega)\}$ is obtained in a linear form as follows;

$$\{H(\omega)\}=[-\omega^2[M_{AA}]+[K^*_{AA}]]^{-1}[-(-\omega^2[M_{AB}]+[K^*_{AB}])\{V\}] \quad \cdots (2)$$

where $\{V\}$: the indicating vector of the direction of input.

(2) First-order expansion of the frequency transfer function with normalized random variables

Assume that Young's modulus E is an only random variable (the mean and the variance are M and σ, respectively) in a system. Then through the transformation of variable in the following,

$$a=\frac{E-M}{\sigma} \quad \cdots (3)$$

the random variable a becomes a normalized random variable of which the mean and the variance are 0 and 1, respectively. The frequency transfer function $H(\omega)$ is expanded into the first-order approximation at the mean point M as follows;

$$H(\omega, E)=H(\omega, M)+\frac{\partial H}{\partial E}(E-M) \quad \cdots (4)$$

Rewriting eq. (4) by a, we obtain the first-order approximation of $H(\omega)$ at the mean point $a=0$ as follows;

$$H(\omega, a)=H(\omega, 0)+\frac{\partial H}{\partial a}a \quad \cdots (5)$$

where

$$H(\omega, 0)=H(\omega, M), \quad \frac{\partial H}{\partial a}=\frac{\partial H}{\partial E}\sigma$$

In a simple expansion, a general expression of eq. (5) can be obtained in the following. Let the total number of elements be N. Then the total number of random variables is $4N$ (Young's modulus E, Poisson's ratio ν, the unit mass ρ and the viscous damping factor h are all random variables), and $H(\omega)$ is expanded into the first-order approximation at the mean point $\{a_i\}=\{0\}$ as follows ;

$$H(\omega, a)=H(\omega, 0)+\sum_{i=1}^{4N}\left.\frac{\partial H(\omega,0)}{\partial a_i}\right|_{a_i=0} a_i \quad (6)$$

(3) Interpolation function of the frequency transfer function

A general functional form of the frequency transfer function implies that we can not expect a highly accurate approximation by eq. (6). In the general frequency response analysis, the frequency transfer function of a total system is interpolated along a frequency axis by using the frequency transfer function of a SDOF system or a 2-DOF system. Then this method is naturally introduced into and applied to the present method in which the frequency transfer functions are interpolated along random variable axes such as Young's modulus or the unit mass. Hence, consider a $(4N+1)$ dimensional hyper-plane in ω and a_i axis $(i=1\cdots 4N)$. Then a function similar to the frequency transfer function of a SDOF system which is given by the following eq. (7) is introduced for interpolation of the function on the a_i-axis.

$$H(\omega)=\frac{-m}{k-m\omega^2+2hki} \quad (7)$$

Refering to eq. (7), the following functional forms (complex fractional functions) are employed for Young's modulus and the unit mass as interpolation functions for the frequency transfer function with each random variable. For the damping factor, a linear function is employed because it has not a serious effect on natural frequencies of a system. For Poisson's ratio, a linear function is inevitably employed because no functional form is available for a spring-mass model, where c_1, c_2 and c_3 are complex constants. Namely,

For Young's modulus ;

$$H(\omega, a_i)=\frac{1}{c_1+c_2a_i} \quad (8)$$

For the unit mass ;

$$H(\omega, a_i)=\frac{c_3+a_i}{c_1+c_2a_i} \quad (9)$$

For the viscous damping factor ;

$$H(\omega, a_i)=c_1+c_2a_i \quad (10)$$

For Poisson's ratio ;

$$H(\omega, a_i)=\frac{c_3+a_i}{c_1+c_2a_i} \quad (11)$$

c_1, c_2 and c_3 can all be computed from the values of $H(\omega, 0)$ and $\partial H(\omega, 0)/\partial a_i$. For example, in the case of the unit mass, c_1, c_2 and c_3 can be determined with the condition that c_2 and c_3 are both real constants as follows ;

$$c_1=\frac{1}{A}a_2(a_1-a_2i),\ c_2=\frac{1}{A}(a_2b_1-a_1b_2),\ c_3=\frac{1}{A}a_2(a_1^2+a_2^2) \quad (12)$$

where i : an imaginary unit

$A=a_1(a_2b_1-a_1b_2)+a_2(a_1b_1+a_2b_2)$

$a_1=Re[H(\omega, 0)]$, $a_2=Im[H(\omega, 0)]$, $b_1=Re[\partial H(\omega, 0)/\partial a_i]$, $b_2=Im[\partial H(\omega, 0)/\partial a_i]$

(4) Statistical value of response

The main objective of this study is not to obtain approximation of response, but to obtain statistical value of response. Hence the following direct method to obtain the moments of each order of random variables is introduced[8].

The first-order moment (the expectation) is obtained as follows ;

$$E[y]=\int_{-\infty}^{\infty} f(x)\,p_x(x)\,dx \quad (13)$$

In the same manner, the second-order moment (the variance) is obtained as follows ;

$$Var[y]=\int_{-\infty}^{\infty}\{f(x)-E[y]\}^2 p_X(x)\,dx \quad (14)$$

Other moments of further order are also obtained in the same manner.

(5) Statistical value of time history response

Denoting the Fourier spectrum of input by $F(\omega)$ and the frequency transfer function by $H(\omega)$, then the response $g(t)$ is obtained by the Fourier inverse transformation as follows ;

$$g(t)=\int_{-\infty}^{\infty} F(\omega)H(\omega)e^{i\omega t}dt \quad (15)$$

Considering that $g(t)$ is a function of a_i, that is,

$$g(a_i;t)=\int_{-\infty}^{\infty} F(\omega)H(a_i;\omega)e^{i\omega t}dt \quad (16)$$

the overall mean $m_T(t)$ and the overall variance $\sigma_T^2(t)$ are theoretically obtained by the following eq. (17) and eq. (18), respectively.

$$m_T(t)=E[g(t)]=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} g(a_1,\cdots,a_n;t)f_{A_1\cdots A_n}(a_1,\cdots,a_n)\,da_1\cdots da_n \quad (17)$$

$$\sigma_T^2(t)=Var[g(t)]$$
$$=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\{g(a_1,\cdots,a_n,;t)-E[g(t)]\}^2 f_{A_1\cdots A_n}(a_1,\cdots,a_n)\,da_1\cdots da_n \quad (18)$$

It is, however, quite difficult to compute eq. (17) and eq. (18) because the statistical behavior of each random variable is discussed only along with each axis in this study.

On the other hand, the mean $m_i(t)$ and the variance $\sigma_i^2(t)$ with each random variable a_i can be obtained by the afore-mentioned method as follows ;

$$m_i(t)=E[g_i(t)]=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} g(0,\cdots,0,a_i,0,\cdots,0;t)f_{A_1\cdots A_n}(a_1,\cdots,a_n)\,da_1\cdots da_n$$
$$=\int_{-\infty}^{\infty} g(a_i;t)\,f_{A_i}(a)\,da \quad (19)$$

where $f_{A_i}(a)$: the probability density function of standard normal distribution, that is,

$$f_{A_i}(a)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{a^2}{2}\right)$$

In the same manner,

$$\sigma_i^2(t)=Var[g_i(t)]=\int_{-\infty}^{\infty}\{g(a_i;t)-E[g_i(t)]\}^2 f_{A_i}(a)\,da \quad (20)$$

And it is supposed that the overall mean and the overall variance are obtained in a simple way, which needs to be verified by MCS, as follows ;

$$m_T(t)=E[g(t)]=\frac{1}{4N}\sum_{i=1}^{4N} m_i(t) \quad (21)$$

$$\sigma_T^2(t)=Var[g(t)]=\frac{1}{4N}\sum_{i=1}^{4N}\sum_{j=1}^{4N}\sigma_i(t)\sigma_j(t)E[a_ia_j] \quad (22)$$

As for the evaluation of correlation, the overall variance can not be obtained as a sum of variance of each random variable using their correlation coefficients because the frequency transfer function which determines dynamic behaviors of a system has a strong non-linearity with the change of Young's modulus and the unit mass. So it is inferred that the overall variance with a correlational effect can be obtained by using a specific interpolation function corresponding to a correlation coefficient. Then based on this inference the following method is intuitively employed. The first-order derivative $\partial H(\omega;0)/\partial a_i$ in eq. (6) is changed into the modified derivative $\partial H^M(\omega;0)/\partial a_i$ to consider the response behavior due to correlation as follows ;

$$\frac{\partial H^M(\omega;0)}{\partial a_i}=\frac{\partial H(\omega;0)}{\partial a_i}+\sum_{j=1(j\neq i)}^{4N}\frac{\partial H(\omega;0)}{\partial a_j}E[a_ia_j] \quad (23)$$

Instead of using eq. (22) with the real value of $E\ [a_i a_j]$, the response behavior due to correlation is evaluated by using eq. (23) to specify the interpolation function and eq. (22) with $E\ [a_i a_j]=\delta_{ij}$, which means that a_i and a_j are independent if $i \neq j$.

3. NUMERICAL EXAMPLES

(1) Verification by Monte-Carlo simulation method

a) A SDOF system

First of all, to verify the basic validity of the proposed method, the response of a SDOF system subjected to the El-Centro accelerogram is computed and compared with the result of Monte-Carlo Simulation (MCS). Input data is summarized in Table 1. Fig. 1 shows the deterministic response corresponding to the mean value of the input data. Fig. 2 shows the acceleration response by MCS. Fig. 3 shows another result of the MCS method in which random variables, k and m, vary separately (this MCS method is named the second MCS in this study). Fig. 4 shows the acceleration response by the proposed method. The rigid line and the broken line mean the mean response and $\pm\sigma$-response (σ is the standard deviation), respectively. Fig. 5 shows their σ-responses.

Fig. 4 shows a satisfactory agreement with Fig. 2 and Fig. 3 and these results show the possibility of the proposed method to analyze stochastic response of structures with uncertain parameters.

b) A 2-DOF system

Next, to examine the applicability of the method to a MDOF system, the response of a 2-DOF system

Table 1 Input data for a SDOF system.

the spring constant	k	2.5×10^5 *kgf/cm* (2.45×10^8 *N/m*)
the mass	m	6.25×10^3 *kgf·sec²/cm⁴* (6.125×10^6 *kg*)
the damping factor	h	0.05
the input earthquake		El-Centro accelerogram(*N-S*)
the time increment	dt	0.02(*sec*)
the duration of input		10.00(*sec*)
the duration of analysis	T	15.00(*sec*)
the coef. of variation	δ_k	0.2
the coef. of variation	δ_m	0.2
trials of M-C simulation	N	400

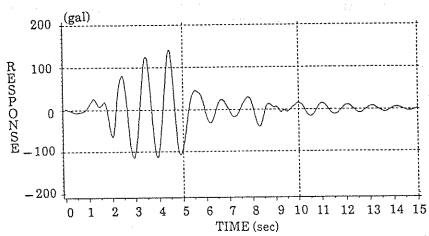


Fig. 1 Deterministic response of the SDOF system.

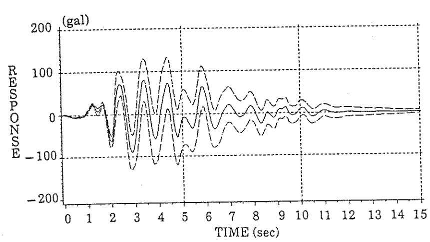


Fig. 2 Mean and $\pm\sigma$-response by MCS.

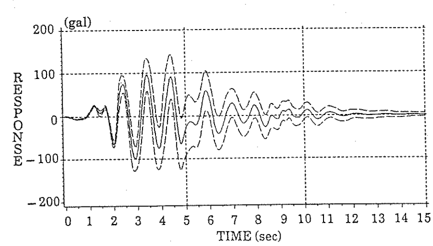


Fig. 3 Mean and $\pm\sigma$-response by the second MCS.

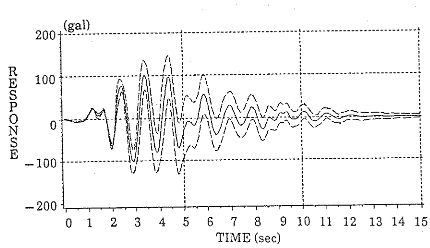


Fig. 4 Mean and $\pm\sigma$-response by the present method.

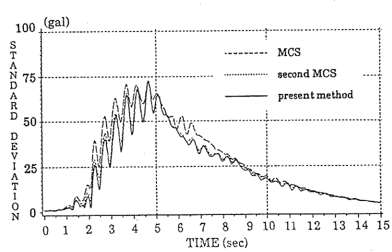


Fig. 5 σ-response by each method.

subjected to the El-Centro accelerogram is computed and compared with the result of MCS just as in the case of a SDOF system. Input data is summarized in Table 2. Fig. 6 shows the deterministic response corresponding to the mean value of the input data. Fig. 7 shows the acceleration response by MCS. Fig. 8 shows the acceleration response by the second MCS. Fig. 9 shows the acceleration response by the proposed method. The rigid and broken line correspond to the mean and σ-response, respectively. Fig. 10 shows each σ-response.

Fig. 9 shows a quite good agreement with Fig. 8 because in present analytical method, it is assumed that random variables change along with each axis just as they do in the second MCS. These results imply that the present method is satisfactory to approximate the behavior of each random variable even if the system is a MDOF system. On the other hand, Fig. 7 shows some difference from Fig. 8 and Fig. 9. This is because the second MCS and the present method can only evaluate the behavior of each random variable along with their axis, and the effect of intersectional area of any pair of random variables is not considered. However, it is shown that if each random variable has a linear effect on the stochastic response, the value of the standard deviation by the second MCS and the present method becomes $1/\sqrt{n}$ of the real value, where n is the total number of a random variable's axis. In this case, n is 4 and the value of the standard deviation of the two should be multiplied by 2.0 $(=\sqrt{4})$ if the spring constant k and unit mass m had a linear effect on the response. The ratio of the maximum value between the two is about 1.3~1.8. Refering to the result of a SDOF system and the present result, n should be reduced by 1 if k and m are

Table 2 Input data for a 2-DOF system.

the spring constant	k_1	5.0×10^5 *kgf/cm* (4.9×10^8 *N/m*)
the mass	m_1	6.25×10^3 *kgf·sec²/cm⁴* (6.125×10^6 *kg*)
the spring constant	k_2	5.0×10^5 *kgf/cm* (4.9×10^8 *N/m*)
the mass	m_2	6.25×10^3 *kgf·sec²/cm⁴* (6.125×10^6 *kg*)
the damping factor	h_1	0.05
the damping factor	h_2	0.05
the input earthquake		El-Centro accelerogram(*N-S*)
the time increment	dt	0.02(*sec*)
the duration of input		10.00(*sec*)
the duration of analysis	T	15.00(*sec*)
the coef. of variation	δ_k	0.2
the coef. of variation	δ_m	0.2
trials of M-C simulation	N	400

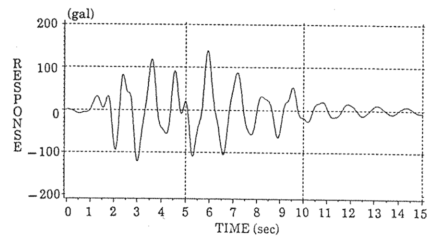


Fig. 6 Deterministic response of the 2-DOF system.

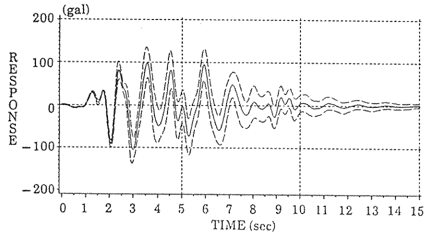


Fig. 7 Mean and $\pm\sigma$-respoose by MCS.

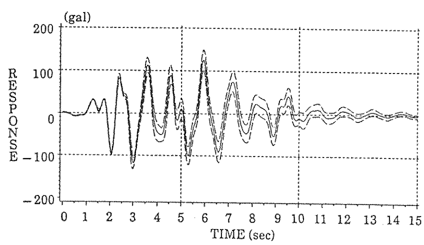


Fig. 8 Mean and $\pm\sigma$-response by the second MCS.

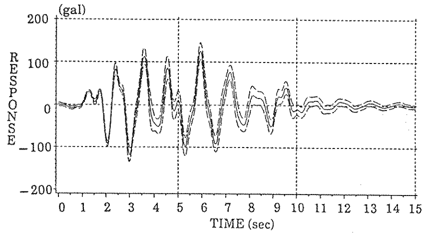


Fig. 9 Mean and $\pm\sigma$-response by the present method.

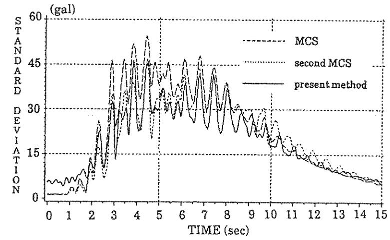


Fig. 10 σ-response by each method.

both random variables simultaneously. Then n becomes 2 in this case. **Fig. 10** shows their σ-responses, two of which are multiplied by $\sqrt{2}$. These results show that it is possible to apply the proposed method to a MDOF system.

(2) Basic examination by finite element method

a) Examination of correlation and mesh division

In this section, four different finite element models of a 2-dimensional area (10 m×10 m) are computed by the proposed method and compared with each other to examine and approve the validity of the method including the examination of correlation and mesh division. The first model is a 1-element model ; the second model is a 4-element model ; the third model is a 16-element model ; and the fourth model is a 100-element model. Input data are summarized in **Table 3**. Each analytical model is shown in **Fig. 11**.

The deterministic response of a left-edge-surface point (shown by an arrow-mark in each figure) in the 100-element model is shown in **Fig. 12**. **Fig. 13** shows the σ-response of the x-directional acceleration response of the point. The correlation coefficients between any pair of elements are set at 1.0 in every model, so it is expected theoretically that the results of each model will show good agreement. The maximum values and their occurrence times are summarized in **Table 4** including their computing time (by HITAC M-280H).

These results show good agreement with each other and there seem to be no serious problems in the present method concerning correlation and mesh division. So it is possible to apply the proposed method to a practical model.

b) Verification by Monte-Carlo simulation

Several cases of MCS are performed to verify the validity of the proposed method. They are summarized

Table 3 Input data for FEM models.

the Young's modulus	E	2.1×10^3 kgf/cm^2 ($2.058\times10^8 N/m^2$)
the Poisson's ratio	ν	0.3
the unit weight	w	2.5×10^{-3} kgf/cm^3 ($2.45\times10^4 N/m^3$)
the damping factor	h	0.05
the input earthquake		El-Centro accelerogram(N-S)
the time increment	dt	0.02(sec)
the duration of input		10.00(sec)
the duration of analysis	T	15.00(sec)
the coef. of variation	δ_E	0.2
the coef. of variation	δ_ν	0.2
the coef. of variation	δ_w	0.2
the coef. of variation	δ_h	0.2
the correlation coef.	ρ	1.0

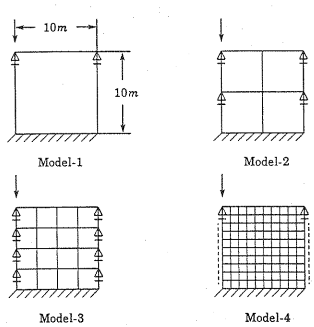


Fig. 11 Each analytical model.

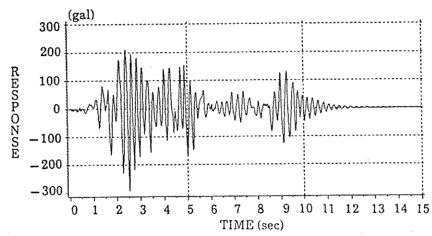


Fig. 12 Deterministic response.

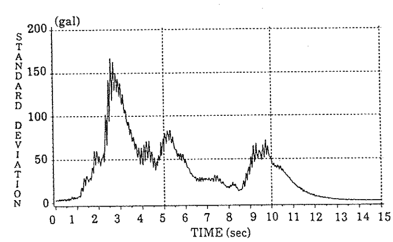


Fig. 13 σ-response of the 100-element model.

Table 4 Maximum mean and σ-response.

model	r.v.	δ	mean/σ max.	CPU	CPU/el.
1-element FEM	E, ν ρ, h	0.2	−235.18(2.54) 86.49(2.60)	4.0(sec)	4.0(sec)
4-element FEM	E, ν ρ, h	0.2	−267.46(2.52) 92.19(2.58)	7.9	1.98
16-element FEM	E, ν ρ, h	0.2	−262.15(2.52) 89.74(2.58)	24.2	1.51
100element FEM	E, ν ρ, h	0.2	−273.59(2.52) 96.42(2.58)	163.3	1.63

in Table 5. The σ-response by MCS in each case are summarized in Table 6 and the corresponding results by the present method are also summarized in Table 6.

These results show a tendency similar to the results of a 2-DOF system. The value in the blanket in Table 6 is $\sqrt{n}$ times the value of the original one. These modified values agree well with the MCS values. To examine the effect of each random variable, those σ-responses in the 1-element model are shown in

Table 5 A list of analytical cases.

case	model	r.v.	mat.	trials
1	1-element	E, ν, ρ, h	1	100
2	4-element	E, ν, ρ, h	1	100
3	16-element	E, ν, ρ, h	1	60
4	4-element	E, ν, ρ, h	2	100
5	4-element	E	2	100
6	4-element	ρ	2	100
7	4-element	E, ρ	2	100

Table 6 Maximum σ-response.

case	MCS	n	present method
1	130.05 *gal*(2.60 *sec*)	3	86.49(2.60)[149.81]
2	145.56(2.58)	3	92.19(2.58)[159.63]
3	164.06(2.70)	3	89.74(2.58)[155.43]
4	165.52(2.58)	6	72.93(2.58)[178.64]
5	144.94(2.60)	8	50.19(2.82)[141.96]
6	156.84(2.84)	8	51.48(2.58)[145.61]
7	170.16(2.58)	6	71.54(2.58)[175.24]

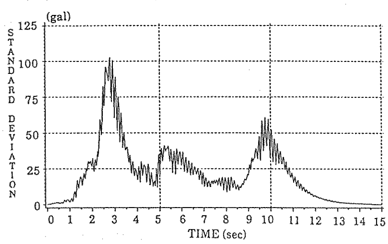


Fig. 14 σ-response by MCS($r.v.$: E).

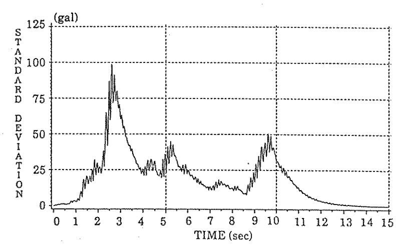


Fig. 15 σ-response by the present method ($r.v.$: E).

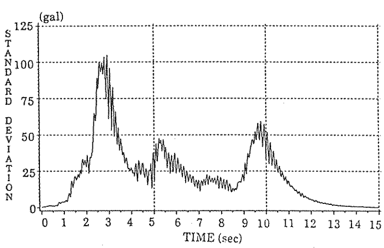
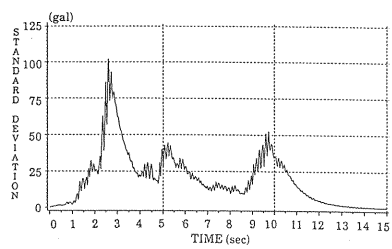


Fig. 16 σ-response by MCS($r.v.$: ρ).



Fig, 17 σ-response by the present method ($r.v.$: ρ).

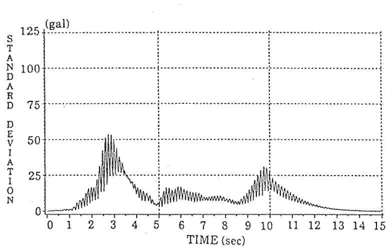


Fig. 18 σ-response by MCS($r.v.$: ν).

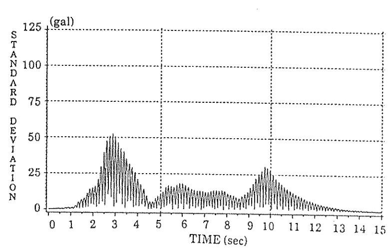


Fig. 19 σ-response by the present method($r.v.$: ν).

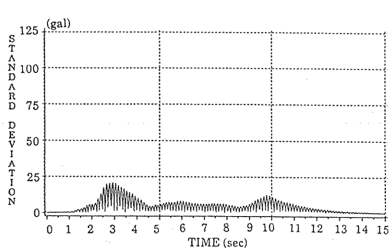


Fig. 20 σ-response by MCS($r.v.$: h).

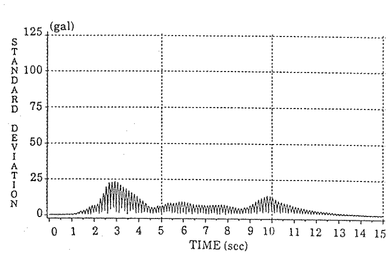


Fig. 21 σ-response by the present method($r.v.$: h).

Fig. 14 through Fig. 21, and the maximum value of their mean and σ-response are summarized in Table 7 (1-element model) and in Table 8 (4-element model).

These results show that the present method agrees well with MCS as long as Young's modulus and unit weight are not random variable simultaneously even if the system has many degrees of freedom and/or several different materials. Even in case of Young's modulus and unit weight being both random variables, the statistical value of response could be obtained with good accuracy by using the afore-mentioned modification factor $\sqrt{n}$. This fact implies that the effect of their simultaneous randomness may not be so serious because they behave together as a dominant frequency $\omega=\sqrt{k/m}$ except for inertia forces.

(3) Numerical example of a practical model

Table 7 1-axis behavior in the 1-element model (Maximum mean and σ-response).

r.v.	δ	MCS	second MCS	present method
E, ν, ρ, h	0.2	−173.43(2.28) 130.05(2.60)	−236.50(2.54) 85.51(2.60)	−235.18(2.54) 86.49(2.60)
E	0.2	−253.67(2.54) 60.61(2.92)		−252.98(2.54) 57.05(2.60)
ν	0.2	−268.82(2.54) 30.90(2.72)		−271.66(2.54) 30.26(2.86)
ρ	0.2	−256.58(2.54) 59.33(2.92)		−253.44(2.54) 58.86(2.60)
h	0.2	−272.41(2.54) 12.11(2.92)		−272.09(2.54) 13.49(2.92)

Table 8 1-axis behavior in the 4-element model (Maximum mean and σ-response).

r.v.	δ	MCS	second MCS	present method
E, ν, ρ, h	0.2	−219.95(2.52) 145.36(2.58)	−272.77(2.52) 93.26(2.68)	−267.46(2.52) 92.19(2.58)
E	0.2	−279.71(2.52) 69.08(3.10)		−278.18(2.52) 61.97(2.70)
ν	0.2	−290.33(2.52) 30.07(3.04)		−290.85(2.52) 31.47(3.04)
ρ	0.2	−284.11(2.52) 60.50(2.68)		−280.14(2.52) 65.79(2.58)
h	0.2	−291.16(2.52) 15.98(3.20)		−290.83(2.52) 14.51(3.08)

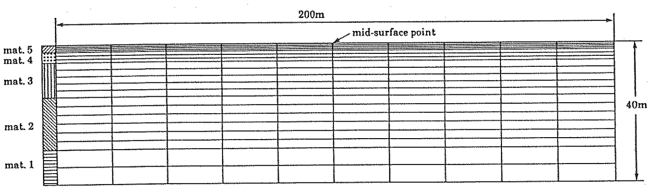


Fig. 22 An analytical model.

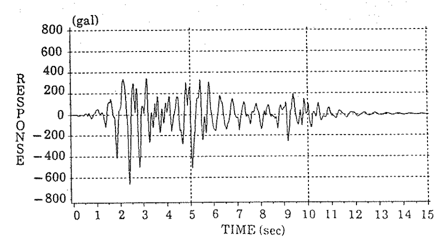


Fig. 23 Deterministic response.

Table 9 Input data for a practical model.

the Young's modulus E_1	2.0×10^4 kgf/cm^2(1.96×10^9 N/m^2)	the Poisson's ratio ν_1	0.30	the input earthquake	El-Centro accelerogram(N-S)
E_2	1.0×10^4 (9.8×10^8)	ν_2	0.35	the time increment dt	0.02(sec)
E_3	2.0×10^3 (1.96×10^8)	ν_3	0.40	the duration of input	10.00(sec)
E_4	1.0×10^3 (9.8×10^7)	ν_4	0.40	the duration of analysis T	15.00(sec)
E_5	2.0×10^2 (1.96×10^7)	ν_5	0.45	the coef. of variation δ_E	0.2
the unit weight w_1	2.5×10^{-3} kgf/cm^3(2.45×10^4 N/m^3)	the damping factor h_1	0.03	the coef. of variation δ_ν	0.2
w_2	2.5×10^{-3} (2.45×10^4)	h_2	0.04	the coef. of variation δ_w	0.2
w_3	2.5×10^{-3} (2.45×10^4)	h_3	0.05	the coef. of variation δ_h	0.2
w_4	2.5×10^{-3} (2.45×10^4)	h_4	0.07	the correlation model	$\rho(dx,dy) = exp[-\{(\alpha\cdot dx)^2+(\beta\cdot dy)^2\}^{\frac{1}{2}}]$
w_5	2.5×10^{-3} (2.45×10^4)	h_5	0.10		

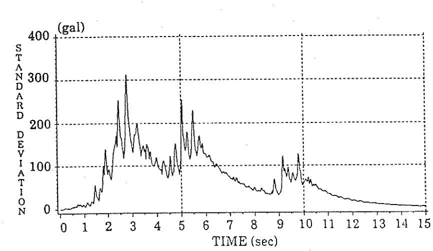


Fig. 24 σ-response by MCS.

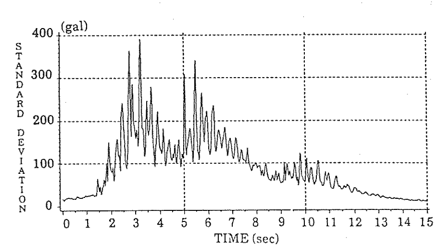


Fig. 25 σ-response by the present method.

Table 10 Results of the practical model.

model	r.v.	δ	mean/σ max.	remarks
10-element FEM	E, ν ρ, h	0.2	−560.81(2.40) 311.91(2.78)	MCS($N=100$)
200element FEM	E, ν ρ, h	0.2	−655.76(2.38) 101.32(3.20)[392.41]	perfectly correlated ($n=15$)
200element FEM	E, ν h	0.2	−656.92(2.38) 74.02(3.20)[331.03]	perfectly correlated ($n=20$)
200element FEM	ν ρ, h	0.2	−658.52(2.38) 73.15(3.20)[327.14]	perfectly correlated ($n=20$)
200element FEM	E, ν ρ, h	0.2	−658.49(2.38) 69.93(3.20)[270.84]	exponentially correlated($n=15$) ($\alpha=\beta=10m$)
200element FEM	E, ν ρ, h	0.2	−658.51(2.38) 69.33(3.20)[268.51]	exponentially correlated($n=15$) ($\alpha=10m, \beta=1m$)

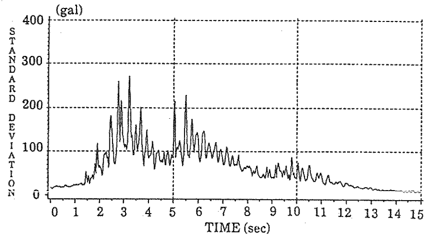


Fig. 26 σ-response with the exponential correlation (α=10 m, β=10 m).

The analytical model shown in Fig. 22 is composed of 200 elements and 231 nodes. The mesh layout is set so simple that the result can be checked by MCS of a small FEM model. Input data are summarized in Table 9. The deterministic response is shown in Fig. 23.

The σ-response at the mid-surface point (shown in Fig. 22) by MCS is shown in Fig. 24 and the corresponding σ-response by the present method is shown in Fig. 25. Other examinations are also performed and those results are summarized in Table 10. These results show a tendendency similar to the above numerical examples. Finally, a practical FEM model with an exponential correlation which is specified by the reduction constants of correlation, α and β, is analyzed by the present method and also summarized in Table 10. The σ-response of the point is shown in Fig. 26.

4. CONCLUSIONS

A new method of frequency response analysis of probabilistic structures using FEM to evaluate the stochastic dynamic behavior of the structures with uncertain parameters is presented. Through several numerical examples, the following conclusions were obtained.

(1) The stochastic response of structures with uncertain parameters can be analyzed quantitatively with a high accuracy by the proposed method when Young's modulus and the unit mass are not random variables simultaneously.

(2) In general, Young's modulus and the unit mass affect the stochastic response to a similar extent. Poisson's ratio and the viscous damping factor affect less. This can be quantitatively examined using the present method.

(3) The present method is available when Young's modulus and the unit mass are both random variables using the modification factor proposed in this paper.

By using the present method, other numerical examinations are possible. Other informative remarks from the practical point of view, such as design and construction, are also possible and research on this will be reported in the near future.

REFERENCES

1) Bellman, R. E. : Perturbation Techniques in Mathematics, Physics and Engineering, Holt, New York, 1964.
2) Hoshiya, M. and Shah, H. C. : Free Vibration of Stochastic Beam-Column, EM. Div., ASCE, Aug., 1971.
3) Hisada, T. and Nakagiri, S. : Stochastic Finite Element Method Developed for Structural Safety and Reliability, Proc. 3rd ICOSSAR, June, 1981, p. 395.
4) Hisada, T. and Nakagiri, S. : Role of the Stochastic Finite Element Method in Structural Safety and Reliability, Proc. 4th ICOSSAR, May, 1985, I-385.
5) Yamazaki, F. and Shinozuka, M., : Response Variability of Stochastic Finite Element System, Proc. of the 7th Japan Earthquake Engineering Symposium, pp. 1447~1452, 1986.
6) Tanaka, Y., Ukon, H. and Matsumoto, T. : Frequency Response Analysis by Stochastic Finite Element Method, Proc. of the

42nd Annual Conference of the Japan Society of Civil Engineers, I-269, 1987.

7) Clough, R. W. and Penzien, J. : Dynamics of Structures, McGraw-Hill, Inc., New York, 1975.

8) Ang, A. H-S. and Tang, W. H. : Probability Concepts in Engineering and Design Volume-I Basic Principles, John Wiley & Sons, Inc., New York, 1975.

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