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BIFURCATION BEHAVIOR OF AN OCTAGONAL TRUSS DOME WITH IMPERFECTIONS

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This paper offers a group theoretic study of the influence of imperfections on bifurcation buckling behavior of an octagonal, reticulated truss dome structure. The dome exhibited diversified bifurcation behavior in association with various imperfection modes chosen to be covariant with subgroups of a dihedral group. Such variations in bifurcation behavior nonetheless did not alter greatly the buckling capacities of the dome. In addition, the concept of symmetry preserving bifuraction is introduced as a potential cause of bifurcation buckling phenomena.

Keywords : *bifurcation behavior, dihedral groups, symmetry, truss dome, imperfection*

1. INTRODUCTION

Symmetry exerts a great influence on various physical phenomena. A number of minimum (or maximum) principles indicate that the optimum state is achieved usually by the highest symmetry. Naturally, dome structures are often constructed to hold point and line symmetric stiffness distributions and geometric configurations. Such symmetric construction reflects the common underlying belief that symmetric structures exhibit excellent appearances, while realizing efficient stiffness distributions against external loads. However, as dome structures tend to be slender in association with the progress in structural engineering, bifurcation buckling type collapes has drawn great concern.

Mathematical studies conducted from a group theoretic standpoint[1] have revealed interrelationships between symmetry and bifurcation behavior. Extending these studies, the authors[2),3)] applied them to the description of bifurcation behavior of a series of reticulated, polygonal-shaped, truss-dome structures. Potential bifurcation modes of the domes were obtained by investigating subgroups of dihedral groups.

This research is undertaken in order to evaluate the influence of symmetry of domes on their bifurcation buckling behavior. For this purpose, the bifurcation behavior of a reticulated, octagonal truss dome was investigated for various imperfection modes chosen through a group theoretic viewpoint. An emphasis is placed on identifying the diffences in qualitative and quantitative aspects of the behavior.

2. INFLUENCE OF GEOMETRIC SYMMETRY ON BIFURCATION BEHAVIOR

In order to investigate the effects of symmetry on bifurcation buckling behavior, case studies are performed on the octagonal, reticulated truss dome shown in **Fig. 1** by employing its configuration as a

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parameter. As reported in Ref. 3, bifurcation modes of this dome under symmetric vertical loadings can be represented by the following seven sugroups of a dihedral group D_8:

$$\left.\begin{array}{lll} D_{8/2}=\langle\sigma_1, \sigma_3, \sigma_5, \sigma_7, \tau\sigma_2, \tau\sigma_4, \tau\sigma_6, \tau\sigma_8\rangle & & \\ D_2^{2j-1}=\langle\sigma_1, \sigma_5, \tau\sigma_{2j-1}, \tau\sigma_{2j+3}\rangle & D_2^{2j}=\langle\sigma_1, \sigma_5, \tau\sigma_{2j}, \tau\sigma_{2j+4}\rangle & j=1 \text{ or } 2 \\ D_1^{2j-1}=\langle\sigma_1, \tau\sigma_{2j-1}\rangle & D_1^{2j}=\langle\sigma_1, \tau\sigma_{2j}\rangle & j=1,2,3 \text{ or } 4 \\ C_2=\langle\sigma_1, \sigma_5\rangle & E=\langle\sigma_1\rangle & \end{array}\right\} \cdots\cdots\cdots\cdots (1)$$

where σ_j denotes the point symmetry regarding a $45\cdot(j-1)$ degree rotation and $\tau\sigma_j$ does the line symmetry in the straight line intersecting with the y-axis at the origin at an angle of $-45/2\cdot(j-1)$ degrees. The level of symmetry of these groups can be represented by 'orders' (see Table 1), which represent the number of elements of a group[2]. As can be seen from this table, these groups possess greater orders and higher symmetry in the sequence advanced in Table 1.

The configuration of the dome was altered by introducing z-directional initial imperfections of nodes 1 through 8 in such a manner that each configuration was represented by one of the subgroups. Table 1 contains these imperfections, which were scaled by a constant value of 0.01 cm.

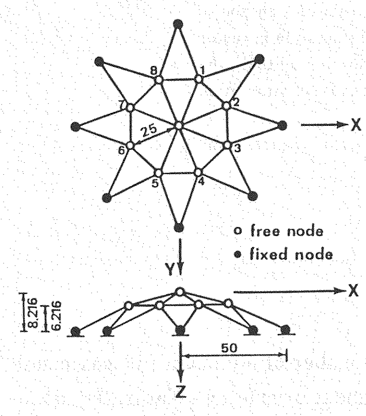

Fig. 1 Octagonal Dome (unit in cm).

Table 1 Initial Imperfection Modes.

Type	Order	Node Number							
		1	2	3	4	5	6	7	8
$D_{8/2}$	8	1	-1	1	-1	1	-1	1	-1
D_2^{2j-1}	4	-1	-1	1	1	-1	-1	1	1
D_2^{2j}	4	0	1	0	-1	0	1	0	-1
D_1^{2j-1}	2	2	1	-1	-2	-2	-1	1	2
D_1^{2j}	2	0	1	2	1	0	-1	-2	-1
C_2	2	2	1	-2	-1	2	1	-2	-1
E	1	1	0	-1	0.5	0	-0.5	0.5	-0.5

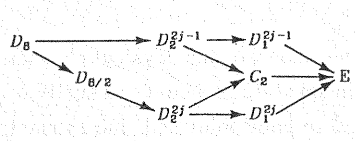

Fig. 2 Inter-group Relationships of D_8.

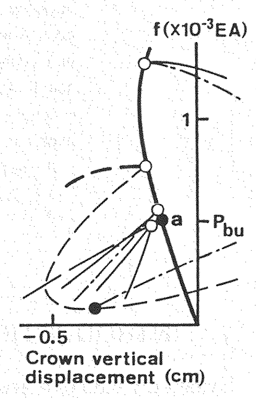

Fig. 3 Equilibrium Paths of Regular Octagonal Dome.

Inter-group relationships of D_8, shown in Fig. 2, are used here to describe bifurcation behavior of these altered configurations. The symbol '$S \rightarrow T$' in this figure indicates that group T is a subgroup of group S; every group always has itself as its subgroup. The inter-group relationship of the group D_8 includes all the subgroups; however, that for C_2 does only C_2 and E. Subgroups with greater orders, in this manner, possessed not only higher symmetry but more subgroups and more complex inter-group relationships. Drastic variations among the relationships imply that these configurations will display diversified bifurcation phenomena.

Bifurcation path tracing analyses were conducted on this dome by means of a finite displacement analysis technique[4] for truss structures. We considered symmetric vertical loading pattern applied to each node with the same intensity, except for the center node, for which a load with the half intensity was applied. Figure 3 shows equilibrium paths obtained for the perfect configuration, whereas Fig. 4 does those for altered configurations. Various kinds of lines denote the types of paths; symbols (●) and (○) express bifurcation point types. Only a bifurcation path is obtaind for each asymmetric bifurcation point for simplicity.

Equilibrium paths obtained for the imperfection modes associated with groups E and D_1^{2j-1} both had only one bifurcation path represented by group E, thus exhibiting very simple behavior. Paths relevant with

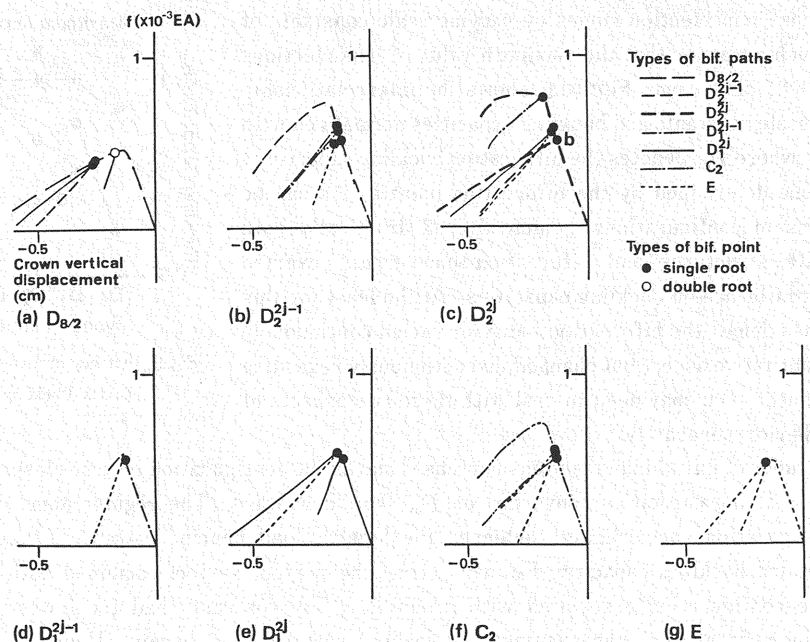

Fig. 4 Equilibrium Paths Obtained for Various Configurations of Octagonal Dome.

group D_1^{2j} were slightly more complex owing to the presence of bifurcation paths represented by group E or D_1^{2j}. Paths for groups D_2^{2j-1}, D_2^{2j} and $D_{8/2}$ had much more complex bifurcation path frameworks because of the presence of a series of branching paths. The paths for all these altered configurations nonetheless were significantly simpler than those for the regular octagonal dome.

As we have seen, slight variations in the dome's configuration greatly altered subgroup structures, thereby leading to completely different bifurcation behavior. Bifurcation behavior of symmetric structures, in general, should be highly sensitive to variations in their configurations. Such a feature implies difficulties involved in tracing bifurcation behavior of actual dome structures, greatly influenced by various factors degrading their symmetry.

A Fujii's finding regarding bifurcation behavior[1] says that the symmetry group of a bifurcation path is a 'subgroup' of the symmetry group of a main path. On the basis of this, the authors succeeded in identifying main and bifurcation paths[2]. At the course of this, we tacitly interpreted that the term 'subgroup' (of a group) did not include the group itself. This stronger sense of interpretation of this finding held for most of the equilibrium paths observed here and in other papers[2),3)]. Exceptional cases, however, arised for the equilibrium paths for D_2^{2j}, D_1^{2j}, C_2 and E. For example, two paths both represented by group D_2^{2j} intersected at the bifurcation point b (see Fig. 4 (c)). For this case, the term 'subgroup' needs to include a group itself so as to satisfy the finding. This term, to be precise, should be interpreted in this manner.

Note that main and bifurcation paths cannot be identified for these two paths, with the same level of symmetry. Two paths with the same type were connected by a bifurcation point. Fujii called such phenomena the symmetry 'preserving' bifurcation behavior. This case does not agree with the concept of symmetry breaking[1),2)], for which a main path and a bifurcation point possess the same level of symmetry but bifurcation paths do the lower one. The symmetry preserving bifurcation phenomena, nonetheless, existed solely for imperfect systems, which are covariant with the (real) subgroups of a dihedral group. It remains to be settled in the future to investigate if such phenomena can exist in bifurcation phenomena covariant with dihedral groups.

In order to investigate a quantitative influence of symmetry, buckling capacities of the dome were obtained for those imperfection modes by varying scale constants of the modes in such a manner that the maximum value of imperfections equals either 0.01 or 0.1 cm. Figure 5 shows the interrelationship between these scale constants and buckling capacities normalized with respect to P_{bu}, where P_{bu} denotes the bifurcation buckling capacity of the regular dome determined by the bifurcation point a. As can be seen, variations in configurations, which altered drastically both bifurcation path structures and bifurcation phenomena, exerted relatively weak influence on buckling capacities. At the least for this reticurated truss dome, the bifurcation behavior varied continuously regarding a quantitative aspect but changed discontinuously regarding a qualitative aspect. One may need to deal with these two aspects of bifurcation behavior separately.

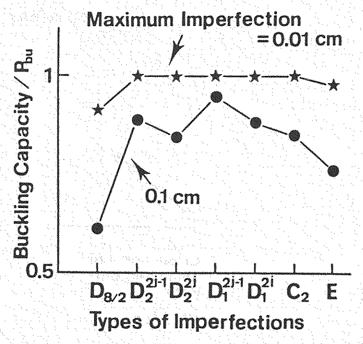

Fig. 5 Effects of Dome's Configuration on Its Buckling Capacity.

As the magnitudes of initial imperfections decreased and as the configuration became closer to that of the regular dome, buckling capacities converged on P_{bu} from downward. The regular dome with the most symmetric configuration, accordingly, achieved the highest load bearing capacity (P_{bu}) and loss in symmetry degraded buckling capacity. For this case, the perfect system combined both strength and symmetry. The configuration associated with group $D_{8/2}$, by contrast, had the lowest capacity and represented the most critical case against the loading pattern used herein. Undesirable geometric imperfection modes against external loads should be identified much more systematically in the design of dome structures so as to realize the most critical situation against them.

3. CONCLUDING REMARKS

This investigation offered case studies on an octagonal, reticulated truss dome performed by varying its geometric configuration. Such variations did not exert a great influence on its buckling capacities. A reduction in the level of geometric symmetry, by contrast, significantly simplified the bifurcation behavior, thus greatly reducing the tasks in the analytical tracing of the behavior. The most symmetric configuration achieved the highest load bearing capacity in spite of the complexity of relevant bifurcation behavior. These facts implied that one may need to deal with quantitative and qualitative characteristics of bifurcation behavior separately. In addition, the concept of symmetry 'preserving' bifurcation was introduced as a potential cause of bifurcation buckling phenomena.

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