

ON MINIMUM WEIGHT DESIGN OF PEDESTRIAN BRIDGES TAKING VIBRATION SERVICEABILITY INTO CONSIDERATION

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Minimum weight design method of pedestrian bridges taking vibration serviceability into consideration is studied. To shorten the computational time for structural analysis, an approximation method of vibration serviceability analysis is studied and applied successfully. Two-level optimization technique is also studied. In structural level optimization, design variables are reduced to the geometrical moments of inertia from the sectional sizes of primal problem. Member level optimization is formulated as an optimization problem easy to solve and the way to find the optimal design is presented for both continuous and discrete plate thickness. Simple, two-continuous and three-continuous pedestrian bridges are designed by the proposed method and the efficiency and the validity are shown.

Keywords: minimum weight design, two-level optimization, vibration of pedestrian bridge, approximation method

1. INTRODUCTION

It is more than 10 years since the vibrations of the pedestrian bridges became a serious problem. Although for short span pedestrian bridges the standard design³⁾ has been completed, the vibrations of the relatively long span pedestrian bridges are important subjects to be solved. One of the subjects is the dynamic stability due to the wind and another is the vibration serviceability to the pedestrian. For the former problem the wind tunnel examinations have been carried out and the safety to the wind is certified by the wind-proof design method. For the latter problem the following items are defined in the Technical Standard for Pedestrian Bridges and Underpasses (abbreviated as Technical Standard after)¹⁾,

1) 2-12 Deflection ; Maximum deflection of the main girder due to live load shall not exceed the 1/600 of the span length of the main girder. But when the bridge is designed with special consideration on the influence to the pedestrian, the maximum deflection may be increased to the 1/400 of the span length.

2) 2-13 Vibration ; The vibration of the main girder due to live load shall not discomfort the pedestrian.

These items are defined, but a concrete method to estimate the vibration serviceability is not defined. However an analysis method for the vibration serviceability of pedestrian bridges has been proposed²⁾ and it is applied to the analysis of vibration serviceability, especially of the pedestrian bridges for those spans where no standard design is prepared.

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On the optimum design of frame structures, application of dual methods⁴⁾ and approximation techniques^{5),6)} were studied. Also the method to determine the sectional sizes of grillages efficiently by the two-level optimization technique is presented⁷⁾. Using the two-level optimization technique proposed in the paper the numbers of design variables and constraints at the stage of the structural level optimization can be reduced significantly and the discrete values of plate thickness can be treated easily.

This paper studies the minimum weight design of the pedestrian bridges taking the vibration serviceability into consideration. An approximation technique for the analysis of the vibration serviceability and the two-level optimization technique are studied and applied. Using these techniques the efficient design method with discrete plate thickness are proposed.

2. MINIMUM WEIGHT DESIGN OF PEDESTRIAN BRIDGES

In this paper the section of each member is I-section symmetry with respect to two axes as shown in Fig. 1. Web height and web thickness are same through the members. The minimum weight design of the pedestrian bridges taking the vibration serviceability into consideration is formulated as follows,

Objective ; $W = \sum_{i=1}^n \rho l_i A_i (b_i, t_i, h) \longrightarrow \min \dots\dots\dots (1)$

Constraints ; $\left. \begin{aligned} g_i^{(1)} &= \sigma_i - \sigma_{ai} \leq 0 \\ g_i^{(2)} &= b_i - b_l \leq 0 \\ g_i^{(3)} &= b_i/32 - t_i \leq 0 \\ t_l &\leq t_i \leq t_u \end{aligned} \right\} (i=1 \sim n) \dots\dots\dots (2)$

$\left. \begin{aligned} h_l &\leq h \leq h_u \\ g^{(4)} &= \delta - \delta_a \leq 0 \\ g^{(5)} &= S - R \leq 0 \end{aligned} \right\} \dots\dots\dots (3)$

Design Variables ; $b_i, t_i (i=1 \sim n), h$

where ρ is the mass density, l_i is the length of the i -th member, A_i is the sectional area of the i -th member, σ_i is the working stress of the i -th member, σ_{ai} is the allowable stress of the i -th member. b_l is the lower limit value of the flange width, t_l and t_u are the lower and the upper limit values of flange thickness respectively, h_l and h_u are the lower and the upper limit values of web height, δ is the deflection at the span center, δ_a is the allowable deflection value, S is the quantity of vibration stimulus to the pedestrian, R is the allowable quantity of vibration stimulus, n is the number of members. $g^{(1)}$ is the constraint on the stress, $g^{(2)}$ is the constraint on the lower limit of the flange width, $g^{(3)}$ is the constraint on the lower limit of the flange thickness, $g^{(4)}$ is the constraint on the deflection and $g^{(5)}$ is the constraint on the vibration serviceability.

The problem formulated above is given as the primal problem.

Finite element analysis is used to calculate the influence lines of stresses and deflection. Live load and dead load are loaded on the lines. For live load the values defined in the Technical Standard is used. For dead load the dead weight of the main girder, the dead load except for the main girder (q_d) and the mark load are considered. The dead weight of the main girder is calculated exactly from the sectional sizes in the process of optimization. The values of the dead load q_d are shown in Table 1 for each slab type and width. They are determined referring to the standard design³⁾.

The allowable stress of steel is also referred to the Technical Standard.

Allowable deflection is 1/600 of span length and, when the

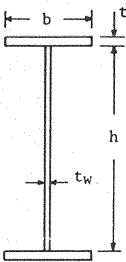


Fig. 1 I-section.

Table 1 Dead load q_d (t/m).

slab type width (m)	deck plate	steel deck plate
1. 5	4. 0	2. 3
2. 1	5. 1	2. 8
2. 2 5		2. 9

ergonomical serviceability analysis of bridge vibration explained later is done, it is 1/400 of the span length.

3. APPROXIMATION METHOD OF VIBRATION SERVICEABILITY ANALYSIS OF PEDESTRIAN BRIDGES

The types of the structures to be analyzed in this paper are simple, two- and three-continuous pedestrian bridges with varying sections. In process of optimization many structural analyses are required, so the following approximation method based on reference 2) is used.

(1) Eigen Vibration of the Beams with Varying Sections

In reference 2) n -th eigen vibration mode $\phi_n(x)$ was expressed by sine series as follows,

$$\phi_n(x) = \sum_{m=1}^{\infty} A_{nm} \cdot \sin(m\pi x/L) \dots\dots\dots (4)$$

where L is the bridge length.

However, the eigen vibration analysis (energy method, etc) of the beams with varying sections by equation (4) is rather troublesome, so the stiffness matrix and the mass matrix are created by the finite element method of beams subjected to bending moments and the eigen frequencies and the eigen vibration modes are calculated by sub-space iteration method. Using the obtained values of the eigen frequencies and the eigen vibration modes the coefficients A_{nm} in equation (4) are calculated by Fourier sine series analysis. The coefficients A_{nm} satisfy the following normalized condition,

$$\sum_{m=1}^{\infty} A_{nm}^2 = 2/\rho^* L \dots\dots\dots (5)$$

where ρ^* is the mass density per unit length and assumed to be constant through the length.

(2) Displacement Amplitude by Maximum Response Spectrum

The eigen vibration modes are expressed by equation (4). When a pedestrian keeps step with the n -th frequency and they are resonant, maximum displacement y_n at point x due to n -th vibration is calculated by the following equation,

$$y_n(x) = (f_0/2) \left(\sum_{m=1}^{\infty} A_{nm} [R_D] \right) \sum_{m=1}^{\infty} A_{nm} \cdot \sin(m\pi x/L) \dots\dots\dots (6)$$

where f_0 is the amplitude of external force expressed by (pedestrian force ratio to static weight α_A) $\times$ (weight of a pedestrian W_b). In this paper α_A is 0.4 and W_b is 60 kgf. $[R_D]$ is the maximum displacement spectrum. In this paper, only the steady state response factor from the periodically moving force is considered and the $[R_D]$ is calculated by the following equation,

$$[R_D] = (1/\sqrt{2}) \{ 1/[\sqrt{\{\varepsilon(2\omega_n + \varepsilon)\}^2 + \{2\eta\omega_n(\varepsilon + \omega_n)\}^2}] + [1/(\sqrt{\{\varepsilon(2\omega_n - \varepsilon)\}^2 + \{2\eta\omega_n(\varepsilon - \omega_n)\}^2})] \} \dots\dots\dots (7)$$

where ω_n is the n -th eigen vibration frequency, v is the velocity of moving load, η is the damping coefficient of a bridge and ε is $m\pi v/L$.

(3) Ergonomical Serviceability Analysis

To analyze the vibration serviceability of a pedestrian bridge three loading conditions are considered. They are single walker, single runner and crowded walker. All of these three loading conditions are not always to be considered for the design of every pedestrian bridge, but only necessary loading conditions should be considered depending on the construction site and the character of the bridge as shown in Table 2.

For each loading condition and the degree of vibration (for k -continuous bridge, k degrees of vibration are considered), the constraints on the vibration serviceability are defined as follows,

Table 2 Locations of pedestrian bridge and load conditions
(○ indicate the loading conditions to be considered).

Condition Location	Single Walking	Crowded Walking	Single Running
Memorial bridge in park	○	○	○
Crossing traffic way in city	○	○	
Crossing river in city	○		○
Others	○		

Table 3 Load conditions and factors²⁾.

Load condition	Single walking	Crowded walking	single running
Frequency range f_B	1.0~3.0 Hz	1.0~3.0 Hz	2.0~4.0 Hz
Acceptance limit R^*		1.7cm/s (RMS) (slightly hard to walk)	2.7cm/s (RMS) (extremely hard to walk)
Response factor r_R	$1 - k_R V_R$	$\lambda T \leq 5$ $\lambda T > 5$	1.0
Required probability and index $p_a, \beta d$	5~10%	5~10%, 1.65~1.3	neglect
Pedestrian number	1 person	Arrival rate λ person/sec	1 person
Laoding factor r_S $T = \theta/v$	1.0	$\lambda T \leq 5$ $\lambda T > 5$	1.0
Pedestrian force f_a (W_B : weight)		0.4 W_B	$f_B = \begin{cases} 2.0 \text{ Hz } 0.4 W_B \\ 2.5 \text{ Hz } 1.0 W_B \\ 3.0 \text{ Hz } 1.6 W_B \\ 4.0 \text{ Hz } 2.0 W_B \end{cases}$ linear approximation between above points
Moving velocity v	0.7 f_B m/s	1.4 f_B m/s	1.4 f_B m/s
Resonance factor r_f	$\sqrt{g(f_B/2-1)/g(0)}$, when $S^* \leq R^*$		1.0
Remarks	$V_R = 0.35, V_R^* = 0.5, \alpha = 0.85, g(\cdot)$: standard normal probability density function $V_R^* = \sqrt{4\lambda T/3(\lambda T+1)^2}$, $k_R = 0.84$ (5%), 0.25 (10%)		

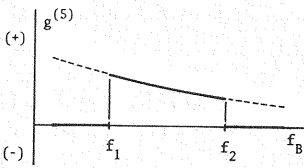


Fig. 2 $g^{(5)}-f_B$ relationship.

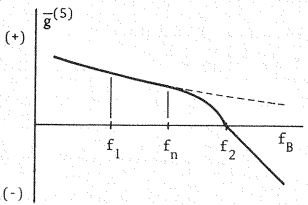


Fig. 3 $g^{(5)}-f_B$ relationship after transformation.

$g^{(5)} = S - R \leq 0$

where

$$\left. \begin{aligned} S &= \gamma_s \cdot \gamma_r \cdot S^* \\ R &= \gamma_R \cdot R^* \end{aligned} \right\} \dots\dots\dots (8)$$

and where

$S^* = \gamma_T \cdot y_n \cdot \omega_n \dots\dots\dots (9)$

and where S^* is the quantity of stimulus to the pedestrian and γ_T is a conversion factor. γ_T is 0.3 in this paper and other factors $\gamma_s, \gamma_r, \gamma_R, R^*$ are shown in Table 3.

As shown in Table 3 the constraints on the vibration serviceability are considered only in the limited range of frequencies. As a result the constraints are discontinuous, not only first derivatives, but function values as shown in Fig. 2. These functions are not suitable for optimization, so they are transformed by the following equation,

i) $0 < f_B \leq f_n$; $\bar{g}^{(5)} = g^{(5)} \dots\dots\dots (10)$

ii) $f_n \leq f_B \leq f_2$; $\bar{g}^{(5)} = \xi \cdot g^{(5)} \dots\dots\dots (11)$

where

$\xi = 1 - [(f_B - f_n)/(f_2 - f_n)]^6 \dots\dots\dots (12)$

iii) $f_B > f_2$; $\bar{g}^{(5)} = f_2 - f_B \dots\dots\dots (13)$

where f_B is the frequency of a bridge, f_1 and f_2 are respectively the lower and upper limits of frequencies and f_n is $(f_1 + f_2)/2$.

By these transformations Fig. 2 is fixed as shown in Fig. 3.

The transformation of equation (10) contradicts the content of Table 3. However, it is practically general that the pedestrian bridges those frequencies are less than f_1 do not satisfy the constraints on the stress or the deflection. So to simplify the problem, equation (10) is used.

4. MINIMUM WEIGHT DESIGN BY TWO-LEVEL OPTIMIZATION

In process of optimization of the primal problem in 2. many structural analyses including eigen value analysis are required. It is difficult to calculate analytically the first derivatives of the constraints of the problem in this paper. They are calculated by finite difference method. When the number of sections of a pedestrian bridge is N in the primal problem, $(2N+2)$ structural analyses are required to calculate a set of the first derivatives. As written before, one structural analysis includes finite element analysis and eigen value analysis, so when the structure and the number of members are big it becomes unpractical to solve the primal problem without any transformation.

However the primal problem consists of structural level optimization and member level optimization. Using a two-level optimization technique the number of design variables and constraints of structural level optimization can be reduced as explained below. As a result the number of structural analyses required to calculate a set of derivatives can be reduced by almost half from $(2N+2)$ to $(N+2)$.

(1) Formulation of Structural Level Optimization

When the vibration serviceability is considered, structural level optimization of a pedestrian bridge is formulated as follows,

$$\text{Objective ; } W = \sum_{i=1}^n \rho l_i A_i(I_i, h) \longrightarrow \min \quad \dots\dots\dots (14)$$

$$\text{Constraints ; } g_i \leq 0 \quad (i=1 \sim n) \quad \dots\dots\dots (15)$$

$$\left. \begin{array}{l} h_l \leq h \leq h_u \\ g^{(4)} = \delta - \delta_a \leq 0 \\ g^{(5)} = S - R \leq 0 \end{array} \right\} \quad \dots\dots\dots (3)$$

Design Variables ; $I_i (i=1 \sim n), h$

where I_i is the geometrical moment of inertia of i -th member, g_i is the constraint concerned with i -th member. The value of g_i is calculated in the member level optimization.

In this structural level optimization, the design variables are reduced to the geometrical moment of inertia of each member from the sectional sizes of each member in primal problem. Structural analyses are done in this level, so this reduction is meaningful as shown in the numerical examples.

(2) Formulation of Member Level Optimization

Member level optimization is carried out for each member. In member level optimization of the i -th member, the values of the geometrical moment of inertia and the web height are given from the structural level optimization, and under these values the sectional sizes to minimize the sectional area and to satisfy the constraints on stress and geometrical relationship between sectional sizes are determined.

For the i -th member, this member level optimization is formulated as follows,

$$\text{Objective ; } A = 2bt + t_w h \longrightarrow \min \quad \dots\dots\dots (16)$$

$$\text{Constraints ; } I, h \longrightarrow \text{given} \quad \dots\dots\dots (17)$$

$$g^{(1)} = \sigma - \sigma_a \leq 0 \quad \dots\dots\dots (18)$$

$$g^{(2)} = b_t - b \leq 0 \quad \dots\dots\dots (19)$$

$$g^{(3)} = b/32 - t \leq 0 \quad \dots\dots\dots (20)$$

$$t_l \leq t \leq t_u \quad \dots\dots\dots (21)$$

Design Variables ; b, t

The shadowed portion in Fig.4 is the general feasible region shaped by equations (18)~(21).

This member level optimization is a two variable problem, but can be transformed into one variable problem expressing b by t as follows,

$$b = (12I - t_w h^3) / 6t(h + t)^2 \quad \dots\dots\dots (22)$$

Substituting equation (22) into equation (16), objective A is expressed as follows,

$$A = (12I - t_w h^3) / 3(h + t)^2 + t_w h \quad \dots\dots\dots (23)$$

As the values of I, h and t_w are constant in this problem, above function is monotonically decreased function for flange thickness t . So, when the value of web height h is given, the optimal design of this problem is immediately X in Fig.4 in so far as t is a continuous variable. That is, in this member level optimization, only an algorithm to solve the nonlinear equation $g(t)=0$ is required. And when only discrete values (for example mm unit) of flange thickness t are to be considered, the optimal value of t is given by omitting fractions of the value of t corresponding to X in Fig.4.

In practice the values of h given from structural level optimization are not always in the range of $h_e \leq h \leq h_c$ in Fig.4. In the process of optimization it may happen that h is less than h_e or h is greater than h_c . Also the shape of feasible region is dependant on the values of the geometrical moment of inertia,

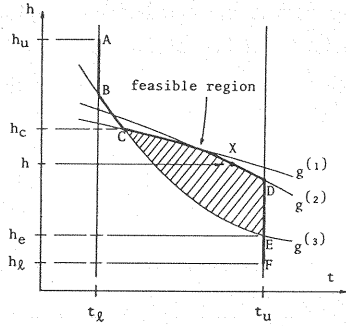


Fig. 4 Feasible region of member level optimization.

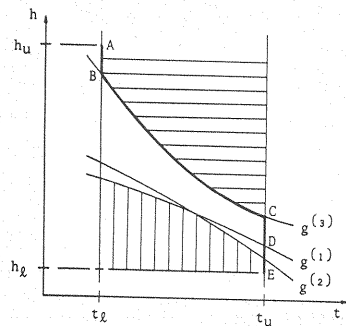


Fig. 5 The case where no feasible region exists.

working moment, kind of steel, and upper and lower limits of flange width and thickness. Some combinations of these values may cause contradictions between constraints, and the problem may have no feasible region as shown in Fig. 5.

In these cases including the case where the feasible design can be found easily, the design and the value of constraint g_i are determined by the following way :

A) When feasible region exists (Fig. 4).

1) $h_c \leq h \leq h_u$; The design on ABC in Fig. 4 corresponding to given h is selected and value of g_i is as follows,

$$g_i = \max [g^{(1)}, 0] + \max [g^{(2)}, 0] + \max [g^{(3)}, 0] \dots \dots \dots (24)$$

2) $h_e \leq h \leq h_c$; The design on CDE in Fig. 4 corresponding to given h is selected and value of g_i is as follows,

$$g_i = \max [g^{(1)}, g^{(2)}, g^{(3)}] \dots \dots \dots (25)$$

3) $h_l \leq h \leq h_e$; The design on EF in Fig. 4 corresponding to given h is selected and value of g_i is determined by equation (24).

B) When no feasible region exists (Fig. 5).

The design on ABCDE in Fig. 5 is selected and value of g_i is calculated by equation (24).

(3) Optimization

In Fig. 6 the flow-chart of minimum weight design of pedestrian bridges by two-level optimization is shown. It is composed of structural level optimization, member level optimization, structural analysis and optimizer. { } means the vector.

Optimizer includes one or several optimization techniques. The values of the design variables I and h are out-put from it and the values of objective and constraints corresponding to the design variables are in-put.

5. NUMERICAL EXAMPLES

As numerical examples simple and two continuous pedestrian bridges are designed by the proposed method in

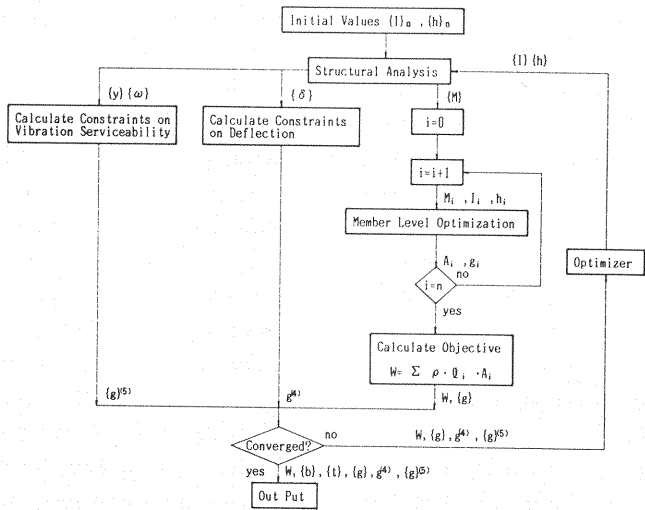


Fig. 6 Flow chart of minimum weight design of pedestrian bridge by two-level optimization.

this paper and the method to solve the primal problem directly (abbreviated as PP). Also a three-continuous pedestrian bridge is designed by the proposed method with discrete plate thickness. These pedestrian bridges are required to have the same vibration serviceability with the memorial pedestrian bridge in park in Table 2. So three loading conditions are all considered. The arrival rate of a crowd of pedestrian is assumed to be 0.5 person/sec. Bridge width is 2.25 m and the kind of steel is SS41. No horizontal stiffeners are used, so when the plate thickness is continuous, web thickness t_w is calculated by the following equation,

$t_w = \max(h/152, 0.9) \text{ (cm)}$ (26)

and when t_w is discrete, the value calculated by equation (26) is raised to mm unit.

Limit values of flange thickness and width are as follows,

$t_l = 1.0 \text{ cm}, t_u = 3.8 \text{ cm}, b_l = 10.0 \text{ cm}$ (27)

The length of compression flange between supports is 300 cm.

To solve the optimal problem of this paper, Sequential Quadratic Programming (SQP)⁹⁾ in the general purpose optimization program ADS⁹⁾ is applied.

In the following examples the numbers of the members for structural analysis and for design are not the same. In general the latter is far less than the former by linking them. The number in parentheses in the following figures are the numbers of the members for design.

The number of structural analysis N in the following tables includes the number of the structural analysis for the first derivatives by finite difference method.

(1) Simple Pedestrian Bridge

A pedestrian bridge with span length 40 m shown in Fig. 7 is designed by the proposed method and PP.

The number of members for structural analysis is 12 and for design it is 2. Three sets of initial values (case 1~case 3) are considered and under these initial values the bridge is designed.

The values of initial total volume V_0 , total volume V , web height h and the number of structural analysis

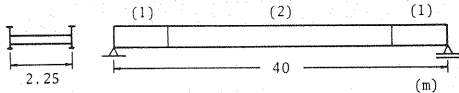


Fig. 7 Simple pedestrian bridge.

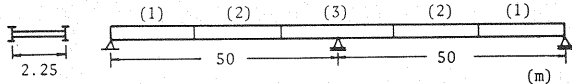


Fig. 8 Two-continuous pedestrian bridge.

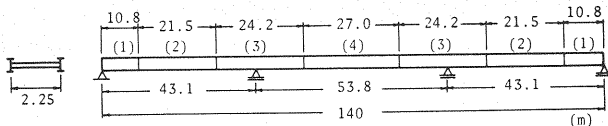


Fig. 9 Three-continuous pedestrian bridge.

Table 4 Results of simple pedestrian bridge.

method	case	$V_0 \text{ (m}^3 \text{)}$	$V \text{ (m}^3 \text{)}$	$h \text{ (cm)}$	N
proposed method	1	7.259	8.839	119.7	56
	2	9.257	8.974	116.4	72
	3	8.309	8.977	116.0	75
primal problem	1	7.259	8.875	121.7	105
	2	9.257	8.979	117.8	98
	3	8.309	9.031	117.4	178

Table 5 Results of two-continuous pedestrian bridge.

method	case	$V_0 \text{ (m}^3 \text{)}$	$V \text{ (m}^3 \text{)}$	$h \text{ (cm)}$	N
proposed method	1	4.130	3.131	187.3	41
	2	3.215	3.143	184.7	33
	3	3.563	3.133	185.7	65
	4	3.017	3.138	182.8	59
	5	3.374	3.131	185.9	65
primal problem	1	4.130	3.141	185.5	120
	2	3.215	3.141	181.4	80
	3	3.563	3.132	186.4	67
	4	3.017	3.133	184.2	89
	5	3.374	3.136	183.0	123

Table 6 Results of three-continuous pedestrian bridge.

design variable	case	$V_0 \text{ (m}^3 \text{)}$	$V \text{ (m}^3 \text{)}$	sectional sizes									N
				b_1	t_1	b_2	t_2	b_3	t_3	b_4	t_4	h	
continuous	1	3.985	3.825	28.14	1.03	31.53	1.13	33.99	1.94	52.14	3.80	133.9	73
discrete	2	3.768	3.837	28.74	1.00	30.06	1.20	40.50	1.50	54.36	3.80	135.0	173

N are shown in Table 4. As shown in the table the value of V and h by each method are almost same and the values of N are quite different. N by the proposed method is far less than N by PP.

The active constraints of this problem were the constraints on stress of member (1), deflection and vibration serviceability for crowded walker.

(2) Two-Continuous Bridge

Two-continuous pedestrian bridge with bridge length 100 m (50 m+50 m) shown in Fig. 8 is designed by the proposed method and PP.

The number of members for structural analysis is 16 and for design it is 3. Five sets of initial values (case 1~case 5) are considered and under these initial values the bridge is designed.

The values of initial total volume V_0 , total volume V , web height h and the number of structural analysis N are shown in Table 5. In the same way as above the values of V and h by each method are almost same and N by the proposed method is far less than N by PP except for case 3.

The active constraints of this problem were the constraints on stress of member (1) and (2) and vibration serviceability of 2nd mode for crowded walker.

(3) Three-Continuous Bridge

Three-continuous pedestrian bridge with bridge length 140 m (43.1 m+53.8 m+43.1 m) shown in Fig. 9 is designed by the proposed method for both cases where the plate thickness is continuous or discrete.

The number of members for structural analysis is 24 and for design it is 4.

The values of initial total volume V_0 , total volume V , all sectional sizes and the number of structural analysis N are shown in Table 6.

The active constraints of this problem were the constraints on stress of member (1), (2) and (3) and vibration serviceability of 2nd and 3rd modes for crowded walker.

6. CONCLUSION

An efficient minimum weight design method of pedestrian bridges taking vibration serviceability into consideration is proposed.

The conclusions are as follows :

- (1) To shorten the computational time for structural analysis, an approximation method of vibration serviceability analysis was studied and applied successfully.
- (2) The minimum weight design of pedestrian bridges is formulated taking vibration serviceability into consideration. The primal problem is divided into two parts, structural level optimization and member level optimization.
- (3) In structural level optimization, the design variables are transformed to geometrical moments of inertia from the sectional sizes of primal problem.
- (4) Member level optimization is formulated as a simple optimization problem. The design variable of this level is flange thickness only and the way to find its optimal value was presented for both continuous and discrete values.
- (5) Three numerical examples were presented and the efficiency and the validity of the proposed method were shown.

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