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FAILURE OF ELASTO-PLASTIC COLUMNS WITH INITIAL CROOKEDNESS IN PARAMETRIC RESONANCE

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The ultimate strength of elasto-plastic columns under the condition of parametric resonance is presented in this paper. Special attention is paid to the influence of the initial crookedness, the magnitude of the periodic and static axial force, slenderness ratio and the Bauschinger effect on the dynamic behavior.

The dynamic characteristics are obtained by using a numerical in-plane dynamic analysis which takes both the geometrical and material nonlinearities into account.

1. INTRODUCTION

When a column is subjected to a periodic axial force under a certain condition, the amplitude of the flexural vibration grows rapidly, which is the well-known phenomenon as parametric resonance. Numerous studies on the dynamic instability of columns under the periodic axial force have been carried out hitherto. For example, Bolotin[1] investigated extensively the dynamic unstable region of the column including the effect of various damping and nonlinearities. However, there are many problems left to be solved in this field.

Evensen and Evan-Iwanowski[2] have performed the extensive experiments of columns taking into account the longitudinal vibration and showed that these experimental results agreed well with the results by Bolotin. Brown et al[3]. showed an analytical procedure to determine the dynamic unstable region by the finite element method and investigated the effect of various end conditions on the unstable region. Iwatsubo et al.[4] studied the effect of end conditions experimentally and examined the existence of combination resonance. Takahashi[5),6] investigated also the unstable region of combination resonance and the dynamic system with non-uniform modal damping using the harmonic balance method.

While these works treated chiefly perfect columns, the dynamic instability of initially crooked elastic column is investigated theoretically by Bolotin[7] and Mettler[8]. They pointed out that the boundary of the unstable region was hardly affected by the initial crookedness of columns. Stevens[9] examined also the effect of the initial crookedness on the lateral response of viscoelastic columns subjected to the periodic axial force. However, the effect of the various magnitude of initial crookedness on the lateral response is not discussed sufficiently. Furthermore, the combined effect of yielding of the material and the initial crookedness on the dynamic instability has been left unknown so far. It may be required to use a numerical

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dynamic analysis in order to clarify these effects at the present time.

In this paper, the dynamic behavior and ultimate strength of elasto-plastic columns with initial crookedness under the periodic axial force are presented. The dynamic characteristics are investigated by using a numerical in-plane dynamic analysis which takes into account the geometrical and material nonlinearities. Two types of columns are considered here, one of which is subjected to static axial forces such as main member of truss bridge. Another is subjected to no static axial force such as lateral member.

Furthermore, a simple procedure which includes the Bauschinger effect in the numerical analysis is proposed. The Bauschinger effect is examined by comparing with the case where the stress-strain relationship of materials is ideal elasto-plastic.

2. METHOD OF ANALYSIS

(1) Analytical Procedure of Dynamic Ultimate Strength

The nonlinear behavior of columns with the effect of the finite displacement as well as yielding of steel material is analyzed by the finite element method and the modified incremental load method. The modified incremental load method implies here the combined use of the dynamic incremental loading and the modified Newton-Raphson method. Newmark's β method ($\beta=1/4$) is employed for the numerical integration of the incremental equations of motion. With the solution known at cartain time, the solution after an incremental time step is obtained in the iterative calculation. If the solution can not be obtained within the initial time interval, the time interval is reduced to one half or quarter of the initial value. When the solution in a iterative step does not converge in spite of reduction of the time interval, or when the deformation of the column increases rapidly, the column is considered to be in the state of collapse. These numerical procedures are fundamentally same as those in Ref. 10) except that the Bauschinger effect is included. The time interval of the numerical integration is taken to be the one-sixtyfourth of the first natural period of the flexural vibration of the column.

For example, when steel materials are unloaded at a certain tensile yielded state and successively loaded in the opposite compressive direction, the compressive yield stress decreases significantly. Namely, the material shows the Bauschinger effect. Yokoo and Nakamura et al.[11),12)] proposed the non-stationary hysteretic stress-strain relationships in the Ramberg-Osgood's representation based on the experimental data. They represented the stress-strain relationships for the reversed path from a yield plateaux in the following expression :

$$\frac{\varepsilon-\varepsilon^{(i)}}{\varepsilon_y}=\frac{\sigma-\sigma^{(i)}}{\sigma_y}\left\{1+(a)^{-r}\left|\frac{\sigma-\sigma^{(i)}}{\sigma_y}\right|^{r-1}\right\} \qquad (1)$$

in which $\sigma^{(i)}$ and $\varepsilon^{(i)}$ are the stress and strain at a reversal point on a yield plateaux and σ_y and ε_y are the yield stress and strain respectively. The parameters a and r are given by a function of the normalized plastic strain amplitude e_p (which is the maximum experienced plastic strain normalized by the yield strain). Since the strain is given as a function of stress in Eq. (1), it is inconvenient to apply this form to the numerical analysis employed here. Therefore, Eq. (1) is converted to the other form which consist of two separate straight lines and one connecting curve as follows :

$$\begin{aligned} &\bar{\sigma}=\bar{\varepsilon} && \bar{\varepsilon}<e_l \\ &\bar{\sigma}=e-C_1e^2+C_2e^3-C_3e^4+C_4e^5+e_l && e_l<\bar{\varepsilon}<e_m \qquad (2). \\ &\bar{\sigma}=2 && \bar{\varepsilon}>e_m \end{aligned}$$

Namely, the curved part in the stress-strain relationships are expressed by a series of strain, in which $\bar{\sigma}$ and $\bar{\varepsilon}$ are the normalized deviatoric stress and strain reversed from a yield state respectively (see **Fig. 1**). e_l is the normalized proportional limit strain in Eq. (1) and $e=\bar{\varepsilon}-e_l$. e_m is the normalized strain when the normalized stress is taken equal to 2 in the second expression in Eq. (2). **Fig. 1** shows the stress-strain curves obtained from Eq. (1) and Eq. (2). The curve by Yokoo and Nakamura is calculated by taking the normalized plastic strain amplitude $e_p=2$ and the proposed curve is for $e_l=1$. Both

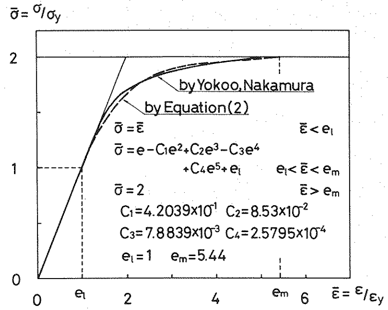

Fig. 1 Stress-strain relationship including Bauschinger effect.

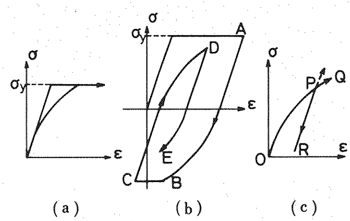

Fig. 2 Various stress-strain paths.

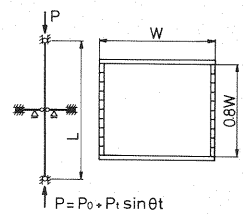

Fig. 3 Analytical model.

stress-strain curves are in good agreement. The following numerical analyses use this stress-strain relationship expressed by one set of parameters given in Fig. 1. Because, one of the purpose of this paper is to investigate the qualitative influence of the Bauschinger effect on the dynamic behavior and the ultimate strength of the columns. The following assumptions are made to include the Bauschinger effect in the stress-strain relationship :

a) When steels did not experience yielding in the past, the stress-strain relationship follows the Hooke's law.

b) When the strain increases after the stress reaches the yield stress, the stress remains at the yield stress level and only the strain increases as shown in Fig. 2(a).

c) Once the strain reverses from a yield plateaux, the stress-strain relationship follows Eq. (2) independent of the sign of the strain and the number of reversing such as the path A–B–C, C–D or D–E in Fig. 2(b).

d) When the strain reverses from the point R in the elastic region and increases beyond the strain level at the preceding reversal point P, the stress-strain path follows the preceding path such as O–P–Q in Fig. 2(c).

(2) Analytical Model and Parameters

The columns analyzed here are simply supported (restrained at the center in the longitudinal direction) and have the box cross sections as shown in Fig. 3. A column whose length is 10 m is divided into 10 equal column elements. Each cross section is divided into 22 segments as shown in Fig. 3 in order to identify yielded zones. Only weak-axis bending is considered here. The periodic axial force applied to the columns is given by

$$P(t)=P_0+P_t\sin\theta t \qquad (3)$$

in which P_0 is the static component, P_t is the amplitude of the varying component and θ is its circular frequency. By introducing $\alpha=P_0/P_{cr}$ and $\tau=P_t/P_{cr}$, Eq. (3) can be rewritten as follows :

$$P(t)=P_{cr}(\alpha+\tau\sin\theta t) \qquad (4)$$

in which P_{cr} is the static crush load of the column with the initial crookedness which is given by a sinusoidal curve with the maximum magnitude at the middle being one-thousandth of the length. The circular frequency θ of the periodic axial force is taken equal to twice the first natural circular frequency of the flexural vibration. This means that the frequency of the periodic axial force is in the principal unstable region. Subsidiary members without the initial static axial compression force ($\alpha=0$) and main members subjected usually to the initial static axial compression force (α is

Table 1 Characteristics of cross section.

Slenderness Ratio	30	60	90	120	150
Dimension of Flange (mm)	970x23	480x17	310x21	233x18	183x14
Dimension of Web (mm)	776x23	384x17	248x21	186x18	146x14
1st Natural Circular Frequency (rad/s)	155.6	79.7	54.4	41.0	32.7
Static Crush Load (MN)	18.30	6.24	4.08	1.79	0.75
Ratio of Crush Load to Yield Axial Force	0.969	0.903	0.740	0.509	0.347

taken as 0.5 here) are analyzed. The slenderness ratio of the columns ranges from 30 to 150 and the columns have about the 1/1 000 initial crookedness except for the case in chapter 3. (1), b). Table 1 shows the dimensions of the flanges and webs, the first natural circular frequency and the ratio of the static crush load with the 1/1 000 initial crookedness to the yield axial force for each slenderness ratio. Young's modulus of steels is 206 GN/m^2, the yield stress is 235 MN/m^2 and the mass per unit volume is 7.85 Mg/m^3. The residual stresses and the viscous and structural damping are not taken into account.

3. NUMERICAL RESULTS

(1) Dynamic Behavior of Columns under Periodic Axial Force (No Static Axial Force)

a) Fundamental dynamic behavior of columns

Fig. 4 shows the lateral displacement response at the middle of a column under the periodic axial force whose amplitude is a half of the static crush load (τ=0.5). The slenderness ratio of the column is 120. The ordinate is the lateral displacement response normalized by the column length and the abscissa is the elapse time normalized by the first natural period of the column. A circle in this figure indicates the inception of yielding in a cross section. The flexural vibration is amplified by parametric resonance, but as yielding starts to spread in the cross sections of the column, the vibration approaches to a steady state. The relationship between the resisting bending moment and the curvature in this case is shown in Fig. 5. The ordinate is the resisting bending moment normalized by the yield bending moment and the abscissa shows the curvature normalized by the yield curvature. This relationship is indicating a stationary hysteretic loop after yielding occurs in the cross section of the column. The area bounded by the loop is the dissipated energy caused by the hysteretic damping in one cycle. If the input energy caused by the external force is balanced to the dissipated energy, the displacement response becomes a steady state. Even in case where the cross section does not yield, there is a possibility of stationary vibration owing to the geometrical nonlinearity. But in this analysis, the flexural vibration reaches a stationary state only by the hysteretic damping caused by yielding of cross sections, not by the geometrical nonlinearity.

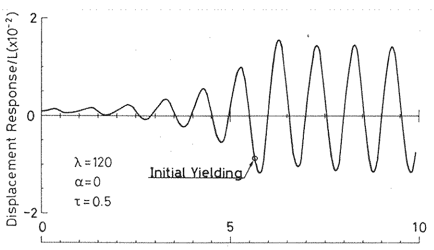

Fig. 4 Lateral displacement response at midheight (τ=0.5).

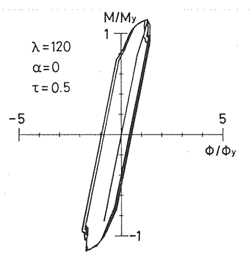

Fig. 5 Resisting bending moment versus curvature relationship at midheight.

b) Influence of the initial crookedness

The columns have the initial crookedness whose deflectional configuration is a half sinusoidal curve here. Fig. 6 shows the lateral displacement response of a column with the initial crookedness whose magnitude at midheight is one-hundredth of the column length. The vibration stimulated by this imperfection becomes rather larger than the vibration stimulated by the initial crookedness of one-

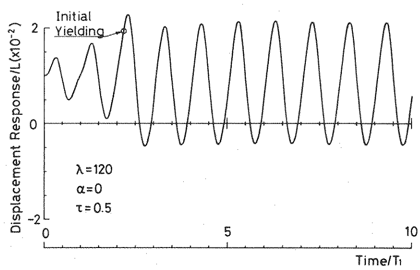

Fig. 6 Lateral displacement response at midheight (initial crookedness $=L/100$).

thousandth of the column length as shown in **Fig. 4**. However, both vibrational amplitudes in a steady state have almost the same value. Therefore, it can be concluded that the stationary vibrational amplitude might be independent of the magnitude of the initial crookedness of the column.

c) Influence of magnitude of periodic axial force

Fig. 7 shows the lateral displacement response at midheight, being taken as $\lambda=120$ and $\tau=1.0$. Within ten cycles after the inception of yielding, the vibrational amplitude of the displacement response remains almost constant. However, the center of the vibration gradually shifts downwards and the vibration becomes a steady state decreasing its amplitude. The vibrational configurations at the both peaks ① and ② are plotted in **Fig. 7**. The middle part of the vibrational configuration ① is swollen in comparison with the first natural mode shape of the flexural vibration. It is because the yielded zones are concentrated in the vicinity of the middle of the column. This plastic deformation results in the residual deflection. When the column is subsequently displaced to the opposite side, the lateral deflection in the vicinity of the middle of the column can not sufficiently develop owing to the residual deflection. Therefore, the vibrational configuration ② in **Fig. 7** becomes similar to the combined one of the first natural mode shape and the third one. **Fig. 8** shows the relationships between the lateral displacement and resisting bending moment, applied axial force and the stress distribution at midheight in one cycle after the cross section starts to yield. The abscissa is the elapse time, while the ordinates are the lateral displacement response and the resisting bending moment normalized by the column length and the yield bending moment respectively. The stress distribution in the cross section is shown below the abscissa and the applied axial force expressed by the mean stress is shown above the abscissa. The left-hand side of the vertical lines representing the cross section or (+) sign indicates tension and the right-hand side or (—) sign indicates compression. The shaded portion shows the yielded zone. When the axial compression force becomes maximum, the lateral displacement and the resisting bending moment do not always reach their peaks. Therefore, the equilibrium state of column does not become critical, even if the column is subjected to the periodic axial force whose amplitude is equal to the static crush load.

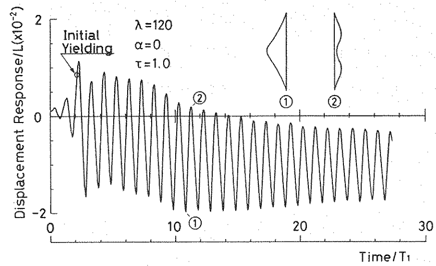

Fig. 7 Lateral displacement response at midheight ($\tau=1.0$).

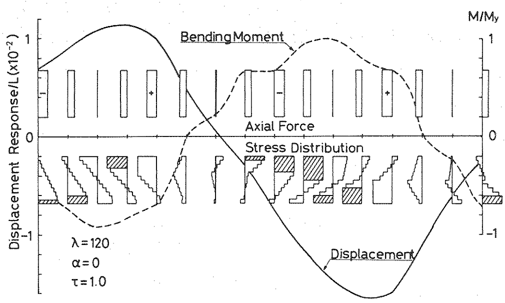

Fig. 8 Applied axial force versus stress distribution at midheight relationship ($\tau=1.0$).

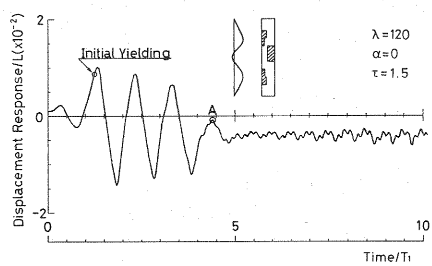

Fig. 9 Lateral displacement response at midheight ($\tau=1.5$).

Fig. 9 shows the lateral displacement response at midheight, being taken as $\lambda=120$ and $\tau=1.5$. The cross sections in the vicinity of midheight start to yield during the second cycle, because the flexural vibration of the column rapidly diverges by parametric resonance. The amplitude of the displacement response becomes rapidly small after the fourth cycle and the vibration with a short period appears. The vibrational configuration and the yielded zones of the column corresponding to point A is given in **Fig. 9**.

The vibrational configuration is different from the first natural mode shape, because the yielded zones spread extensively into the cross sections and are concentrated in the vicinity of the middle of the column. The change of the vibrational configuration causes the additional yielded zones as shown in Fig. 9. The extensive development of yielded zones results in the great hysteretic damping effect and a change of the stiffness of the column, which reduce the amplitude of the lateral displacement rapidly. The above results indicates that the amplitude of the lateral displacement in the steady state is much affected by the magnitude of the periodic axial force. However, the vibrational amplitude in the steady state does not always increase with increment of the magnitude of the periodic axial force owing to the difference in the spreading manner of the yielded zones.

d) Influence of slenderness ratio

Fig. 10 shows the lateral displacement response at the middle of the column for $\lambda=30$. The periodic axial force equal to the static crush load is applied to the column. In this case the resultant force of the applied force and the inertia force in the longitudinal vibration causes yielding over the entire cross section in the vicinity of midheight. Because the difference between the static crush load and the yield axial load of short columns becomes very small. However, owing to the hysteretic damping caused by this yielding, the flexural vibration does not develop even under the condition of parametric resonance. The vibrational amplitude is smaller than the initial crookedness, the period is equal to that of the applied axial force, and the cross sections start to yield by the slight development of the flexural vibration. When the slenderness ratio of the columns is greater than or equal to 60, the flexural vibration diverges by parametric resonance and becomes a steady state. The larger the slenderness ratio becomes, the larger the amplification factor and the vibrational amplitude are in a steady state. Consequently the collapse does not occur for the elasto-plastic columns with slenderness ratio ranging from 30 to 150 when they subjected to the periodic axial force whose magnitude is equal to the static crush load.

e) Influence of Bauschinger effect

The resultant force of the applied and the inertia force in the longitudinal vibration causes yielding over the entire cross sections in the vicinity of the middle of the column in the range of $\lambda=30$ and $\tau=1.0$ before the periodic axial force reaches the initial peak of compression. Fig. 11 shows the stress-strain diagram of a flange at midheight when the Bauschinger effect is taken into account. When the compressive yielding occurs first, the plastic strain amplitude is smaller than the yield strain. As the stress-strain relationship is affected by the Bauschinger effect for the paths reversed from the yield state, the strain amplitude becomes very large on the subsequent tensile path. During the subsequent compressive path, the column collapses, because the yielded zones spread into the cross sections and the stiffness of the column is deteriorated significantly. For $\lambda=60$ and $\tau=1.0$, the column also collapses for the same reason. In both cases, however, the plastic strain amplitude produced by the initial yielding is much smaller than twice the yield strain. Therefore, the Bauschinger effect expressed by Eq. (2) seems to estimate the strength of columns conservatively. Fig. 12 shows the lateral displacement response at midheight, being taken as $\lambda=$

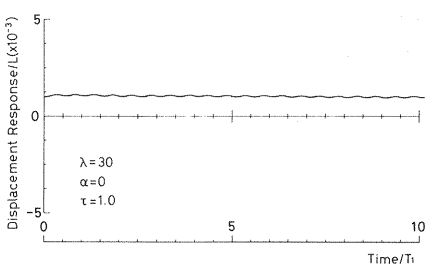

Fig. 10 Lateral displacement response at midheight ($\lambda=30$).

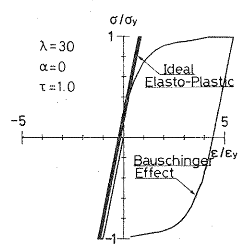

Fig. 11 Stress-strain relationship at midheight ($\lambda=30$).

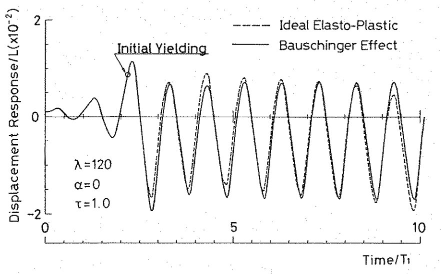

Fig. 12 Lateral displacement response at midheight including Bauschinger effect ($\lambda=120$).

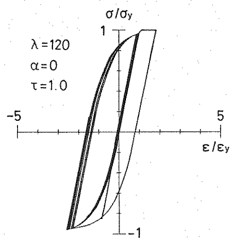

Fig. 13 Stress-strain relationship at midheight including Bauschinger effect ($\lambda=120$).

120 and $\tau=1.0$. The solid line presents the case with the Bauschinger effect and the dashed line presents the case for the ideal elasto-plastic relationship. The initial yielding occurs in the positive region during the third cycle. As the subsequent stress-strain path is affected by the Bauschinger effect, the strain amplitude becomes larger than that without the Bauschinger effect. Therefore, the displacement response becomes larger in this case than that in the case for the ideal elasto-plastic one. Fig. 13 shows the stress-strain diagram of the flange at midheight in the case where the initial yielding occurs in the tensile path. In the subsequent compressive path, the large plastic strain develops owing to the Bauschinger effect. But the column does not collapse and the stress-strain diagram forms a stationary hysteretic loop. In the case where $\lambda=120$, $\tau=1.2$ and $\lambda=150$, $\tau\geqq 0.6$, however, the spread of the yielded zone by the Bauschinger effect results in the significant reduction of the flexural rigidity and columns are brought out to collapse.

(2) Dynamic Behavior of Columns under Periodic and Static Axial Force

a) Fundamental dynamic behavior

In order to discuss the characteristic dynamic behavior of the main compression members of truss bridges, columns subjected to a half of the static crush load ($\alpha=0.5$) are investigated here. Fig. 14 shows the lateral displacement response at midheight for $\lambda=90$ and $\tau=0.5$. The amplitude of the flexural vibration increases by parametric resonance, and yielding of the cross section occurs in the vicinity of the middle of the column. In this figure, a circle shows the time when the initial yielded zone appears. The residual displacement occurs owing to yielding of the cross sections, and the center of the lateral displacement response moves gradually to the upward direction. When the magnitude of this movement reaches a certain magnitude, the period of the displacement response changes from that of the first natural mode. After the sequential increment of the dynamic residual displacement, the lateral displacement diverges rapidly and the column collapses. The monotonic shift of the displacement response is caused by the energy supplied by the static axial force to the system as mentioned in Ref. (10). If the column deforms first plastically in one direction, the supply of the potential energy to the system by the static axial force

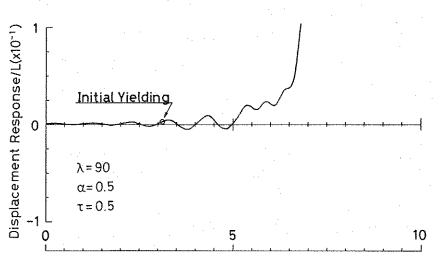

Fig. 14 Lateral displacement response at midheight ($\alpha=0.5$, $\lambda=90$).

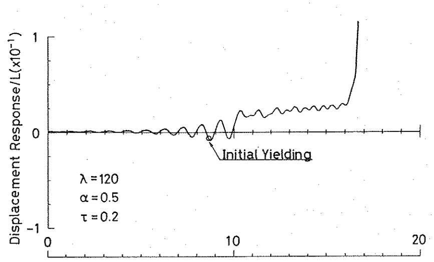

Fig. 15 Lateral displacement response at midheight ($\alpha=0.5$, $\lambda=120$).

promotes the deformation in this direction. If the column deforms in the opposite direction of the residual displacement, the absorption of the energy caused by the static axial force reduces the amplitude of the displacement response. Therefore, the center of the displacement response shifts to only one direction decreasing gradually its amplitude. Even if columns do not collapse within 20 cycles of loading of the periodic axial force, the subsequent periodic forces are considered to lead the column collapse. Fig. 15 shows the lateral displacement response at midheight for $\lambda=120$ and $\tau=0.2$. After the eleventh cycle, the period of the column changes from the period of parametric resonance into that of the applied periodic axial force. The center of the displacement response shifts to only the upward direction and the column collapses.

Fig. 16 shows the relationships between the magnitude of the applied axial force and the critical lateral displacement at midheight of the column of $\lambda=90$ and 120. The magnitude of the applied force is normalized by the yield axial force, and the lateral deflection is normalized by the column length. Both the triangles for $\lambda=90$ and the crosses for $\lambda=120$ indicate the critical lateral displacement at which the vibration begins to diverge under the dynamic forces (P_o+P_t). The solid line shows the maximum applied axial force and the corresponding lateral deflection, when the column is loaded only by the static axial force. The dynamic ultimate state comes right above the static one. This means that if the vibrational lateral deflection becomes larger than the static ultimate deflection, the vibrational lateral displacement increases rapidly and the column collapses. The column sometimes suddenly collapses without the change of the period. The reason is that the dynamic equilibrium between the applied axial force and the resisting bending moment fails right after the static equilibrium fails before thd period of the displacement response changes.

Generally speaking, the period of the displacement response changes under the following three conditions,

1) When the column is subjected to the static axial force.
2) When yielding of cross sections occurs.
3) When the center of the vibration moves to one direction.

The first condition combined with the second condition leads the third condition by the energetic effect of the static axial force as described above. Then, the residual displacement and yielding of the cross sections should be considered to be associated with the dynamic behavior of the column. Namely, the dynamic residual displacement and yielding alter the natural frequency of the column affected by the significant reduction of the stiffness. Therefore, the natural frequency of the column shifts to the outside of the principal unstable region.

b) Influence of Bauschinger effect

Fig. 17 shows the lateral displacement response at midheight for $\lambda=90$, $\tau=0.5$ under the Bauschinger effect. After the flexural vibration develops by parametric resonance, the center of the lateral displacement response moves upward sequentially and the period of the displacement response changes. However, the column does not collapse within 20 cycles of loading of the periodic axial force. Fig. 18 shows

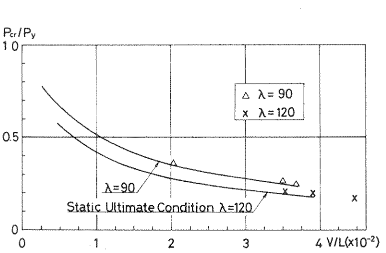

Fig. 16 Lateral displacement at midheight versus magnitude of applied axial force.

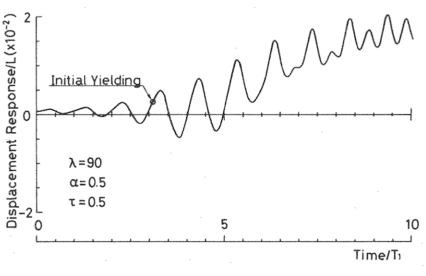

Fig. 17 Lateral displacement response at midheight including Bauschinger effect ($\lambda=90$).

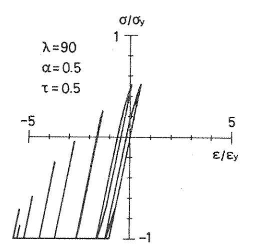

Fig. 18 Stress-strain relationship at midheight including Bauschinger effect ($\lambda=90$).

the stress-strain diagram of the flange at midheight. This curve forms a hysteretic loop owing to the Bauschinger effect after the stress reaches the yield state. Consequently, the hysteretic damping occurs and the shift of the lateral displacement response is smaller than that without the Bauschinger effect. This implies that the Bauschinger effect does not always decrease dynamically the ultimate strength.

(3) Comparison between Dynamic and Static Ultimate Strength

Influences of the initial crookedness, magnitude of the periodic axial force, slenderness ratio, the Bauschinger effect and the static axial force on the dynamic behavior and the ultimate strength of columns subjected to the periodic axial force were investigated here. The results are summarized in Fig. 19. Fig. 19 (a) shows the results without the static axial force, and Fig. 19(b) with it. The ordinate shows the maximum magnitude of the periodic axial force including the static component normalized by the yield axial force. The abscissa shows the slenderness ratio and slenderness ratio parameter. The circles show the case where the stress-strain relationship is the ideal elasto-plastic one, while the triangles show the case where the Bauschinger effect is taken into account. Closed symbols indicate that columns are brought out to collapse within 20 cycles of loading of the periodic axial force. Namely, the open and closed symbols roughly estimate the boundary whether columns are brought out to collapse or not. The solid line shows the static crush load of columns with the 1/1 000 initial crookedness and the dashed line shows the yield axial force and Euler's curve.

When columns are not subjected to the static axial force, columns with the ideal elasto-plastic stress-strain relationship do not collapse under the periodic axial force whose maximum magnitude is equal to or below the static crush load. The periodic axial force whose maximum magnitude is greater than the Euler load does not always lead the column to failure.

When the Bauschinger effect is taken into account, the column collapses because of the reduction of the stiffness of the column for $\lambda \leqq 60$ and $\lambda \geqq 120$.

When columns are subjected to the static axial force equal to the half of the static crush load, the dynamic ultimate strength is smaller than the static one for $\lambda \geqq 90$ if the material is the ideal elasto-plastic. The dynamic ultimate strength of the columns with the stress-strain relationship including the Bauschinger effect is a little greater than that without the Bauschinger effect. But for $\lambda \geqq 120$, the dynamic ultimate strength is smaller than the static one.

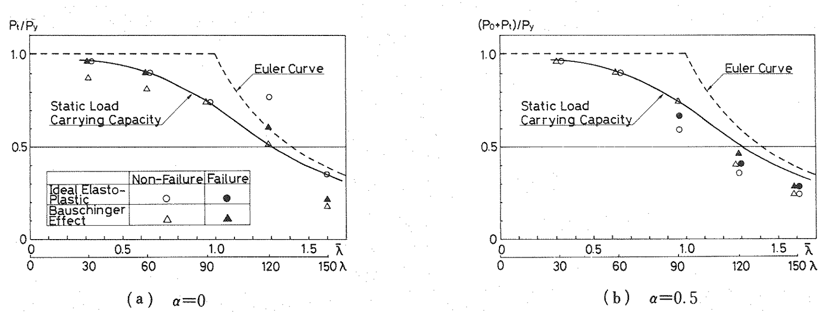

Fig. 19 Comparison between dynamic ultimate strength within 20 cycles of loading of periodic axial force and static one.

4. CONCLUSIONS

In this paper, the dynamic behavior and ultimate strength of columns subjected to the periodic axial force with or without the static axial force are investigated.

Some important findings are summarized as follows :

(1) The case where columns are not applied by static axial force

a) When columns are subjected to a certain magnitude of the periodic axial force, the flexural vibration of the columns grows by parametric resonance and a certain cross section starts to yield, but the columns do not always collapse owing to the hysteretic damping effect.

b) The amplitude of the lateral displacement response at parametric resonance is not affected by the magnitude of the initial crookedness, but affected by the magnitude of the periodic axial force and the slenderness ratio.

c) The dynamic ultimate strength is slightly greater than the static one and is not affected by the Bauschinger effect, except when the slenderness ratio is very large.

(2) The case where columns are applied by the static axial forces

a) When columns are subjected to the periodic and static axial force, a certain cross section starts to yield and the center of the lateral displacement response gradually moves to a direction and the columns are brought out to collapse.

b) When the lateral displacement response rapidly increases and the column collapses, the period of the lateral displacement is occasionally transferred to the period of the applied axial force.

c) The dynamic ultimate strength of columns made of materials with the Bauschinger effect is larger than that without the Bauschinger effect.

d) The dynamic ultimate strength of columns within 20 cycles of loading of the periodic axial force is smaller than the static one when the column has the relatively large slenderness ratio.

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