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PROBABILITY DISTRIBUTIONS BY STANDARDIZED VARIATE

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It is very advantageous to represent various probability distributions by respective standardized variates with zero mean and unity variance, as well as the normal distribution. Then, useful figures and tables on them can be prepared, and a probability distribution can be compared numerically and visually with the others based on the same standard. Some figures are shown for distributions commonly encountered in structural reliability. Characteristics of these distributions are briefly presented and the characteristic value defined by the normal distribution in the limit state design is discussed on validity of its use for the other distributions.

1. INTRODUCTION

Some figures are shown for distributions commonly encountered in structural reliability such as : (1) Weibull ; (2) beta ; (3) Gumbel ; and (4) logarithmic normal distributions, and some considerations are presented[1].

Relationship between a random variable, x, of i distribution and its standardized variate, u_i, is expressed as

$$x=\overline{X}(1+u_i V) \qquad (1)$$

in which $\overline{X}$ and V are mean and coefficient of variation of x. $G_i(u_i)$ and $g_i(u_i)$ denote the cumulative distribution function and the probability density function of the standardized variate for the i distribution, respectively. In the case where a probability distribution is defined by three or more parameters such as bounds of the random variable, they can not be represented uniquely as the normal distribution. Thus, properly chosen indices consistent with the standardized variate must be used.

2. FIGURES BY STANDARDIZED VARIATE

Figs. 1~8 give $g_i(u_i)$ and $G_i(u_i)$ for the i distribution with those for the standard normal distribution. In these figures, λ is an index which indicates distance from the mean to the lower limit, x_l, of x devided by the standard deviation, defined by the following equation,

$$x_l=\overline{X}(1-\lambda V) \qquad (2)$$

and μ is used to define the upper limit, x_u, of x as

$$x_u=\overline{X}(1+\lambda \mu V) \qquad (3)$$

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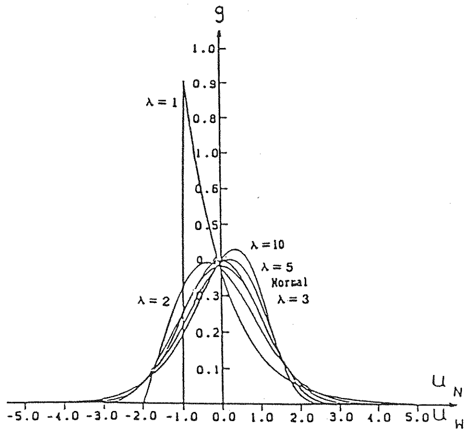


Fig. 1 Probability density function of Weibull distribution by standardized variate.

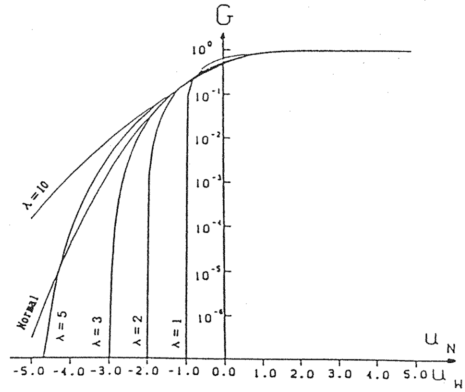


Fig. 2 Distribution function of Weibull distribution by standardized variate.

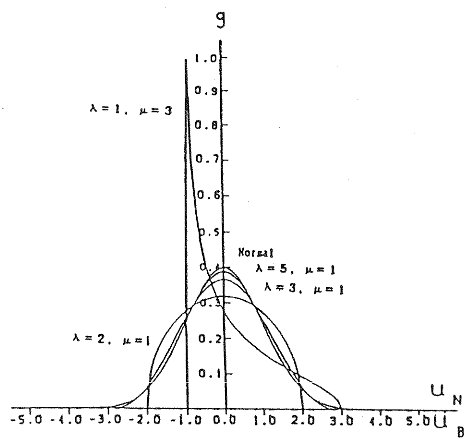


Fig. 3 Probability density function of beta distribution by standardized variate.

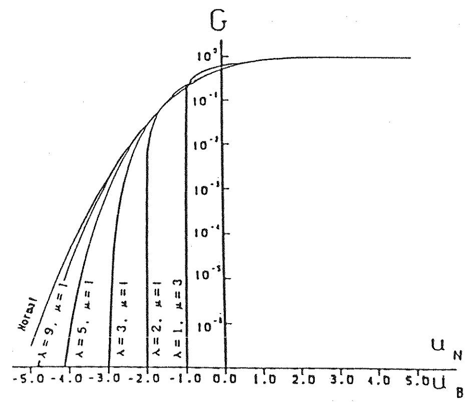


Fig. 4 Distribution function of beta distribution by standardized variate.

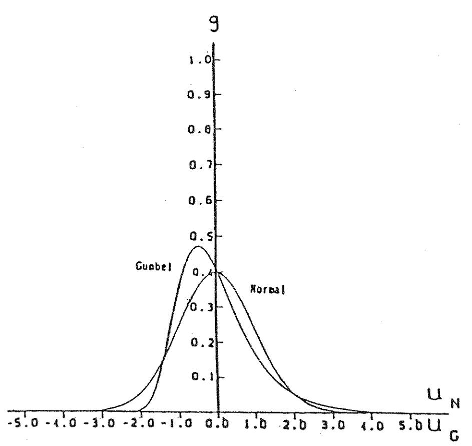


Fig. 5 Probability density function of Gumbel distribution by standardized variate.

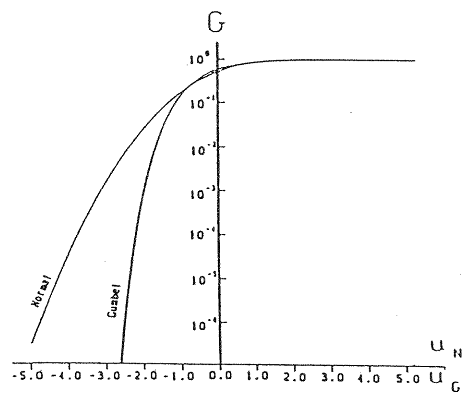


Fig. 6 Distribution function of Gumbel distribution by standardized variate.

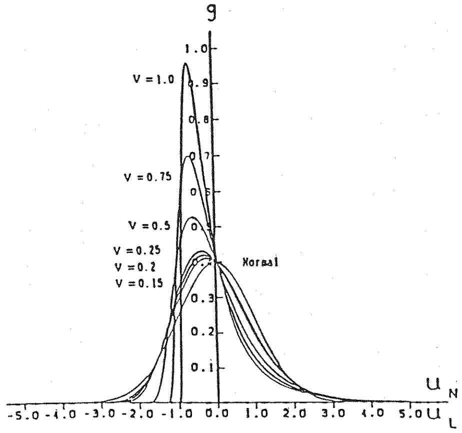


Fig. 7 Probability density function of LN distribution by standardized variate.

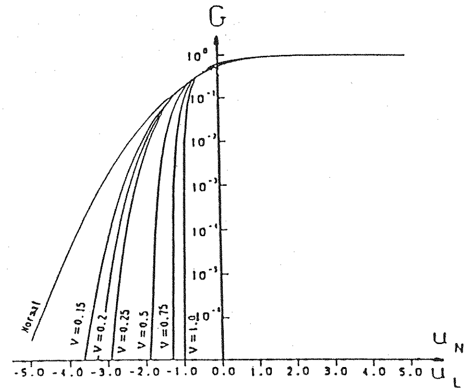


Fig. 8 Distribution function of LN distribution by standardized variate.

3. SOME CHARACTERISTICS OF THE DISTRIBUTIONS

(1) Weibull distribution

When λ is less than 3 or greater than 6, the degree of the skewness of the Weibull distribution tends to be extreme with its value apart from the foregoing values. On the other hand, in the case where λ takes on values from 3—5, the features of the distribution are similar to the normal distribution for the most part.

(2) Beta distribution

It can be observed that the beta distribution approaches to the normal distribution with increasing of λ when μ holds unity.

(3) Gumbel distribution

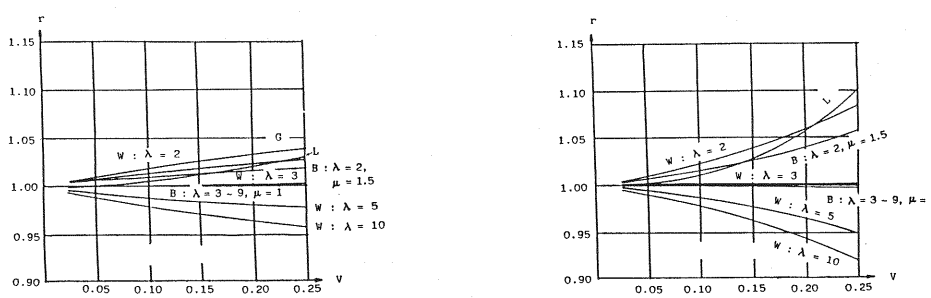
Figure of cumulative distribution function shows illustratively that there is considesrable difference between the Gumbel distribution and the normal distribution, especially when the cumulative probability, p, or the probability of exceedance, $1-p$, is small, as the result of its nature.

(4) Logarithmic normal distribution

We observe that the degree of the positive skewness of the logarithmic normal distribution increases and the distribution is apart from the normal distribution with the coefficient of variation, V.

4. CHARACTERISTIC VALUE OF THE LIMIT STATE DESIGN

The characteristic value[2] of the material strengths or the loads in the limit state design is usually



(a) to (1-1.645 V) (b) to (1+1.645 V)

Fig. 9 Correction factor, r, of characteristic value.

defined by the equation

$$x_k=\overline{X}(1+kV) \quad (4)$$

where x_k is the characteristic value and k is -1.645 (for the strength), or 1.645 (for the load), which implies that the cumulative probability for the strength or the probability of exceedance for the load is 5 % regarding these random variables as the normal distribution. A correction factor, which gives the validity of the application of Eq. (4) for any other i distribution, can be defined by

$$r=\frac{1+u_{ik}V}{1+kV} \quad (5)$$

in which u_{ik} is the standardized variate of i distribution for the probability of 5 %.

Fig. 9 shows the results of Eq. (5) for some distributions, in which W, B, G, and L denote the Weibull, beta, Gumbel and logarithmic normal distributions, respectively. From Fig. 9 the following comments can be written.

As there are, at present, few statistical data of the strengths and the loads, the use of Eq. (4) is well valid, except special cases where it is considered that the coefficient of variation is greater than 0.25 and λ is very small in the Weibull or beta distribution. This indicates that the influence of the type of the distribution on the characteristic values, which correspond to relatively large probability p or $1-p$, opposed to the failure probability, is little and it is not necessarily restricted to the normal distribution to apply Eq. (4).

REFERENCES

1) Koyama, K. and Chou, T. : Various Probability Distribution expressed by standardized variate, Proc. of 39 th Annual Conference of JSCE, 1984 (in Japanese).

2) Japan Society of Civil Engineers : Guide to Limit State Design of Concrete Structures, JSCE, 1983 (in Japanese).

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