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A NON-ITERATIVE OPTIMUM DESIGN METHOD FOR CABLE-STAYED BRIDGES

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A computationally efficient optimum design method is proposed. Unlike conventional optimization methods, all the optimized properties are obtained without conducting iterations. In this method, redundant forces of a statically indeterminate structure are computed from the condition of geometrical compatibilities, the members are fabricated so as to introduce prestresses which nullify the incompatibilities. Such an introduction of prestressing is compatible with conventional erecting process of cable-stayed bridges.

A series of analyses have verified that this method is practical and makes it easier to determine economical structural types of cable-stayed bridges.

1. INTRODUCTION

Cable-stayed type of bridges, which combine economy and beauty, have been extensivelly employed in constructing middle scale long-spanned bridges. It has been, however, problematic to select proper structural types in designing these bridges since the bridges have many design alternatives, including : the configuration of cables, the number of cables, and the support condition of towers, etc.

In order to make the selection easier, considerable research works have been conducted on the cable-stayed bridges. These research works can be divided into two general categories. Firstly, there are studies regarding the structural types[1),2)]. Although these papers would be impotant in designing them, the papers fail to compare their structural types from economical standpoints. In addition, these methods are not easily applicable to the design of the briges, especially to those with eccentric span ratios.

Sencondly, there are a series of research works concerning the optimum design of cable-stayed bridges[3)~5)]. Such research works are primarily concerned with the optimization of cross-sectional properties, while demanding the optimum design technique necessitating huge and complex computer analyses. The optimization of the structural types, however, has been somewhat disregarded in this technique.

This paper introduces an optimum design method, conveniently applicable to the selection of economical structural types of cable-stayed bridges. The optimum design method proposed herein extends the methods advanced in Ref. 6 to two-dimentional frame structures. This method is easily applicable to the design and does not necessitate proficiency in numerical analysis technique, unlike the conventional optimization

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technique developed on the basis of mathematical programing. While it is mandatory to compute redundant forces in statically indeterminate structures from geometrical compatibility relationships, the new method determines these forces using the condition that the objective function for the optimum design problem is minimized. The optimum member forces and cross sections are computed based on the redundant forces determined in this manner. The validity of the method is verified on the basis of simple analyses on hypothetical cable-stayed bridges and their desirable structural types are advanced.

2. FORMULATION OF NEW OPTIMUM DESIGN METHOD

(1) Basic concept of new optimum design method

The basic concept of the optimum design method proposed is explained below with the use of a simple example structure shown in Fig. 1. This structure, a simple girder suspended at its midspan by a cable, has a single redundancy. Choosing the cable tension, X, as a redrndant force, the member forces of this structure are represented as follows :

$$\begin{aligned} M &= w(L_g \cdot x - x^2)/2 - x \cdot X/2 \quad \text{(girder)} \\ N &= X \quad \text{(cable)} \end{aligned} \qquad (1)$$

where M expresses the bending moment, N is the axial force, w denotes the uniformly applied vertical loads, and x expresses the coordinate. In conventional design methods, the redundant forces are calculated so as to satisfy geometrical cmpatibilities. For example, the redundant force for this example structure can be computed from the condition that the girder's deflection at its span center is equal to the cable's elongation, and is given as follows :

$$X = \frac{5w \cdot L_g^4/384EI_g}{L_g^3/48EI_g + L_c/EA_c} \qquad (2)$$

Because Eq. (2) contains the member stiffnesses EA_c and EI_g, the procedure of optimizing this structure involves iterations in arriving at a solution. Mathematical programing or other technique is used for the iterations in conventional optimum design methods.

On the contrary, the design method proposed herein does not attempt to fulfill the geometrical compatibility conditions but redundant forces are determined from the condition that the objective function for the optimum design is minimized.

Now, the objective function may be arbitrary defined according to the demand of the problem. Square summation of the main girder, i. e.

$$f = \int_0^{Lg} M^2 \, dx \qquad (3)$$

is utilized as an example. This objective function can be minimized by the following redundant force value.

$$X^* = 5w \cdot L_g/8 \qquad (4)$$

Figure 2 shows the member forces computed for this value. The cross-sectional properties are designied according to these member forces. Now, the member stiffnesses can be computed from the condition that the limits for allowable stresses must not be exceeded. The stiffness values are :

$$EI/L_g^3 = 1 \quad \text{(girder)}, \quad EA_c/L_c = 192 \quad \text{(cable)} \qquad (5)$$

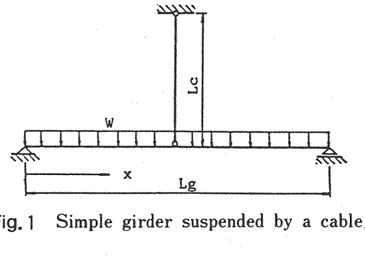

Fig. 1 Simple girder suspended by a cable.

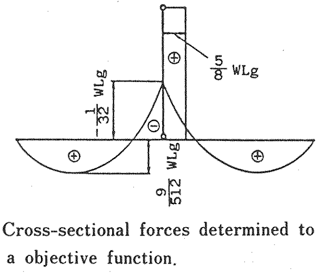

Fig. 2 Cross-sectional forces determined to minimize a objective function.

It must be noted that the geometrical compatibility condition cannot be satisfied by the redundant force and the member stiffness obtained in this fashion.

The redundant force which satisfies the compatibility condition Eq. (2) is equal to :

$$X^c = 4w \cdot L_g / 8 \quad (6)$$

and the corresponding member forces are shown in Fig. 3. There exists the following difference between the X values obtained in Eq. (4) and in Eq. (6) :

$$X^p = X^* - X^c = w \cdot L_g / 8 \quad (7)$$

Also, there exist differences in member forces for these two cases, as shown in Fig. 4. Hence the direct use of the redundant force and member stiffnesses computed using this method will cause geometrical incompatibilities.

The existence of incompatibilities, however, does not limit the usefulness of the method proposed as it is possible to satisfy the compatibility by introducing the corrective prestress, X^p. Prestress is automatically introduced to the main girder by applying additional cambers to the shop cambers of the members at the stage of their fabrication. A question, however, may arise regarding the validity of the introducing of such prestresses. In erecting cable-stayed bridges, it is common to prestress the main girders by jacking up or down the saddles of cables or inserting shim plates between the sockets of the cables and the anchor frames. Hence the optimum design method, which necessitates corrective prestresses to be introduced, does not significantly disorder the conventional construction processes, at the least for this type of bridges. It may rather be stated that the erecting processes will be simplified if this design method is employed since such processes will not require the jacking up of the saddles. Furthermore, the members usually have shop cambers so that the introduction of additional cambers suggested herein does not require any additional efforts in the fabrication process.

For this example structure, the extra cambers required are computed from Eq. (7) as follows (see Fig. 5) :

$$\Delta_g = X^p \cdot L_g / 48EI_g \quad \text{(girder)}, \quad \Delta_c = X^p \cdot L_g / EA_c \quad \text{(cable)} \quad (8)$$

Fabricating the members in this manner will introduce the prestress shown in Fig. 5, thereby completely satisfying the geometrical compatibility of the structure after erection.

(2) General formulation of the optimum design method

General formulation of the optimum design method is introduced. The modified volume, taking the cost of materials into acount, is adopted here as the objective function and the limits of member stresses are employed as the constraints. Then, the problem of optimum design will read :

$$f = \sum_{i=1}^{n} c_i \cdot A_i \cdot L_i \quad (9)$$

$$g = \sigma_c - \sigma_a \leqq 0 \quad (10)$$

in which c_i is the cost coefficient, A_i denotes the cross-sectional area, L_i expresses the member length, σ_c is the actual stress, σ_a denotes the allowable stress and n expresses the number of members. For the members under the influence of axial forces and bending moments, Eq. (10) can be rewritten as follows :

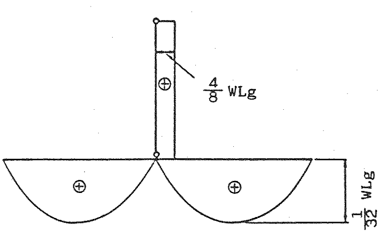

Fig. 3 Cross-sectional forces computed from geometrical compatibility.

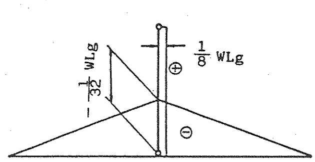

Fig. 4 Cross-sectional forces caused by prestress.

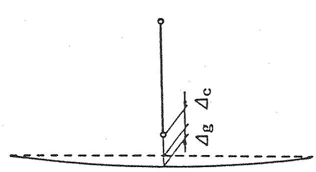

Fig. 5 Amount of prestrains.

$$g_1=\sigma_c/\sigma_{ca}+\sigma_b/\sigma_{ba}(1-\sigma_c/\sigma_{ea})-1\leqq 0$$
$$g_2=\sigma_c/\sigma_{cal}+\sigma_b/\sigma_{bal}(1-\sigma_c/\sigma_{ea})-1\leqq 0 \quad \cdots\cdots (11)$$

where σ_b is the bending stress, σ_{ea} expresses the allowable stress considering the Euler buckling, σ_{ca} denotes the allowable compressive stress, σ_{cal} is the allowable compressive stress taking local buckling into account and σ_{ba} expresses the allowable bending stress. From these formula, the active constraint can be obtained as follows :

$$g=\sigma_c/\sigma_{Na}+\sigma_b/\sigma_{Ma}-1=0 \quad \cdots\cdots (12)$$

in which

$$\sigma_{Na}=\sigma_{ca} \quad \text{or} \quad \sigma_{Na}=\sigma_{cal}$$
$$\sigma_{Ma}=\sigma_{ba}(1-\sigma_c/\sigma_{ea}) \quad \text{or} \quad \sigma_{Ma}=\sigma_{cal}(1-\sigma_c/\sigma_{ea})$$

As we will see in Sec. 3. (1), the section modulus Z_i of a member 'i', can be expressed in terms of the following linearized expression concerning the sectional area A_i :

$$Z_i=\alpha_i\cdot A_i-\beta_i \quad \cdots\cdots (13)$$

Substituting Eq. (13) into Eq. (12), the minimum area A_i can be expressed as a function of member forces :

$$A_i=a_i\cdot N_i/\sigma_{Nai}+b_i\cdot M_i/\sigma_{Mai}+d_i \quad \cdots\cdots (14)$$

in which the constants a_i, b_i and d_i used in this equation are named the coefficients of configuration and expressed as :

$$a_i=1-\beta_i/(\alpha_i\cdot A^0_i),\quad b_i=1/\alpha_i,\quad d_i=\beta_i/\alpha_i \quad \cdots\cdots (15)$$

where A^0_i will be refered to in Sec. 3. (1). Substituting Eq. (15) into Eq. (9), the objective function is rewritten as follows :

$$f=\sum_{i=1}^{n} c_i\left\{a_i\cdot\frac{N_i}{\sigma_{Nai}}+b_i\cdot\frac{M_i}{\sigma_{Mai}}+d_i\right\}L_i \quad \cdots\cdots (16)$$

The minimizing process of this equation can be interpreted as a problem of weighted residual method with the residuals N_i and M_i, and weights $c_ia_iL_i/\sigma_{Nai}$ and $c_ib_iL_i/\sigma_{Mai}$. When the multiples of weights by residuals are introduced as new weights, Eq. (16) can be further transformed.

$$f=\sum_{i=1}^{n} c_i\left\{a_i\cdot\frac{N_i^2}{\sigma_{Nai}}+b_i\cdot\frac{M_i^2}{\sigma_{Mai}}+d_i\right\}L_i \quad \cdots\cdots (17)$$

This transformation, based on the method of least squares, seems to be valid in that such a method is commonly used at present. Furthermore, the transformation is advantageous from a structural standpoint as well, since the distribution of the members' cross section is leveled due to the property of the revised objective function. The cross-sectional forces N_i and M_i in Eq. (17) can be expressed as follows :

$$N_i=N_{i0}+\sum_{j=1}^{m} N_{ij}X_j,\quad M_i=M_{i0}+\sum_{j=1}^{m} M_{ij}X_j \quad \cdots\cdots (18)$$

where N_{i0} and M_{i0} are the axial force and the bending moment at member i caused by a unit load at point j of the statically determinate system, respectively ; X_j denotes the redundant force at point j and m expresses the degree of redundancy.

As we have seen in Sec. (2), the redundant forces X_j are employed herein as the design variables since their values can be controlled by introducing prestresses. Accordingly, the optimum design problem advanced above is identical with the problem of minimization without constraints. It is possible to achieve this minimization by solving the simultaneous equations that are obtained by differentiating the objective function with respect to each redundant force and setting the resulting derivatives equal to zero, i. e.,

$$\frac{\partial f}{\partial X_j}=\frac{\partial}{\partial X_j}\left|\sum_{i=1}^{n} c_i\left\{a_i\frac{N_i^2}{\sigma_{Nai}}+b_i\frac{M_i^2}{\sigma_{Mai}}+d_i\right\}L_i\right|=0 \quad \cdots\cdots (19)$$

By the way, the following solution strategy would be more advisable from both theoretical and practical standpoints. The principle of least work, based on the energy method, can be expressed as follows :

$$\frac{\partial U}{\partial X_j}=\frac{\partial}{\partial X_j}\left|\sum_{i=1}^{n}\left\{\frac{N_i^2}{2EA_i}+\frac{M_i^2}{2EI_i}+\frac{\kappa\cdot Q_i}{2A_iG_i}\right\}^2 L_i\right|=0 \quad (20)$$

in which U is the strain energy. Noting that the variable d_i is constant and that the contribution of the shear forces on strain energy is usually negligible in the analyses of girder systems, and setting.

$$EA_i=\sigma_{Nai}/2c_i a_i \quad \text{and} \quad EI_i=\sigma_{Mai}/2c_i b_i \quad (21)$$

one can show that Eq. (19) and Eq. (20) are identical. One consequence of this is that the optimum member forces can be computed with the aids of conventional structural analysis technique with the use of the member stiffnesses defined by Eq. (21), instead of solving Eq. (19) directly.

The section properties can be computed from the optimum member forces obtained in this manner. The objective function, which indexes the economy of structure, can be calculated on substituting the results of the element optimazation into Eq. (9). Prestresses and extra cambers are determined by performing structural analyses based on the optimized cross sections. The method for calculation will be discussed in Sec. 3. (2).

The optimum design method proposed here does not require repeated structural analyses in determining cross-sectional properties and other values, and is performed with the use of general-purpose, finite-element structural analysis programs. This design method, therefore, can be practically applicable to the selection of economical structural types of cable-stayed bridges.

3. APPLICATION TO OPTIMUM DESIGN OF CABLE-STAYED BRIDGES

(1) Computation of coefficients of cable stayed-bridges

Prior to applying the optimum design method to cable-stayed bridges, the coefficients used in Eq. (21) must be computed. These coefficients consist of the shape coefficients a_i and b_i, cost coefficients c_i, and allowable stresses σ_{Nai} and σ_{Mai}.

a) Shape Coefficients

The shape coefficients are computed based on the assumption that the girder has a trapezoidal box section and the tower has a regular box section (see Fig. 6). Actually, the cable-stayed bridges with these types of sections are widely used in Japan because such sections generally provide large torsional stiffnesses and excellent stabilities against wind. In computing the shape coefficients, the following assumptions are used:

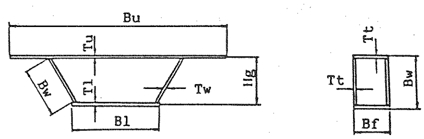

Fig. 6 Cross-section of girder and tower.

1. All parts of the cross-section are effective.
2. The influence of the longitudinal libs is considered in determining the geometric properties.
3. Each of the deck and the web plate thicknesses of a girder is decided according to the requirement of the minimum thickness.
4. The flange and web plate thicknesses of a tower are identical.

The section moduli Z for the girders (see Eq. (13)) are computed by accounting for only the bottom flange plates, while those for the towers by considering the flange plates of both sides. The constants, α, β and A^0 are computed as follows:

$$\alpha=(6B_uT_u+3B_wT_w+H_gT_w)H_g/6(B_uT_u+B_wT_w)$$
$$\beta=(B_u^2+2B_uT_uB_wT_w+B_w^2T_w^2)H_g/(B_uT_u+B_wT_w) \quad \text{(girder)} \quad (22)$$
$$A^0=2B_uT_u+B_wT_w$$

$$\alpha=(3B_f+B_w)B_w/6(B_f+B_w)$$
$$\beta=0 \quad \text{(tower)} \quad (23)$$
$$A^0=2(B_f+B_w)T$$

The shape coefficients a and b regarding the girders or towers can be computed by substituting these values into Eq. (15). Note that the value of A^0, which is actually not constant, is assumed above to be constant since its value does not have a large influence on the results. The shape coefficients for cables, under the influences of only axial forces, are equal to :

$a=1,\ b=0$ ………………………………………………………………………… (24)

b) Cost Coefficients

The cost coefficients reflect the difference in costs of materials on the objective function. Their values are assumed to be : 1.00 for SS 41, 1.12 for SM 50, 4.85 for PWS. These values are obtained by estimating the average prices through the past decade.

(2) Prestressing

The design method proposed attempts to adjust the geometrical compatibilities by introducing prestresses. The amount of prestresses and extra cambers can be computed with the use of the procedure explained in Sec 2. (1). First, the redundant forces X_j^* are computed such that the objective function is minimized. This computation is achieved by following the optimum design formulation introduced in Sec. 2 (2). Next, the optimum member stiffnesses A_i^* and I_i^* are computed. Finally, the redundant forces X_j^c are computed from the geometrical compatibility condition defined based on the member properties A_i^* and I_i^*. Then the amount of requisite prestresses are computed as follows :

$X_j^p=X_j^*-X_j^c \quad (j=1,2\cdots,m)$ ………………………………………………………… (25)

The amount of additional camber of the girders and the change of cable length are computed from these values. As can be seen from Fig. 7, these corrective distorsions, which cancel the incompatible displacements caused by the redundant forces X_j^*, must be accounted for in designing the bridges.

(3) Problem of optimum design method

As we have already seen, the optimization method introduced herein is able to perfectly satisfy the geometrical compatibility by imposing prestresses on structures for a single load case. In general, bridges are designed on the basis of several load cases. It would not be possible to satisfy all the compatibility conditions. As a consequence, the various properties of the bridge computed may be slightly deviated from the optimum ones for several load cases since only a set of prestresses can be introduced. This problem can be avoided if the severest loading condition can be found previously. It is empirically known that the design of the long-spanned bridges, such as cable-stayed bridges, is mainly governed by dead loads. For this reason, the variation of loadings, which is not expected to greatly influence the optimized properties, is not considered here.

4. NUMERICAL CASE STUDIES

(1) Calculation model

Radial type and harp type cable-stayed bridges with a three span continuous beam system are employed here as analytical examples for case studuies. Their general configurations are shown in Fig. 8. Dead loads are 10.0 t/m (98 N/m) and live loads are 3.5 t/m (34.3 N/m). Live loads are applied to the whole main span.

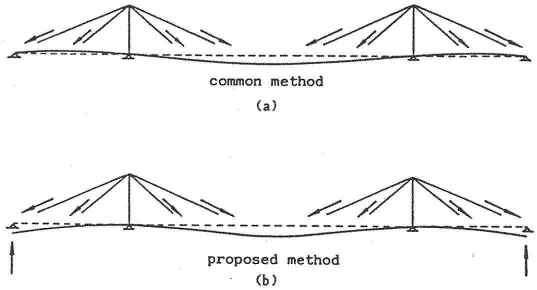

Fig. 7 A method of prestressing for cable-stayed bridge.

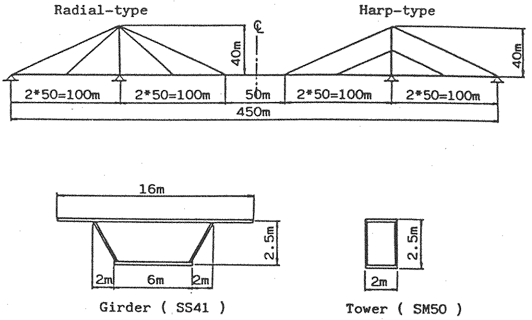

Fig. 8 Calculation model of cable-stayed bridge.

(2) Optimum configurations

The solid and dotted lines in Fig. 9 show the optimized configurations obtained by using the new optimum design method and those by SLP method, respectively. The maximum difference between the design variables (plate thicknesses) computed using these two methods was about 20 percent. Nevertheless, the difference between the objective functions for them was less than one percent. The extra camber of girders and the corrective length of cables to be introduced are shown in Fig. 10.

(3) Suggestion of desirable structural types of cable stayed bridges

The new optimum design method is employed here for the study of economical structural types of cable-stayed bridges with radial or harp type. Economical structural types are identified on the basis of the objective function values. The prameters utillized for a series of structural types for the bridges is shown in Table 1. For simplicity, cables are installed with a same interval and symmetric about the tower. The parameters G, H and F used in this table express the support condition that the towers are fixed to the girder, fixed to the substructure, and pinned on the substructure, respectively. The cables are connected to the tower for all of these cases.

The influence of these parameters on the objective function is shown in Fig. 11 (a), (b), and (c). As can be seen, the values of objective function computed for the radial type were smaller than those of the harp type for all parameter values. For example, their difference is about two percent in the case where $N_c=4$, $H_t=50$ and S_t. The objective function tended to become smaller as the number of cables increased, but the decrease ratio of the objective function attenuated as the increase proceeded. The objective function was minimized when the height of the tower was approximately equal to from 45 to 50 m for the radial type and from 50 to 55 m for the harp type. For these two cases, the ratio of tower height to main

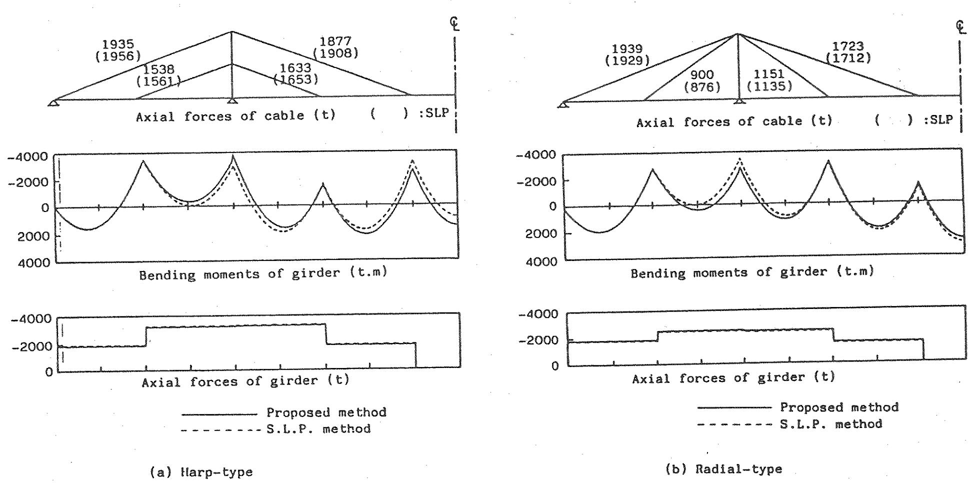

Fig. 9 Confignrations of the Optimized sections.

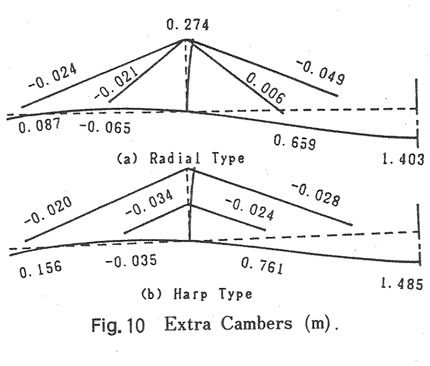

Fig. 10 Extra Cambers (m).

Table 1 Parameters assoslated with a structural type.

Shape of cables : Sc	Radial-Type, Harp-Type
Number of cables : Nc	2, 3, 4, 5
Height of tower : Ht	40, 45, 50, 55, 60
Support condition of tower : St	G, H, F

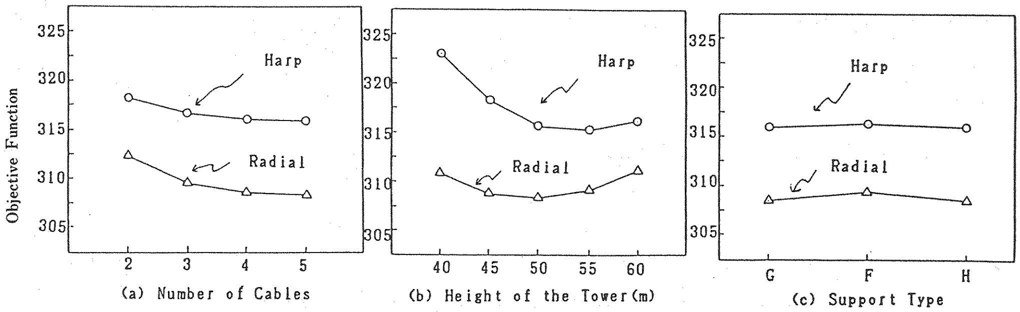

Fig. 11 The Influence on Obj. Function by Parameters.

span length dropped in the range from 0.18 to 0.20 and from 0.20 to 0.22, respectively. There were only small variations in the values of the objective function with respect to the support condition of the tower, especially for the support types G and H. The optimum values for F type were greater than those for the other two types. Since F type bridges must furnish expensive pin joints, it is not preferable to adopt this type. Judging from the fact that the detailed structure of the G type bridge is simpler than those of other types, it can be concluded that this type is the most ecconomical. The validity of this conclusion can be insured from the fact that G type is often adopted in cable-stayed bridges.

Based on these observations, one can state that a radial type with as many cables as possible and 45-50 m tall towers fixed to the main girder can serve as the most desirable structural type. This economical type proposed herein is obtained based on the optimization method developed in this report. Since the method is based on several assumptions, the economical type may slightly differ from the ideal one. However, it was confirmed that the economical type was in very good accordance with those from references and actual examples. Hence this method appear to be valid and reliably applicable to the design of cable-stayed bridges.

5. CONCLUSION

It is important but difficult to appropriately select the structural types of cable-stayed bridges. This paper proposed an optimum design method easily applicable to the slection of an economical structural type. Unlike conventional structural analysis technique which necessitates iteratrions so as to satisfy geometrical compatibilities, redundant forces of structures are computed from the condition that the objective function is minimized without conducting any iterations. Consequently, the new method was far more computationally efficient than conventional techniques. In order to satisfy the geometrical compatibility, the amount of incompatibility is computed based on the optimized cross sections, and the members are fabricated so as to introduce prestresses which cancel the incompatibility. The validity of this method was insured from the fact that it is mandatory in erecting cable-stayed bridges to introduce prestresses by implementing extra camber to girder members.

For the verification of the new method, it is applied to the optimum design of cable-stayed bridges. The optimized objective function values of the bridges obtained by the method were compared with those obtained by the SLP method. There was no more than one percent diffenence between their values so that the new method is likely to present as accurate optimum values as the SLP method does. In addition, the authors determined an economical structural type of a cable-stayed bridge based on the comparison of the values of objective function for various structural types. The economical structural type derermined in this manner tended to be similar to what was reported in relevant references or designs. This optimum design method would be practical enough and simplify the comparison design for cable-stayed bridges. Currently, this method is not applicable to multiple load cases. It is a problem settled in the future to make this model applicable to such load cases.

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