

INTERACTION CURVE OF THIN-WALLED BOX STUB-COLUMN SUBJECTED TO COMPRESSION AND BENDING FOR APPLYING TO OVERALL BUCKLING ANALYSIS OF COLUMNS

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A theoretical study on the ultimate strength of thin-walled box stub-columns subjected to compression and bending is performed by using a finite element method (F. E. M.) on the basis of the elasto-plastic large deflection theory in order to develop an interaction curve at the ultimate state. A modified column method to calculate the ultimate strength of thin-walled box columns considering the local buckling of Plate elements is proposed by using the interaction curve of stub-column. Through numerous calculations by the modified column method, an approximate formula for predicting the ultimate strength of thin-walled box columns is given in this paper.

1. INTRODUCTION

The steel structural design codes in Japan (JSHB)¹⁾, U. K.²⁾, West Germany³⁾ and U. S. A.⁴⁾ permit the use of slender plates as the elements of thin-walled box member subjected to compression and bending.

A sophisticated analysis considering the interactions between local buckling of the plate elements and overall buckling of the member is necessary to calculate the true ultimate strength of thin-walled box members. Therefore, various approximate approaches for predicting the ultimate strength have been proposed hitherto. For example, there are a few methods developed by Little, Usami-Fukumoto and so on.

Little^{5), 6)} calculated the relationships between bending moment, M , and corresponding curvature, ϕ , for the short segments of columns with a square cross-section subjected to specified compressive force by using the mean compressive stress-strain curves from the elasto-plastic and large deflection analyses for the isolated compression plates, instead of the stress-strain curves for the material of segments. Then, he applied these $M-\phi$ curves to the ultimate strength analysis of columns. This method, however, underestimates the rigidity of plate elements under the inclined inplane stress distribution.

Usami-Fukumoto⁷⁾ have proposed a design formula for the ultimate strength of thin-walled box columns on the basis of not only the experimental studies, but also the analytical ones by considering the local buckling derived from the concept of effective width.

There is not still any powerful method which considers the interactive behaviors of neighboring plate elements of a short segment of box member subjected to compression and bending.

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In this study, accordingly, an interaction curve is derived by the F.E.M. parametric studies for calculating the ultimate strength of box stub-columns subjected to compression and bending. A modified column method to estimate the ultimate strength of box columns with slender plate elements is also proposed by applying this interaction curve. Moreover, a practical and approximate formula for evaluating the ultimate strength of columns is given through numerous calculations by the modified column method.

2. F. E. M. ANALYSIS OF THIN-WALLED BOX STUB-COLUMN

(1) Analytical Model

Let us deal with a thin-walled box stub-columns having doubly symmetrical cross-section subjected to compressive force and bending moment with respect to the centroidal axis-Y as shown in Fig. 1. A part of the column meshed in Fig. 1 is only taken as an analytical model due to the symmetry of cross-section, where the dimension, *a*, is the half wave length of initial and subsequent deflection of plate elements which makes the ultimate strength most conservative. According to Ref.9), it can be taken as 0.6 times the longer of width *B* or *D*.

(2) Initial Imperfection and Residual Stress

The shape of initial imperfection modes in flange and web plates are also indicated in Fig. 1. The maximum initial imperfections, *w*₀ and *v*₀, are respectively taken as *B*/150 and *D*/150 of which values correspond to the fabrication tolerance in JSHB.

A residual stress distribution, shown in Fig. 2, is adopted in the analysis. The compressive residual stress, *σ*_{rc}, is taken as 0.3 times the yield stress, *σ*_y, owing to Ref.13).

(3) Boundary Condition

The boundary conditions on loaded edges are shown in Fig. 3(a).

In order to obtain the relationships between compressive force, *P*_r, bending moment, *M*_r, compressive displacement, *U*_r, and rotational angle, *θ*_r, with respect to a point R in the cross-section ①, ③, ⑤, and

⑦ of the F.E.M. model in Fig. 1, the end of cross-section is assumed to keep a plane after loading through inserting an imaginary rigid plate as shown in Fig. 3(b). The boundary conditions at the point R can, thereby, be categorized as follows;

Uniaxial compression :
*θ*_r=0, *e*=0, *P*_r=*P* (1)_{a-c}

Combined actions of compression and bending :
*M*_r=0, *P*=*P*_r, *e*=*M*/*P* (2)_{a-c}

Pure bending :
e=0, *P*_r=0, *M*_r=*M* (3)_{a-c}

where *P* and *M* : applied compression force and bend-

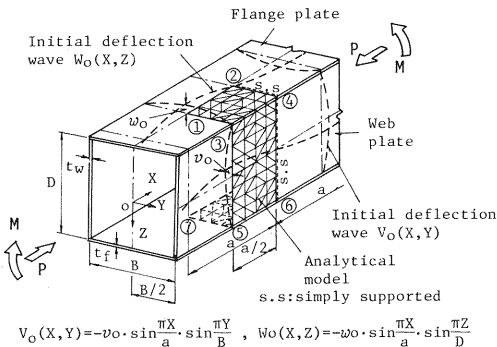


Fig. 1 Analytical model for F.E.M.

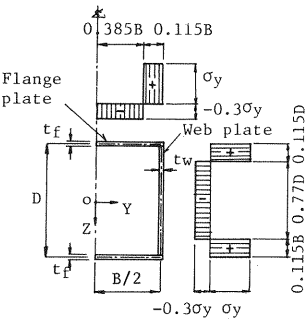


Fig. 2 Residual stress distribution.

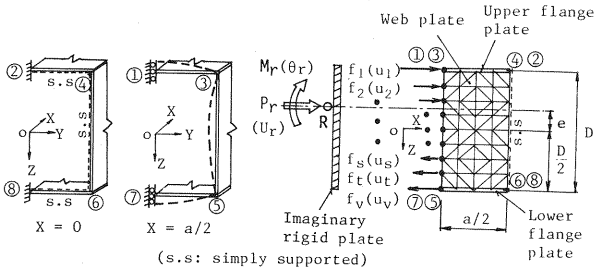


Fig. 3 Boundary conditions and loading method.

ing moment, e : distance between point R and centroid of cross-section (See Fig.3(b)).

(4) Non-Dimensional Plate Slenderness

The plate slendernesses, R_f , R_w and R_{fw} , are used as parameters to represent the sensitivity of stub-columns to buckling, which can be defined by the following equations;

$$\left. \begin{aligned} \text{Flange plate} : R_f &= \frac{B}{t_f} \cdot \sqrt{\frac{12(1-\mu^2)}{k\pi^2}} \cdot \sqrt{\frac{\sigma_{fy}}{E}} \\ \text{Web plate} : R_w &= \frac{D}{t_w} \cdot \sqrt{\frac{12(1-\mu^2)}{k\pi^2}} \cdot \sqrt{\frac{\sigma_{wy}}{E}} \\ \text{Stub-columns} : R_{fw} &= \frac{B}{t_f} \cdot \sqrt{\frac{12(1-\mu^2)}{k_{fw}\pi^2}} \cdot \sqrt{\frac{\sigma_Y^*}{E}} \end{aligned} \right\} \dots\dots\dots (4)_{a-c}$$

where B , D : widths of flange and web plate, t_f , t_w : thicknesses of flange and web plate, σ_{fy} , σ_{wy} : yield stresses, μ : Poisson's ratio ($=0.3$), E : Young's modulus ($=2.06 \times 10^5$ MPa), k : buckling coefficient of isolated plate ($=4.0$), and σ_Y^* is the mean yield stress, which can be defined by;

$$\sigma_Y^* = (A_f \cdot \sigma_{fy} + A_w \cdot \sigma_{wy}) / A \dots\dots\dots (5)$$

where A_f : cross-sectional area of flange plate ($=B \cdot t_f$), A_w : cross-sectional area of web plate ($=D \cdot t_w$), A : half of cross-sectional area of stub-columns ($=A_f + A_w$).

A parameter k_{fw} is the buckling coefficient of stub-column, which can be given by the following equations⁹⁾.

$$k_{fw} = \frac{(\alpha_f + 1/\alpha_f)^2 + (t_w/t_f)^3 \cdot (D/B) \cdot (\alpha_w + 1/\alpha_w)^2}{1 + (D/B)^3 \cdot (t_w/t_f)} , \quad \alpha_f = a/B, \quad \alpha_w = a/D, \quad a = B \cdot \left\{ \frac{1 + (D/B)^3 \cdot (t_w/t_f)^3}{1 + (B/D) \cdot (t_w/t_f)^3} \right\} \dots\dots\dots (6)_{a-d}$$

3. PARAMETRIC STUDY ON ULTIMATE STRENGTH OF STUB-COLUMN

The stub-columns, shown in Table 1 (Cases 1 and 2), were analyzed by altering the ratio of cross-sectional forces, β , i. e.

$$\beta = (M/P_y) / (P/P_y) \dots\dots\dots (7)$$

where M_p and P_y are the fully-plastic moment and squash force, respectively, which can be given by the following equations;

$$\left. \begin{aligned} M_p &= A_f \cdot \sigma_{fy} \cdot D + D^2 \cdot t_w \cdot \sigma_{wy} / 2 \\ P_y &= 2 \cdot \sigma_Y^* \cdot A \end{aligned} \right\} \dots\dots\dots (8)_{a,b}$$

The values of β are taken as 0, 0.385, 0.822, 1.642 and ∞ for Case 1, and 0, 0.426, 0.852, 1.703 and ∞ for Case 2.

(1) Collapse Behavior

a) Bending moment M - rotational angle θ_r curve

Fig.4 shows the relationships between bending moment, M , and rotational angle, θ_r , which are non-dimensionalized by M_p and $(2a/D) \cdot (\sigma_Y^*/E)$, respectively. The initial yield moment, M_y , of flange plates by the elastic beam theory is given as follows;

$$M_y/M_p = 1/[1/\beta + M_p/(\sigma_Y^* \cdot W)] \dots\dots\dots (9)$$

The corresponding locations of M_y/M_p are indicated by the symbol, \bigcirc , on the M - θ_r curves in this figure, where W is the section modulus of stub-columns.

Table 1 Dimensions and plate slendernesses of analytical models.

Items Case	B (mm)	t _f (mm)	D (mm)	t _w (mm)	R _f	R _w	R _{fw}
1	360	12	480	12	0.616	0.821	0.733
2	480	12	360	12	0.821	0.616	0.733

Notes: $\sigma_y = 314$ MPa, $a = 288$ mm

It can be seen from these curves that the ultimate bending moments and the corresponding rotational angle become small in accordance with the decrease of β . The stub-columns in the range where $\beta \geq 0.822$ for Case 1 and $\beta \geq 1.703$ for Case 2 have the ultimate bending moments greater than the initial yield moments

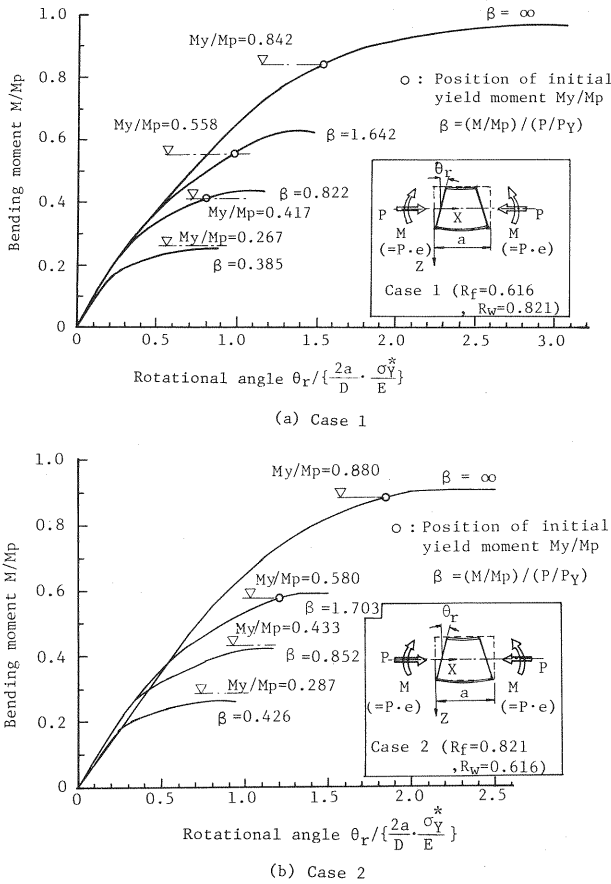


Fig. 4 Bending moment-rotational angle curves.

by 2~12 %, while the stub-columns where $\beta \leq 0.385$ for Case 1 and $\beta \leq 0.852$ for Case 2 reach to the ultimate state prior to the initial yield state. This indicates that the influence of local buckling upon the ultimate strength of stub-columns becomes significant in accordance with the decrease of β .

b) Behavior of flange plate

Fig. 5 shows the relationships between non-dimensionalized compressive force, P_f/P_{fy} , induced to the upper flange plates and the corresponding average strains, $(u/a) \cdot (E/\sigma_{fy})$, where P_{fy} designates the fully-plastic force of flange plate ($=B \cdot t_f \cdot \sigma_{fy}$). The symbols, ●, on the P_f-u curves indicate the locations of ultimate states of stub-columns. The ultimate strengths of flange plates analyzed as the simply supported plates, P_{fu}^u/P_{fy} , are also shown in this figure.

As is seen from this figure, the stiffness of flange plate or the slope of P_f-u curves is independent of the ratio β up to $P_f/P_{fy} \approx 0.6$ in both the Cases 1 and 2. Whereas, the stiffness somewhat depends on β in the range where $P_f/P_{fy} > 0.6$ and the P_f-u curve in case of larger β is situated above the curve with smaller β . This means that the flange

plate becomes stiffer and stronger, because the web plate more or less restrains the flange plate from rotating at the unloaded edges. This tendency seems to increase as the ratio β increases, i. e. ; the case where the applied bending moment is predominant. This figure also shows that the flange plates reach to the ultimate state in all the cases except $\beta=0$ prior to the ultimate state of stub-columns, but the difference of P_f/P_{fy} at two ultimate states is so small that this phenomenon can be ignored to develop the approximate method for evaluating the ultimate strength of stub-columns.

c) Ultimate strength of flange plate

The comparisons of the ultimate strength of flange plate, P_{fu}/P_{fy} , with that of isolated plate (Fig. 5) show the following points. (i) For the pure compression ($\beta=0$), the former is a little bit smaller than the latter in Case 1 ($R_f=0.616$) and vice versa in Case 2 ($R_f=0.821$). This tendency coincides with the result of Ref. 9) i. e. ; a slender plate element in a stub-column is strengthened by other stocky plates and the latter is weakened by the former. (ii) In case where $\beta \neq 0$, P_{fu}/P_{fy} is larger than P_{fu}^u/P_{fy} by 3~6 %, because of the stiffening effects due to the web plates.

(2) Interaction of Compressive Force and Bending Moment

The ultimate compressive force, P_{su}/P_y , and ultimate bending moment, M_u/M_p , of stub-column are summarized in Table 2 and Fig. 6. In this figure, the curve I means the fully-plastic state of stub-column and the curve II corresponds to the curve I divided by the factor k_{pl} which is a non-dimensionalized ultimate strength of stub-column subjected to pure compression and given by a function of parameter R_{fw} as follows⁹⁾;

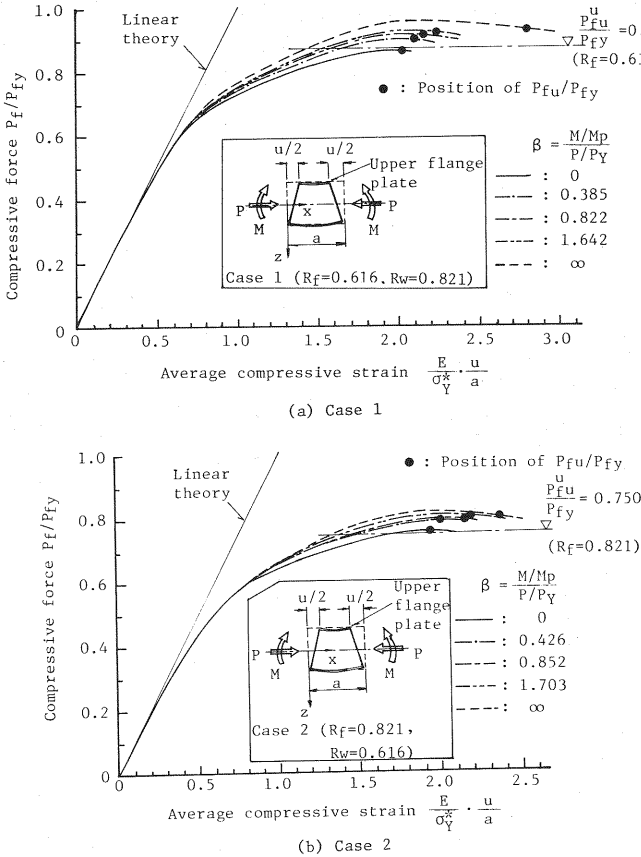


Fig.5 Compressive force-strain curves of upper flange plate.

Table 2 Analytical results of P_{su}/P_y and M_u/M_p .

Items	β ($= \frac{M/M_p}{P/P_y}$)	Analytical results			Eq. (11) Distance from (0,0) to $(\bar{M}_u, \bar{P}_{su})$ $l_u^* = \frac{l_u}{\sqrt{(1)^2 + (2)^2}}$	Errors $\frac{l_u - l_u^*}{l_u}$ (%)
		Ultimate compressive strength $\frac{P_{su}}{P_y} : (1)$	Ultimate bending moment $\frac{M_u}{M_p} : (2)$	Distance from (0,0) to $(\bar{M}_u, \bar{P}_{su})$ $l_u = \sqrt{(1)^2 + (2)^2}$		
1	0	0.802	0	0.802	0.812	-1.2
	0.385	0.660	0.254	0.707	0.683	3.4
	0.822	0.532	0.438	0.689	0.663	3.8
	1.642	0.384	0.630	0.738	0.711	3.7
	∞	0	0.966	0.966	0.812	15.8
2	0	0.802	0	0.802	0.813	-1.4
	0.426	0.626	0.267	0.680	0.661	2.8
	0.852	0.496	0.422	0.651	0.640	1.7
	1.703	0.348	0.592	0.687	0.686	0.1
	∞	0	0.906	0.906	0.812	10.4

numerical results by about 16 % and 10 % in Cases 1 and 2, respectively.

These results suggest that the curve II can be utilized for predicting the ultimate strength of thin-walled stub-columns.

(3) Approximate Interaction Curve

From these results, the ultimate strength of thin-walled stub-columns can approximately be predicted by modifying the interaction curve with respect to the fully-plastic one as follows;

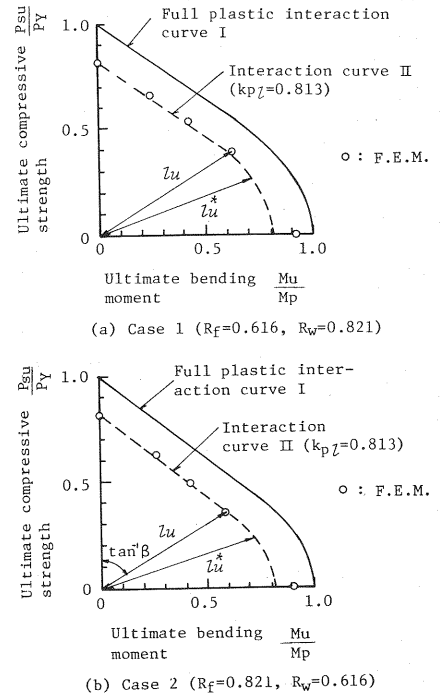


Fig.6 Relationships between P_{su}/P_y and M_u/M_p .

$$\left. \begin{aligned} k_{pt} &= 1.0, (R_{fw} \leq 0.3) \\ &= 0.542 \cdot R_{fw}^3 - 1.249 \cdot R_{fw}^2 \\ &\quad + 0.412 \cdot R_{fw} + 0.968, \\ &\quad (0.3 < R_{fw} \leq 1.3) \end{aligned} \right\} \dots \dots \dots (10)_{a, b}$$

In Table 2, l_u means the distance between the origin (0, 0) and a point $(M_u/M_p, P_{su}/P_y)$ calculated by the F. E. M. analysis, l_u^* being the distance between the origin and a point on the curve II as shown in Fig.6.

The slight differences between l_u and l_u^* in these table and figure show that the numerical results are close to the curve II except the case of pure bending ($\beta = \infty$). For the case of pure bending, the values on the curve II give the conservative ones compared with the

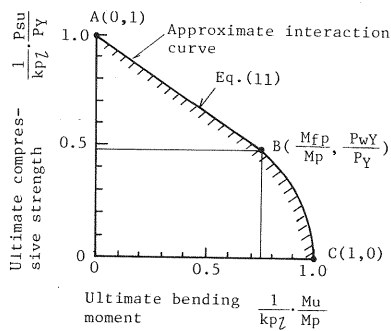


Fig. 7 Approximate interaction curve of box stub-column.

$$\left. \begin{aligned} \bar{M}_u &= (k_{pl} - \bar{P}_{su}) \cdot (\bar{M}_{fp} / \bar{P}_{fy}), \quad \bar{P}_{su} \geq k_{pl} \cdot \bar{P}_{wy} \\ &= \{ 2(1 - \bar{M}_{fp}) - (\bar{M}_{fp} \cdot \bar{P}_{wy}) / \bar{P}_{fy} \} \cdot (\bar{P}_{su} / \bar{P}_{wy})^2 / k_{pl} \\ &\quad + \{ 3(1 - \bar{M}_{fp}) - (\bar{M}_{fp} \cdot \bar{P}_{wy}) / \bar{P}_{fy} \} \cdot (\bar{P}_{su} / \bar{P}_{wy})^2 / k_{pl} - k_{pl}, \\ \bar{P}_{su} &< k_{pl} \cdot \bar{P}_{wy} \end{aligned} \right\} \dots\dots\dots (11)_{a, b}$$

where

$$\left. \begin{aligned} \bar{M}_u &= M_u / M_p, \quad \bar{P}_{su} = P_{su} / P_Y, \quad \bar{P}_{fy} = P_{fy} / P_Y, \quad \bar{P}_{wy} = P_{wy} / P_Y \\ P_{fy} &= 2 A_f \cdot \sigma_{fy}, \quad P_{wy} = 2 A_w \cdot \sigma_{wy}, \quad \bar{M}_{fp} = M_{fp} / M_p, \\ M_{fp} &= P_{fy} \cdot (D/2) \end{aligned} \right\} \dots\dots\dots (12)_{a \sim h}$$

Eq. (11) can be plotted in Fig. 7 as an interaction curve for $(1/k_{pl}) \cdot (M_u/M_p)$ and $(1/k_{pl}) \cdot (P_{su}/P_Y)$.

Although this approximate interaction curve is comparatively conservative in the case where the bending moment is predominant, it will be useful for evaluating the ultimate strength of thin-walled box columns subjected to larger compressive force in comparison with bending moment.

4. ULTIMATE STRENGTH OF CENTRALLY LOADED COLUMN

(1) Modified Column Method for Analyzing Ultimate Strength

It is assumed in a modified column method proposed herein that the interaction between local buckling and overall buckling can be neglected and the thin-walled box columns reach to the ultimate state when either the stress-resultants M_i and P of a cross-section (Fig. 8(c)) satisfy Eq. (11) as is proposed in the above for the local buckling or the column reduces to the unstable state without accompanying the local buckling of plate elements.

The centrally loaded columns, shown in Fig. 8(a), were analyzed by the modified column method, which was formulated by the transfer matrix method¹⁰⁾ on the second order and elasto-plastic theory. The initial imperfections and residual stress distributions are illustrated in Fig. 8(a) and (b).

(2) Verification of Modified Column Method

In order to verify the validity of the above modified column method, experimental results⁸⁾ on the interactive buckling strength of fifteen box columns are compared with those by the modified column method as shown in Fig. 9. The modified column method gives good results and their errors are within $\pm 5\%$

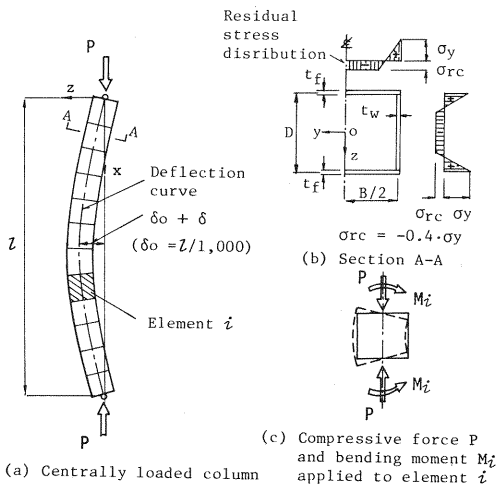


Fig. 8 Analytical model for centrally loaded columns with initial imperfections.

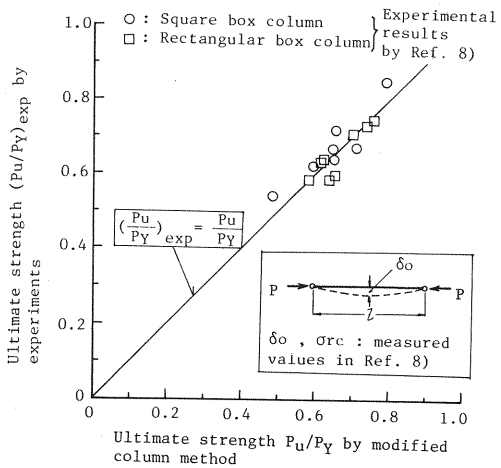


Fig. 9 Comparison of ultimate strength P_u/P_Y by modified column method with $(P_u/P_Y)_{exp}$ by experiments.

against the experimental ones.

(3) Parametric Study

The box columns with the dimensions, $B=D=500\text{ mm}$ and $t_f=t_w=t$, which are listed in Table 3, were analyzed by the modified column method. The thickness, t , and column length, l , are decided so as to fulfill that the plate slenderness, R_{fw} , and the column slenderness, $\bar{\lambda}_y$, are within the ranges $0.3\sim 1.3$ and $0.15\sim 1.05$, respectively, in which $\bar{\lambda}_y$ is given by;

$$\bar{\lambda}_y = \frac{l}{r_y} \cdot \frac{1}{\pi} \cdot \sqrt{\frac{\sigma_y^*}{E}} \dots\dots\dots (13)$$

where r_y : radius of gyration of column.

a) P - δ curve

The relationships between the non-dimensionalized compressive force, P/P_y , and central deflection, δ , of the columns with $R_{fw}=0.7$ are shown in Fig. 10 with the variations of $\bar{\lambda}_y$. The symbol, \bigcirc , on the P - δ curves indicates the location where the maximum stress-resultants M and P satisfy the approximate interaction curve Eq. (11) for the local buckling. Thus, the value of P at this location is taken as the ultimate strength of thin-walled box column, P_u .

It is obvious from this figure that the ultimate strength is affected by the local buckling in the columns where $\bar{\lambda}_y \leq 0.9$ and column buckling in the slender column where $\bar{\lambda}_y \geq 1.05$.

b) Ultimate strength

The ultimate strength curves of columns for the plate slenderness $R_{fw}=0.3\sim 1.3$ are plotted in Fig. 11 as a function of $\bar{\lambda}_y$, in which the curve for $R_{fw}=0.3$ well coincides with the Shultz's curve¹¹⁾ or the ultimate strength curve of box columns without local buckling. This curve is, then, referred to as a basic column curve.

It can be seen from this figure that the ultimate strength of columns decreases from the basic column curve in accordance with the increase of R_{fw} and decrease of $\bar{\lambda}_y$. The ultimate strength curves approach to the basic column curve as $\bar{\lambda}_y$ becomes large, because the ultimate strength becomes independent of the plate slenderness, R_{fw} , in accordance with the increase of $\bar{\lambda}_y$.

Table 3 Plate thickness and plate slenderness R_{fw} of box section of columns.

$t(\text{mm})$	34.24	20.54	14.67	11.41	9.34	7.90
R_{fw}	0.3	0.5	0.7	0.9	1.1	1.3

Notes: $B=D=500\text{ mm}$, $t_f=t_w=t$, $\sigma_y=314\text{ MPa}$.

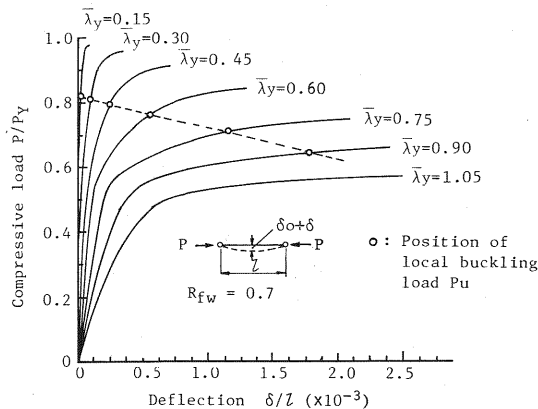


Fig.10 Compressive load P/P_y -deflection δ/l curves.

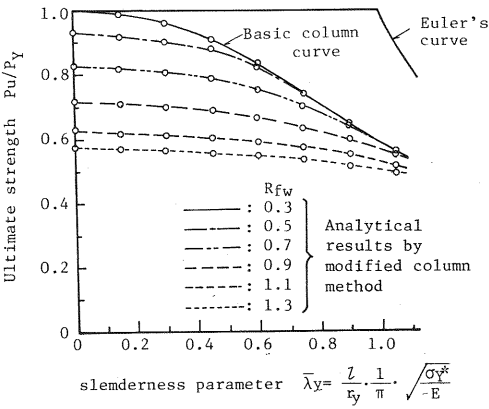


Fig.11 Column strength curves considering local buckling.

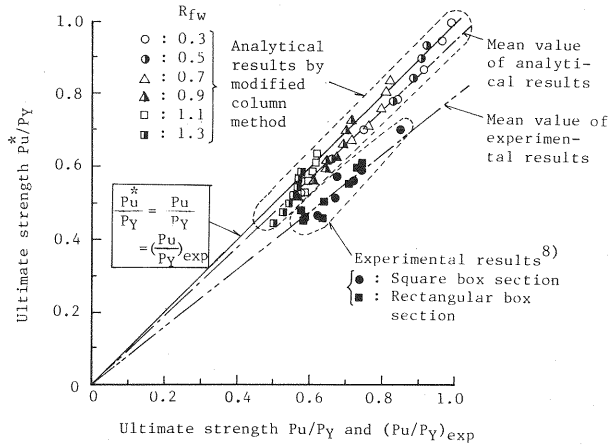


Fig. 12 Comparison of ultimate strength P_u^*/P_Y calculated by Eq. (14) with analytical results P_u/P_Y and experimental results⁸⁾ $(P_u/P_Y)_{exp}$.

(4) Approximate Formula for Ultimate Strength

The results shown in Fig. 11 lead to the following approximate formula for calculating the ultimate strength of box columns having the possibility of local buckling of plate elements;

$$\left. \begin{aligned} P_u^*/P_Y &= P_{u0}/P_Y, & k_{pl} &\geq 1 - (0.48 \cdot \bar{\lambda}_y)^2 \\ &= P_{u0}/P_Y \cdot \{k_{pl} + (0.48 \cdot \bar{\lambda}_y)^2\}, & k_{pl} &< 1 - (0.48 \cdot \bar{\lambda}_y)^2 \end{aligned} \right\} \dots\dots\dots (14)_{a, b}$$

where P_{u0}/P_Y is the ultimate strength on the basic column curve. With regard to a formula of P_{u0}/P_Y , the column strength curve of JSHB¹⁾

$$\left. \begin{aligned} P_{u0}/P_Y &= 1.0, & \bar{\lambda}_y &\leq 0.2 \\ &= 1.0 - 0.545(\bar{\lambda}_y - 0.2), & 0.2 &< \bar{\lambda}_y \leq 1.0 \\ &= 1/(0.773 + \bar{\lambda}_y^2), & 1.0 &< \bar{\lambda}_y \end{aligned} \right\} \dots\dots\dots (15)_{a \sim c}$$

can be recommended in stead of the basic column curve for the sake of simplicity.

The ultimate strengths, P_u^*/P_Y , calculated from Eq. (14) are compared with the results by the modified column method, P_u/P_Y , and with the experimental ones, $(P_u/P_Y)_{exp}$, obtained by Fukumoto et al⁸⁾, as shown in Fig. 12. From this figure, P_u^*/P_Y is nearly equal to P_u/P_Y and their errors fall within $\pm 5\%$. The approximate formula gives the conservative ultimate strength compared with $(P_u/P_Y)_{exp}$ by about 20% on average, because Eq. (14) is derived on the basis of ultimate strength of columns with the severer initial imperfections, $\delta_0 = 1/1000$, and residual stress, $\sigma_{rc} = -0.4 \cdot \sigma_y$, than $\delta_0 = 1/3850$ and $\sigma_{rc} = -0.2 \cdot \sigma_y$ of test columns on average.

In practical design, the approximate formula proposed in the above can be useful for predicting the ultimate strength of thin-walled box columns having the possibility of premature failure by local buckling, because of the fact that the approximate ultimate strength is conservative in comparison with the test results.

5. CONCLUDING REMARKS

(1) Through the preliminary study by the F.E.M. analysis, the approximate interaction curve of thin-walled box stub-columns subjected to compression and bending is defined as the fully-plastic curve divided by the factor k_{pl} which is derived by the non-dimensionalized ultimate strength of stub-column under uniaxial compression.

(2) The modified column method to calculate the ultimate strength of thin-walled box columns is proposed on the basis of a column analysis considering the local buckling of plate elements by using the approximate interaction curve with respect to an ultimate state of cross-section of columns.

(3) The approximate formula for evaluating the ultimate strength of thin-walled box columns is also proposed for the use of practical design.

(4) The modified column method and approximate formula are verified by the test results.

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