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A CONCISE AND EXPLICIT FORMULATION OF OUT-OF-PLANE INSTABILITY OF THIN-WALLED MEMBERS

(*By Akio HASEGAWA, Kithsiri LIYANAGE, Toyohito IKEDA and Fumio NISHINO*)

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Discussion

By Seizou USUKI (*Akita Univ.*)

The purpose of this discussion is certainly not to detract from the value of the authors' exellent, clear and powerful work. I hope that the authors will emphasize the differences between the authors' work and other works.

(1) The authors have based on the displacement field of Eqs. (3) and the strains of Eq. (4·b) and (6), for developing the explicit stiffness equation of instability. On the other hand, Roberts et al.[15),16)] have also presented the differential equation and the stiffness equation of thin walled bars. In these works, they have used the shear strain considering second order terms of displacements of beam axis. In addition, the stiffness equation includes the effects of deflection of beam axis prior to buckling. What are the differences between the authors' fundamentals and Roberts' results.

(2) Barsoum and Gallagher[17)] and Kawai[18)] have derived the stiffness equations based on strains obtained from linearized displacement field. The stiffness equations derived by the authors and by the above are, then, considered to be different from each other. The differential equations governing the lateral-torsional instability of thin walled beams subject to constant axial force N^0 and constant bending moment M_y^0 and M_z^0 are, however, considered equivalent each other in Ref. 17), 18) and the authors. Then, the stiffness equations presented in Ref. 17), 18) and by the authors are also considered to be equivalent, in spite of the differences of the strain components between Ref. 17), 18) and the authors.

(3) The stiffness equations of instability of curved beams have been presented in Ref. 11). What are the differences from Ref. 11), when the radius R of a curved beam becomes infinitely large.

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Closure

By Akio HASEGAWA (*Univ. of Tokyo*), *Kithsiri LIYANAGE* (*Univ. of Tokyo*), *Toyohito IKEDA* (*Univ. of Tokyo*), *and Fumio NISHINO* (*Univ. of Tokyo*)

The writers would like to thank Usuki for his interest in the paper and for his valuable comments. Followings are the closure remarks for discusser's respective comments

(1) Reference 15) has presented certainly an analytical scheme based on the potential energy theorem

to determine buckling loads of thin-walled members, however, the method presented in 15) is applicable only for uniform straight members with limited boundary conditions for which acceptable trial functions of buckling modes are available for cases such as simply supported beams or cantilever beams. As a natural extention from 15), Rrference 16) has dealt with the finite element formulation of the same problem, and has given the geometical stiffness matrix in terms of prebuckling displacements, not of stress resultants. The essential strategy proposed in 15) and 16) seems to be a total Lagrangian-like approach which is a clear contrast to the formulation of linearized finite displacements which the writers has used to derive a concise and explicit stiffness equation in the paper. It should be noted that 1) sum of small displacement stiffness and geometrical stiffness happens to be identical to the tangent stiffness at a reference state of arbitrary equilibrium position, and 2) the tangent stiffness matrix can only be obtainable consistently from the linearized finite displacement theory of continua. For this reason, as formulated in the paper, the second order shear strain components should not appear in the evalution of geometrical stiffness matrix, and prebuckling displacements should be excluded from the reference state of interest. Prior deformatons including prebuckling displacements can adequately be incorporated by successive uses of the elemental stiffness equation (19) helped by the cordinate transformations for respective elements of the assemblage of straight members. By the way, References 15) and 16) have not given the differential equations of the problem, which the discusser may have misunderstood.

(2) In conjunction with the question made by the discusser, consider the simplest case of in-plane beam-column which is typical of the linealized finite displacement formulation. The well-known displacements, strains and stiffness equation of beam-column can of course be extracted from Eqs. (3), (4·b), (6) and (19) in the paper. On the other hand, the same stiffness equation is known to be obtainable through a combined use of the kinematic field of small displacements and the equilibrium consideration at a deformed configuration. As clear from this explanation for the beam-column, References 17) and 18) have obtained their stiffness equations based on the equilibrium after deformation, or the equivalent considerations in terms of external energy expressions, although the kinematic field of small displacements has been utilized for the internal energy expression. While, the proposed formulation presented in the paper has derived the stiffness equation through the consistent application of the virtual work equation of linearized finite displacements of continua for thin-walled members. Therefore, as far as theoretical formulations are concerned irrespective of the displacement fields employed, it is not surprising that the quoted references 17) and 18) including 8) may produce the same results as the present paper for particular illustrations. However, it may be worthwhile to mention that the present formulation including the final expression of stiffness equation appears to be general and much more concise and comprehensive compared with those in quoted literature. Since the kinematic fields and the stiffness equations are given in a vague and complicated way in 17) and 18), it is neither worthwhile nor possible to make a full comparison with the present paper.

(3) Reference 11) has only presented the differential equation for curved members based on the potential energy theorem, but no derivation was made for the corresponding stiffness equation, which the discusser may have misunderstood. Therefore, no comparison is possible in terms of stiffness equation between 11) and the present paper. It is noted from the writers' point of view that the formulations of curved members as they are either for differential equations or for stiffness equations tend always to be tedius and cumbersome, and accordingly the resulting equations turn to be rather complicated as evident from 11), even if the derivations are successful. For this reason, as stated in the paper, the utilization of the idea of the assemblage of straight members seems useful and advantageous and is recommended to apply particularly for practical purposes.

Those described above are the closure intended to correspond to the discusser's valuable comments. Stated hereinafter is to supplement the whole contexts of the original paper and discussion. As explained in the paper and the above closure, the stiffness equation (19) which consists of small displacement $\boldsymbol{K}_E$ and geometrical $\boldsymbol{K}_G$ matrices is obtainable from the virtual work eqtation of linealized finite diiplacements, and accordingly is interpreted as the incremental equation measured from an arbitrary reference state which is supposed to be a known equilibrium state. Thus, $\boldsymbol{K}_E+\boldsymbol{K}_G$ of Eq. (19) is indentical to the tangent stiffness $\boldsymbol{K}_T$ at the state. Therefore, the derivation of $\boldsymbol{K}_E+\boldsymbol{K}_G$ is the most important not only for buckling evaluation but also for tracing the nonlinear finite displacement behavior of structures. Under the circumstances of computer application, the stiffness equation should play the same role of the differential equation with the associated boundary conditions on solution procedures. As always desired for mathematical expressions, such a key note equation should be presented as clearly and concisely as possible. On reviewing the available literature including references quoted in the discussion in this respect, it is evident that the stiffness equation (19) and the underlying formulation are much more clear, comprehensive and reliable helped by its efficient and consistent treatment compared with others.

Last but not least in the closure, the writers would like to present the differential equations and their associated boundary conditions which are exactly consistent with the stiffness equation (19) and are derived with common variational procedures from the virtual work equation (14) of linearized finite displacements for thin-walled members. The resulting differential equations are expressed as

$$
\begin{aligned}
&EAu_c''=0\\
&EI_{yy}v_s^{(4)}-N^0v_s''+(-z_sN^0+M_z^0)\phi''=0\\
&EI_{zz}w_s^{(4)}-N^0w_s''+(y_sN^0-M_y^0)\phi''=0\\
&EI_{\omega\omega}\phi^{(4)}-GJ\phi''+(-z_sN^0+M_z^0)v_s''+(y_sN^0-M_y^0)w_s''\\
&\quad-(r_s^2N^0+\beta_yM_y^0+\beta_zM_z^0+\beta_\omega M_\omega^0)\phi''=0
\end{aligned}
\qquad \cdots\cdots (29\cdot\text{a}\sim\text{d})
$$

with their boundary conditions as

$$
\begin{aligned}
&u_c=u_{ck} && \text{or } n_xEAu_c'=F_{xk}\\
&v_s=v_{sk} && \text{or } n_x[-EI_{yy}v_s'''+N^0v_s'+(z_sN^0-M_z^0)\phi']=F_{yk}\\
&-v_s'=-v_{sk}' && \text{or } n_x[-EI_{yy}v_s''-M_z^0\phi]=D_{yk}\\
&w_s=w_{sk} && \text{or } n_x[-EI_{zz}w_s'''+N^0w_s'+(-y_sN^0+M_y^0)\phi']=F_{zk}\\
&-w_s'=-w_{sk}' && \text{or } n_x[-EI_{zz}w_s''+M_y^0\phi]=D_{zk}\\
&\phi=\phi_k && \text{or } n_x[-(EI_{\omega\omega}\phi'''-GJ\phi')+z_sN^0v_s'-y_sN^0w_s'\\
& && \quad+(r_s^2N^0+\beta_yM_y^0+\beta_zM_z^0+\beta_\omega M_\omega^0)\phi']=C_{Tk}\\
&-\phi'=-\phi_k' && \text{or } n_x[-EI_{\omega\omega}\phi'']=C_{\omega k}
\end{aligned}
\qquad \cdots\cdots (30\cdot\text{a}\sim\text{g})
$$

in which $k=i, j$ for both ends as shown in Fig. 2, and $n_x=-1$ for $i-$ end and $n_x=1$ for $j-$ end.

It should be noted for the future trend of structural mechanics that the governing equation of any problem of concern can be expressed by an alternative form either of the differential equation with its associated boundary conditions or of the stiffness equation. The former is useful for analytical solutions, and the latter is appropriate for computational purpose. With the premise of computer proliferation, the expression of stiffness equation which includes the associated boundary conditions automatically tends to play more and more important role on practical applications, and therefore should be versatile and concise, consistent with the related differential equation.

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