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A CONSIDERATION ON THE EQUIVALENT LINEARIZATION OF RESTORING FORCE CEARACTERISITICS OF STRUCTURES

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Concerning the dynamic response analysis based on the equivalent linearization of the restoring force characteristics of structures, some examinations taking the mechanism of the analytical procedure into consideration have been conducted with one-degree-of-freedom system. As the results, the extent of difference between the solutions by equivalent linear method and exact ones for both steady and unsteady conditions have been clarified. The causes of difference have been also clarified. With these results the limit of validity in the application of the equivalent linear solutions to the seismic design of structures has been shown clearly. Throughout the process of above consideration some of new knowledge concerning the numerical procedure for non-linear dynamic analysis have been obtained and reported.

1. INTRODUCTION

It is well known fact that the force-deflection relations of structures show non-linear hysteretic curves during strong motion earthquakes without regard to the materials such as steel, reinforced concrete and soils. Seismic responses of structures beyond yield point have been investigated since Tanabashi[1].

Loading tests of completely reversed tension and compression beyond yield stress on steel specimens are conducted frequently as the low cycle fatigue tests aiming at measuring thermal fatigue[2], and many studies are performed trying to formulate hysteretic stress-strain relations[3]. From the viewpoint of predicting the response behaviors of steel structures during strong motion earthquakes, Nakamura et al.[3] propose rather general constitutive equations of steel beyond yield point. One of the authors performs an experimental study to examine the effect of strain rate on the hysteresis loops of mild steel in main frequency range of earthquake motions[4]. Instead of steel specimens frames, columns etc. are used in plastic region repetitive loading tests by many researchers such as Igarashi et al.[5], Hanson[6] etc.

Lots of studies concerning the restoring force characteristics of reinforced concrete frames (abbreviated as RC in the following) have been performed here and abroad. Odaka and Saito[7] show that every shape of hysteretic curves of RC frames changes from bi-linear type to medium type between bi-linear and double bi-linear[8] as applied axial force is changed. Clough and Johnston[9] propose Degrading Stiffness system of a multi-linear type for RC frames. Aoyama et al[10] investigate the behaviors of RC portal frames subjected to reversal of horizontal forces under vertical loading and clarify the mode of failure, frame stiffness and its reduction. Shiga et al.[11] compare static hysteretic load-deflection loops of RC frames with dynamical ones

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and show that the static loop is very similar to the dynamical one. They investigate also experimentally damping and stiffness of two story RC frames[12]. Funabashi et al.[13] perform static repetitive horizontal loading tests and forced vibration tests on a steel frame with steel braces and syporexes as aseismic elements and propose a load-deflection curve of double bi-linear type. Abe et al.[14] propose Degrading Siffness Model for RC frames. Tani et al.[15] propose a unique normalized hysteretic curve for RC frames normalizing the load-deflection relations with both maximum load and deflection and name it Normalized Characteristic Loop. Nielsen et al.[16] propose Degrading Bi-Linear System of a multi-linear type of which stiffness is reduced when the response deflection exceeds the preceding maximum response.

The dynamical stress-strain relationships of soils are usually described by the secant moduli defined with extreme points of stress-strain hysteretic curves and the equivalent viscous damping constants. Hardin and Drnevich[18] formulate strain amplitude dependence of above dynamic constants by hyperbolic model etc. Formulation of constitutive equations for sands has been almost brought to completion[19]. There may be, however, only a little papers where shapes of stress-strain hysteresis loops are investigated. Kitaura[20] performs dynamic loading tests on a model structural foundation-surface layer system and shows that the shapes of restoring force-relative displacement relationships of sand layer are similar to the inverse letter of S and formulates the restoring force curves with a modified Ramberg-Osgood model(abbreviated as R-O model in the following). Hara et al.[21] measure the dynamical properties of Kwanto-Loam and show that no frequency dependence is observed in damping constants. One of the authors proposes bi-linear model for dynamic stress-strain relationships of crushed stones[22].

As mentioned above the actual states of restoring force characteristics are being clarified for each sort of structural elements or material specimens. In applying these restoring force characteristics to seismic response analysis of structures we have to express them as simple as possible because every structure consists of lots of elements. We have a equivalent linearization method as one of the simplification procedure of restoring force characteristics. A fundamental principle of this method is that every hysteretic curve is first replaced approximately by that of visco-elastic Voigt model with a certain assumed spring constant. Then critical damping ratio of one-degree-of-freedom system is defined so as to make the loss energy per a cycle of original hysteresis be equal to that of Voigt body. Iemura et al.[23] show that dynamic constants thus derived from loss energy balance coincide with those of the least mean square error method. The equivalent linearization technique is devised originally for the system of which non-linearity is not so strong, so that the spring constant is established primarily as initial tangent modulus of hysteretic curve. Consequently equivalent viscous damping constant has a upper limit such as 15.9 % for both bi-linear and R-O models[24,25]. In soils, however, maximum of usually described damping constant is larger than above and attains to 20 ~30%. This discrepancy comes out of the difference between both values of elastic potential energy[26]. Jacobsen[27] proposes to adopt "Work Area Under Skelton "as potential energy recognizing that there is an ambiguity about its definition. Kokusho et al.[28] clarify nonlinear response behaviors of a model sand layer excited on a shaking table and simulate the experimental results with seismic response analysis applying dynamic constants of sand obtained by above equivalent linearization technique in soils and show that calculated acceleration responses to the excitation of recorded accelerograms agree considerably well with observed responses in both peak amplitudes and phases but the discrepancy appears remarkably in the smaller parts. On the other hand Tani et al.[29] examine numerically seismic response behaviors of one-degree-of-freedom system with various sorts of multi-linear hysteresis loops and point out that there exists an influence of configuration of slip type in hysteresis loop on the calculated responses. Thus it seems that the equivalent linearization technique is practically valid, but its applicability seems to be limitted. In this paper, taking the mechanism of equivalent linearization technique into consideration, it is examined with one-degree-of-freedom system whether the equivalent linearization technique is valid or not, how much discrepancy develops between exact solutions and equivalent linear solutions both in steady and random responses and what is the reason for above discrepancy if it appears.

2. COMPARISON OF EQUIVALENT LINEAR SOLUTIONS AND EXACT ONES IN STEADY STATE RESPONSES

(1) Summary of solutions based on the method by Kryloff and Bogoriuboff

As for steady state responses of single-degree-of-freedom oscillators having hyseretic force-deflection relationships to sinusoidal excitations Jennings[24] shows that approximate analytical solution based on Kryloff-Bogoriuboff method (abbreviated K-B solution in the following) agrees well with exact one. In this paper we confirmed it too as shown in Fig. 10. So that, it will be valid for us to compare the equivalent linear solutions with K-B ones. The equation of motion for the forced vibration of a mass m mounted on a spring of which restoring force is expressed by the function $F(x,t)$ of both time t and relative displacement x is

$$m\ddot{x}+F(x,t)=f_0\cos\omega t \tag{1}$$

Let

$$k/m=\omega_0^2,\quad \omega_0 t=\tau,\quad x/x_y=y,\quad F_y=kx_y,\quad \omega/\omega_0=\eta,\quad f_0/F_y=f \tag{2}$$

where k represents the spring constant in initial or micro-displacement and x_y represents a yielding or a characteristic displacement. Substituting Eq. (2) into Eq. (1) we obtain following expression.

$$d^2y/d\tau^2+F(y,\tau)/F_y=f\cos\eta\tau \tag{3}$$

where y,τ represent normalized displacement and time respectively, f represents amplitude of normalized forced acceleration and η represents normalized frequency. Solution of Eq. (3) for the steady state response is obeained as follows

$$y(\tau)=y_0\cos(\eta\tau+\phi_0) \tag{4}$$

$$X=\eta^2-1=C(y_0)/y_0-1\pm\sqrt{(f/y_0)^2-\{S(y_0)/y_0\}^2} \tag{5}$$

$$\tan\phi_0=S(y_0)/\{C(y_0)-y_0\} \tag{6}$$

$$C(y_0)=\frac{1}{\pi}\int_0^{2\pi}\frac{F(y_0\cos\theta,\tau)}{F_y}\cos\theta\,d\theta \tag{7}$$

$$S(y_0)=\frac{1}{\pi}\int_0^{2\pi}\frac{F(y_0\cos\theta,\tau)}{F_y}\sin\theta\,d\theta \tag{8}$$

where $S(y_0)$ is in proportion to the area bounded by the hysteresis loop. In resonance following equations have been derived from Eq. (5) to determine the resonant frequency and the maximum amplitude.

$$\eta_{res}^2=C(y_0)/y_0,\quad S(y_0)=f \tag{9}$$

(2) Mechanism of equivalent linearization technique

Let k_{eq} and c_{eq} be the equivalent spring constant and viscous coefficient respectively and let

$$k_{eq}/k=\kappa^2,\quad h_{eq}=c_{eq}/(2\sqrt{mk_{eq}}) \tag{10}$$

then Eq. (3) becomes

$$\frac{d^2y}{d\tau^2}+2h_{eq}\kappa\frac{dy}{d\tau}+\kappa^2y=f\cos\eta\tau \tag{11}$$

Substituting Eq. (4) into Eq. (11) resonance curve and phase angle are obtained in the following expressions.

$$y_0^2\{(\kappa^2-\eta^2)^2+4h_{eq}^2\kappa^2\eta^2\}=f^2 \tag{12}$$

$$\tan\phi_0=-2h_{eq}\kappa\eta/(\kappa^2-\eta^2) \tag{13}$$

Letting ΔW be the area bounded by the hysteresis loop and letting $W=k_{eq}y_0^2/2$, equivalent viscous damping constant h_{eq} is given in the following expression.

$$h_{eq}=\Delta W/(4\pi W) \tag{14}$$

ΔW derived for general hysteresis loop is found to be in proportion to $S(y_0)$, and following expression is obtained for Eq. (14).

$$h_{eq}=-S(y_0)/(2\kappa^2y^2) \tag{15}$$

The parameters κ^2 and h_{eq} in Eq. (11) correspond to dynamic constants of soils such as G/G_0 and h

respestively[19], and y_0 corresponds to γ/γ_γ in soils[17]. Eq. (11), for an example of the simplest simulation of general structure, is solved first numerically with given initial values of κ^2 and h_{eq}. Then defining new values from the constitutive equations, that is, relationships of $\kappa^2 \sim y_0$ and $h_{eq} \sim y_0$ with above solution y_0, Eq. (11) is solved again. Above operations should be iterated untill the solution comes to be compatible with the constitutive equations. In the present simplified simulation, however, the solution for steady state response of the equivalent linear system to sinusoidal excitation can be obtained analytically such as Eqs. (12), (13) and (15). With similar form to K-B solution, the equation of resonance curve is derived from Eqs. (12) and (15) as follows

$$X=\eta^2-1=(1-2h_{eq}^2)\kappa^2-1\pm\sqrt{f^2/y_0^2-S(y_0)^2(1-h_{eq}^2)/y_0^2} \quad \cdots\cdots (12)'$$

When the equivalent viscous damping is sufficiently small, that is, $h_{eq}^2 \ll 1$ above equation becomes

$$X=\eta^2-1 \fallingdotseq \kappa^2-1\pm\sqrt{(f/y_0)^2-\{S(y_0)/y_0\}^2} \quad \cdots\cdots (16)$$

On the other hand, equating the loss energy due to hysteresis loop to the one due to viscousity of the equivalent linear system, following equation is obtained.

$$h_{eq}=-S(y_0)/(2\kappa\eta y_0) \quad \cdots\cdots (17)$$

Comparing Eq. (17) with Eq. (15) it is reconfirmed that above two equations coincide only in resonant condition. Moreover, noting that Eq. (16) comes rigorously into extence when Eq. (17) is substituted into Eq. (12), it may be said that in numerical analysis with the equivalent viscous damping constant defined with Eq. (15) the loss energy balance holds accurately even in any frequency range out of resonant point if the value of h_{eq} is small to the extent as $h_{eq}^2 \ll 1$. The resonant frequency and the maximum amplitude are derived from Eq. (16) and are given in similar form to K-B solutions (9) as follows

$$\eta_{res}^2=\kappa^2, \quad S(y_0)=f \quad \cdots\cdots (18)$$

It is evident from Eq. (15) that the equivalent viscous damping constant though defined from a unique hysteresis loop gives different value if the definition of equivalent linear spring constant κ^2 is different. As is evident from Eq. (18), however, the response amplitude in resonance remains constant independently of definition of κ^2 because it is to be determined only with the area bounded by hysteresis loop and the term of external disturbance.

(3) Comparison of equivalent linear solutions and both K-B and exact ones

The equivalent linear solutions linearized with secant moduli of hysteresis loops in the steady state responses were derived for bi-linear model, R-O model and 4 sorts of tri-linear models which show various restoring force characteristics according to the path of maximum response amplitude. Besides, K-B solutions in the steady state responses were derived for above 4 sorts of tri-linear models. These results are tabulated in Table 1 and calculated resonant curves are shown in Fig. 1. This indicates that resonant amplitudes in both solutions coincide perfectly for every model and yet both resonant curves are very similar to each other except for a slight difference between resonant frequencies. K-B solution with R-O model was compared with exact one which will be gone into details later. In any way Fig. 10 (b) indicates that both resonant curves coincide almost perfectly.

3. COMPARISON OF EQUIVALENT LINEAR SOLUTIONS WITH EXACT ONES IN THE RESPONSES UNDER RANDOM EXCITATIONS

(1) Comparisons of responses and these Fourier Spectra to earthquake excitations

The term of earthquake excitation $f_0(t)$ is to be used instead of sinusoidal term in Eq. (1). Normalizing as Eq. (2) and substituting following non-dimensional acceleration $f(\tau)$ for $f_0(t)$ in Eqs. (3) and (11) we have the equations of motion to obtain both exact and equivalent linear solutions.

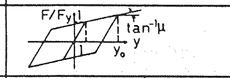
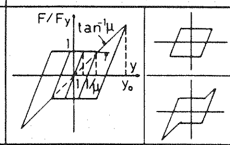
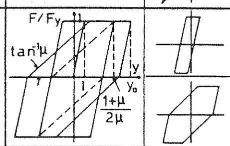
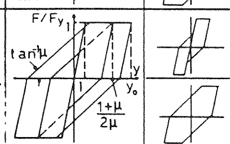
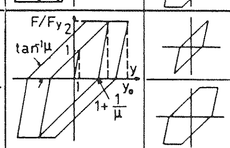
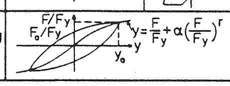
$$f(\tau)=f_0(t)/F_y \quad \cdots\cdots (19)$$

When $f_0(t)$ is given in time interval Δt, time interval of $\Delta\tau$ is to be given as follows

$$\Delta\tau=\omega_0\Delta t \quad \cdots\cdots (20)$$

We used linear acceleration method in step-by-step integration and stress transfer method in non-linear

Table 1 Equivalent Linear Solutions and K-B Solutions.

Sort of Model	Hysteresis Loop	heq	Sort of Solution	η_{res}^2	$X=\eta^2-1$
Bi-Linear		$\frac{2}{\pi}\cdot\frac{1-\mu}{1+\mu(y_0-1)}\cdot\frac{y_0-1}{y_0}$	K-B	$\mu+(1-\mu)(\Theta-0.5\sin 2\Theta)/\pi$	Kryloff & Bogoriuboff $\eta_{res}^2-1\pm\sqrt{(\frac{f}{y_0})^2-(2\varkappa^2 h_{eq})^2}$ Equivalent Linear $\varkappa^2-1\pm\sqrt{(\frac{f}{y_0})^2-(2\varkappa^2 h_{eq})^2}$
			E-L	$[1+\mu(y_0-1)]/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta=1-2/y_0,\ 1\leq y_0$		
Tri-Linear 1		$\frac{2}{\pi}\cdot\frac{y_0-1}{y_0}$	K-B	$(\Theta-0.5\sin 2\Theta)/\pi$	
			E-L	$1/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta=1-2/y_0,\ 1\leq y_0\leq 1/\mu$		
		$\frac{1}{2\pi}(1-\mu)(1+\frac{1}{\mu y_0})^2$	K-B	$\mu+[\Theta_1-0.5\sin 2\Theta_1-\mu(\Theta_2-0.5\sin 2\Theta_2)]/\pi$	
			E-L	$\mu\ (=\varkappa^2)$	
		Remarks	$\cos\Theta_1=1-\mu-1/\mu,\ \cos\Theta_2=-1/\mu y_0,\ 1/\mu\leq y_0$		
Tri-Linear 2		$\frac{2}{\pi}\cdot\frac{y_0-1}{y_0}$	K-B	$(\Theta-0.5\sin 2\Theta)/\pi$	
			E-L	$1/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta=1-2/y_0,\ 1\leq y\leq 0.5(1+1/\mu)$		
		$\frac{1}{2\pi}\frac{\mu(4y_0-3)-1}{\mu y_0}$	K-B	$[(1-\mu)(\Theta_1-0.5\sin 2\Theta_1)+\mu(\Theta_2-0.5\sin 2\Theta_2)]/\pi$	
			E-L	$1/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta_1=1-1/y_0,\ \cos\Theta_2=1-1/y_0-1/\mu y_0,\ 0.5(1+1/\mu)\leq y_0$		
Tri-Linear 3		$\frac{1}{2\pi}\cdot\frac{3-\mu(2+y_0)}{1-\mu}\cdot\frac{y_0-1}{y_0}$	K-B	$[(1-\mu)(\Theta_1-0.5\sin 2\Theta_1-\Theta_2+0.5\sin 2\Theta_2)+\Theta_3-0.5\sin 2\Theta_3]/\pi$	
			E-L	$1/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta_1=(2y_0-3+\mu)/2(1-\mu)y_0,\ \cos\Theta_2=-1/2y_0,\ \cos\Theta_3=-1/y_0,\ 1\leq y_0\leq 0.5(1+1/\mu)$		
		$\frac{1}{2\pi}\frac{\mu(4y_0-3)-1}{\mu y_0}$	K-B	$[(1-\mu)(\Theta_1-0.5\sin 2\Theta_1)+\mu(\Theta_2-0.5\sin 2\Theta_2)]/\pi$	
			E-L	$1/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta_1=1-1/y_0,\ \cos\Theta_2=1-1/y_0-1/\mu y_0,\ 0.5(1+1/\mu)\leq y_0$		
Tri-Linear 4		$\frac{2}{\pi}\cdot\frac{1-\mu}{1+\mu(y_0-1)}\cdot\frac{y_0-1}{y_0}$	K-B	$\mu+(1-\mu)(\Theta-0.5\sin 2\Theta)/\pi$	
			E-L	$[1+\mu(y_0-1)]/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta=1-2/y_0,\ 1\leq y_0\leq 1+1/\mu$		
		$\frac{1}{\pi}\frac{\mu(2y_0-3)-1}{\mu y_0}$	K-B	$[(1-\mu)(\Theta_1-0.5\sin 2\Theta_1)+\mu(\Theta_2-0.5\sin 2\Theta_2)]/\pi$	
			E-L	$2/y_0\ (=\varkappa^2)$	
		Remarks	$\cos\Theta_1=1-2/y_0,\ \cos\Theta_2=1-2(1+\mu)/\mu y_0,\ 1+1/\mu\leq y_0$		
Ramberg & Osgood		$\frac{2}{\pi}\frac{r-1}{r+1}(1-\varkappa^2)$	K-B	$\frac{2}{\pi y_0}\int_{-F_0/F_y}^{F_0/F_y}\sqrt{y_0^2-y^2}\,d(\frac{F}{F_y})$	
			E-L	$\frac{F_0}{F_y}/y_0\ (=\varkappa^2)$	

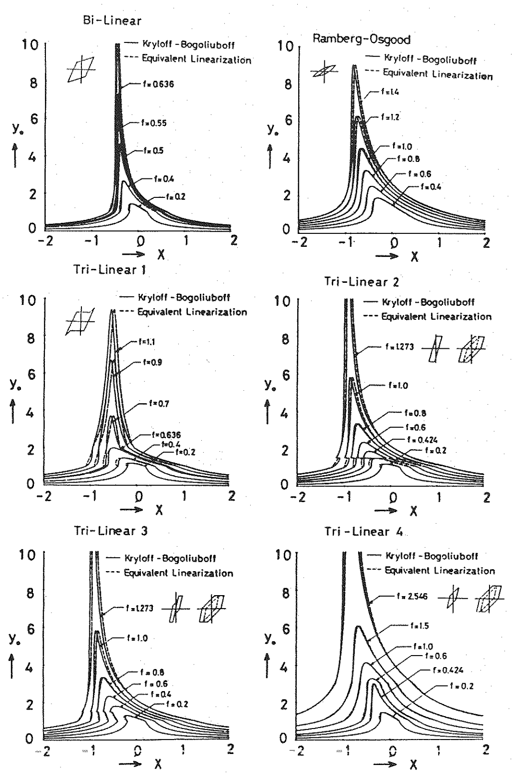


Fig. 1 Resonant Curves by Both Equivalent Linear and K-B Solutions.

analysis[30]. The value of ω_0 in single-degree-of freedon system was adopted to be 10 (rad /s) so as to make its natural frequency come to the range of 1.0 to 2.5(Hz). The maximum value of $f(\tau)$ was set to be 2.0, that is, two times of yielding point. R-O model was applied in the calculation of exact solutions with parameters of $\alpha=0.1$ and r $=7$. Relationships $\varkappa^2\sim y_0$ and $h_{eq}\sim y_0$ in Table 1 were used for constitutive equations in calculating equivalent linear solutions. Effective amplitude by which new dynamic constants in each iterative calculation should be determined was adopted as 2/3 of maximum response displacement as comonly used. We named this coefficient as "Coefficient of Effective Amplitude" temporally in this paper. Several examples of the response acceleration, velocity and displacement are shown in Fig. 2 to Fig. 4 and these Fourier Spectra are shown in Fig. 5 to Fig. 7.

Acceleration responses in above figures indicate that the equivalent linear solutions agree considerably well with exact ones, especially better in near maximum value of every case. As a whole similar tendency to the one reported by Kokusho et al. [28] is recognized. As for velocity responses the coincidence in near maximum value is comparatively well too in each case, however, exact solutions become considerably larger in the parts of small responses. Moreover, in these parts smooth responses of even period like in steady states appear and components of shorter periods seem to diminish. This fact suggests that in exact solutions the restoring force corresponding to the parts of small responses still remains near to linear range so that the component of resonant frequency of the range in input accelerograms comes to be dominant easily because of small damping. Concerning displacement responses the tendency like to velocity becomes more remarkable. Above all, we sometimes find that exact solutions come to be considerably larger than equivalent linear

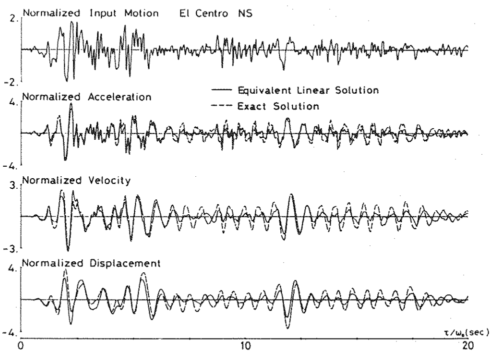


Fig. 2 Response Waves by Both Equivalent Linear and K-B Solutions(EL Centro).

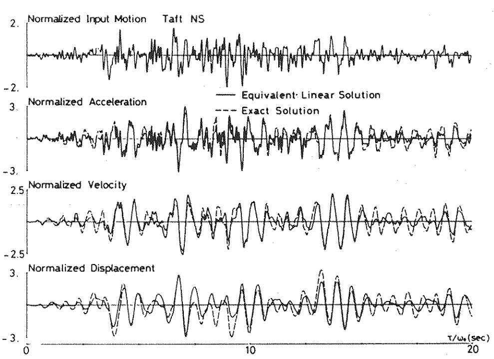


Fig. 3 Response Waves by Both Equivalent Linear and K-B Solutions(Taft).

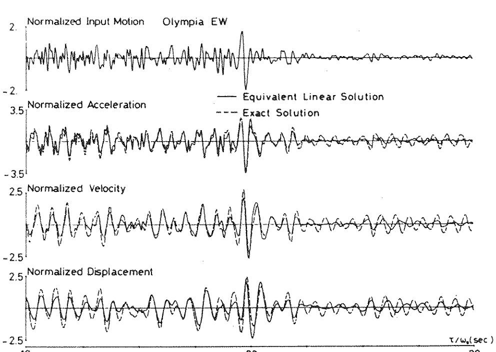


Fig. 4 Response Waves by Both Equivalent Linear and K-B Solutions(Olympia).

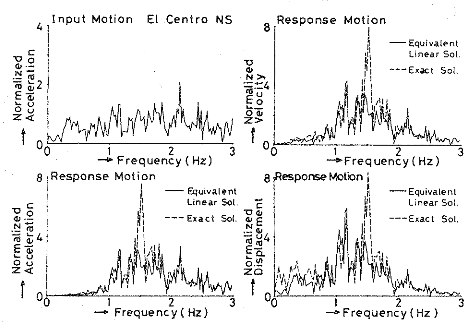


Fig. 5 Fourier Spectra of Response Acceleration (EL Centro).

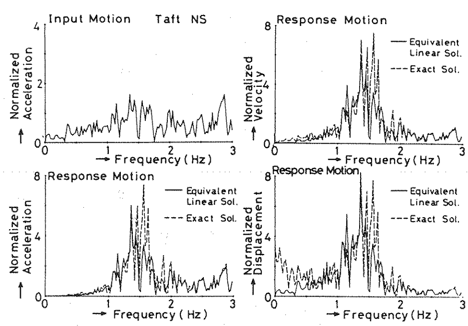


Fig. 6 Fourier Spectra of Response Acceleration (Taft).

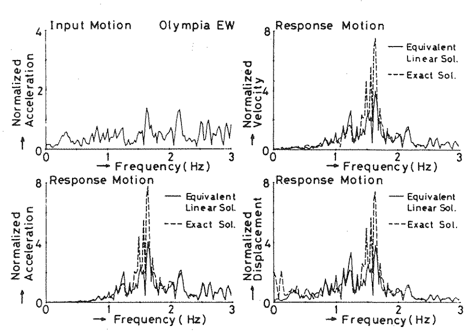


Fig. 7 Fourier Spectra of Response Acceleration (Olympia).

ones even in near about maximum responses. This fact suggests that damping constant has some possibility to be rather overestimated.

Fourier Spectra of responses come to be more smooth as a whole than those of input accelerograms and show such tendency that only remarkably dominant components of input accelerograms remain in responses. Specially, we can find that Fourier Spectrum of every response such as acceleration, velocity and displacement in exact solutions predominates remarkably near at the frequency of 1.6 (Hz) which is the natural frequency of calculated single-degree-of-freedom system with initial stiffness of hysteresis loop. At the frequencies except for above natural frequency where Fourier Spectra of input accelerograms predominate too, every one of Fourier Spectra of equivalent linear solutions coincides nearly with corresponding one of exact solutions. There are not a few structures where response accelerations near about maximum value are important in engineering judgement. For such cases it may be said from above facts that equivalent linear solutions are practically valid even to earthquake excitations. As for displacement responses, however, maximum values are sometimes underestimated in equivalent linear solutions rather than in exact ones, so that equivalent linear ones are sometimes possible to provide data in dangerous side from viewpoint of design. The reason why such phenomenon comes out is not known, however, it is considered that "Coefficient of Effective Amplitude" previously named temporally is possible to be a cause.

(2) Influence of the coefficient of effective amplitude on equivalent linear solutions

Expressing the coefficient of equivalent amplitude as ξ the converged values y_0 in equivalent linear solutions were calculated with various values of ξ and compared with corresponding maximum response displacements in exact solutions. The results are shown in Fig. 8. The solutions of y_0 except for those to El Centro EW and Taft EW show a tendency to decrease because of the increase of damping as ξ is increased. As for the response to Taft EW, for an example, above tendency seems to be stable for the value of ξ greater than 0.6, but it comes to be reversed when ξ is put to 0.5. Moreover, putting ξ to 0.4, y_0 begins to jump one after the other between two values at different times (t=4.48 sec and t=7.81sec in the original accelerogram in this case). This phenomenon comes out from that the response at one time point calculated with dynamic constants defined by maximum response at the other time point in previous calculation becomes to be newly maximum. Even in tne responses to the other input accelerograms which show stable tendency in Fig. 8 the same phenomena appear when the value of ξ is decreased extremely. We sometimes experience also similar jumping phenomena in seismic response analyses with common value 2/3 of ξ for practical design. Concerning a countermeasure for these phenomena it will be thought up to take such an expedient that among several maximum responses jumping one after the other we should select only one considered to be most suitable and attempt the convergence of dynamic constants with it. In addition to above phenomena Fig. 8 shows that every value of y_0 calculated to every input accelerogram with $\xi=2/3$ is considerably smaller than exact one. This fact indicates that damping constants are overestimated in equivalent linear solutions because of an excess of effective amplittudes estimated with $\xi=2/3$. There may be no value for the coefficient of effective amplitude coming into common use to every input earthquake motion, however, some definite value will be necessary from the stand point of aseismic design of structures. Averaging all values for ξ with which y_0 in every equivalent linear solution agrees with exact one in this paper 0.48 was obtained though the number of used accelerograms was only 5. Then we would like to propose the value 0.5 as the coefficient of effective amplitude which

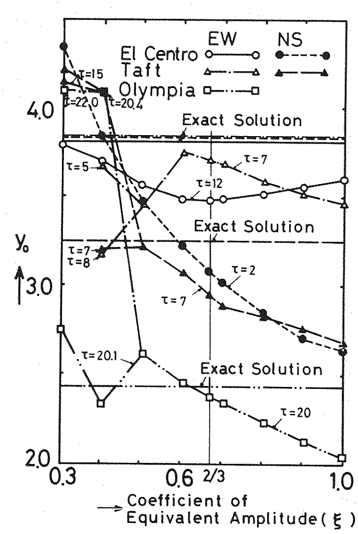


Fig. 8 Effects of Coefficient of Equivalent Amplitude.

may be somewhat better than 2/3.

4. SOME PROBLEMS IN NON-LINEAR SEISMIC RESPONSE ANALYSIS AND A CONSIDERATION ON THEM

(1) Effect of formulation of hysteresis law applied to calculation

In above calculations for exact solutions R-O model was used in obedience to following hysteresis law. We have only to specify the rule to determine a new hysteretic curve which is to be traced after the direction of restoring force reverses at any time on arbitrary point because we start from a skelton curve and so we have always initial hysteretic curve memorized as K_0 in the sequential calculation. We have only to specify above rule in either one of ascending or descending stage of loading because the skelton curve is symmetry with respect to the origin. Following five hysteresis curves $K_1 \sim K_5$ include every cases. Fig. 9 shows the following hysteresis law.

a) Letting skelton curve be K_5 and its initial tangent be K_5' the latest hysteretic curve coming over K_5 in ascending state or under K_5 in descending state should be memorized as K_0 in computer.

b) Whether an alternating load is in ascending state or in descending state should be decided (suppose ascending state in the following).

c) When reversed point is situated out of the loop bounded by K_0 and K_5 as well as over K_5', new hysteretic curve being to be traced hereafter should be determined as follows. Letting the coordinates on K_5 where the new hysteretic curve is to intersect be unknown parameters the simultaneous equations consisting of one equation of above new hysteretic curve into which the coordinatres of reversed point are substituted and another equation of K_5 should be solved wiht Newton-Raphson method etc. Letting the solutions be starting point on K_5 new hysteretic curve should be determined and memorized as K_1.

d) In above c) when reversed point is situated too close to K_5' above unknown parameters can not be obtained easily. Then we had better to obtain the unknowns approximating the new hysteretic curve as linear. The new hysteretic curve thus obtained should be memorized as K_4 though it is a sort of K_1.

e) When reversed point is situated within the loop bounded by K_0 and K_5 or under K_5', the new hysteretic curve should be determined by letting reversed point be the starting point and memorized as K_2 in the former and K_3 in the latter.

f) When the responses of restoring force and displacement tracing along K_1 cross over K_5 or reversed point exists over K_5 from begining K_1 should be memorized as new K_0 hereafter. When above responses attain to K_0 or K_5 tracing along K_2 or attain to K_5 along K_0 or K_3 the new hysteretic curve which is to be traced hereafter should be K_0 or K_5 in the former and K_5 in the latter.

Above rule c) is so important that the response displacements do not become symmetry even in the steady state solutions if it were neglected. Fig. 10 shows, for example, the steady state solutions with R-O model to sinusoidal excitations. Upper two figures in Fig. 10 indicate that in neglecting above rule the resonant

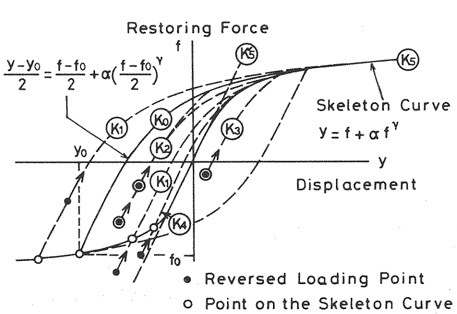


Fig. 9 Rule of Transit on Hysteresis Curve.

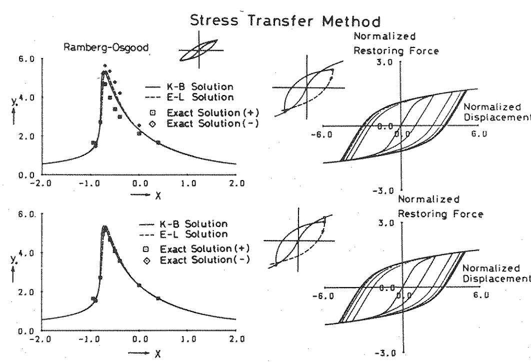


Fig. 10 Removement of Error in Numerical Procedure.

curve obtained from each amplitude in plus and minus sides is separated to upper or lower part away from those of K–B solution and equivalent linear one. Lower two figures in **Fig.** 10 show that the error has been removed by applying the rule c), and besides K–B solutions coincide almost perfectly with exact ones.

(2) Influence of configuration of Slip Type in Hysteresis Loop

In order to examine the influence of configuration of slip type in hysteresis loop pointed out by Tani et al. the steady state responses to sinusoidal excitations were calculated with a Tri–Linear model T–1 shown in Table 1 which is simplified from load–deflection curves observed in bolted connections in suspension tower and shows elasto-plastic behavior in the range from yield point to a certain displacement followed by strain hardening. Some examples of calculated results are shown in **Figs.** 11 and 12. The left hand side responses show input motion, restoring force, acceleration, velocity and displacerment in order from lower part to upper respectively. The right hand side figures show the loci which displacement and each of acceleration, restoring force and velocity draw. It is evidently observed in the figures that response acceleration changes suddenly as soon as restoring force comes in plastic range. The reason, as is evident from Eq. (3), is that surplus part of external force which can not be supported by restoring force changes into response acceleration in reversed direction. **Fig.** 11 is a result of response to sinusoidal excitation of which frequency is smaller than natural one of the system, so that the phases of excitation and response are reversed. On the other hand, as shown in **Fig.** 12, the locus with frequency larger than natural one becomes milder and we obtain more smooth acceleration responses. From above examination it has been revealed that the configuration of slip type in hysteresis loop does evidently influences on acceleration responses.

(3) Comparison of stress transfer method and incremental strain method

Numerical integrations were carried out with stress transfer method[30] and incremental strain method in order to examine the degrees of accuracy in them. Previously used R–O model was used again.

As shown in **Figs.** 13 and14 all responses calculated by both methods with time mesh defined so as to make the discrepancy between both solutions become within 1 % coincide very good in both configuration and magnitude. As for operating time much more time will be needed in incremental strain method in order to obtain above extent of accuracy, however, permitting the error up to several per cent operating time in incremental strain method comes to be relatively less than in stress transfer method.

5. CONCLUSION

The following conclusion can be derived from the previous statesments.

(1) Equivalent viscous damping constant, though defined from a unique hysteresis loop, gives different value if the definition of equivalent linear spring constant is different, however, the response amplitude in resonance becomes constant independently of definition of spring constant because it is to be

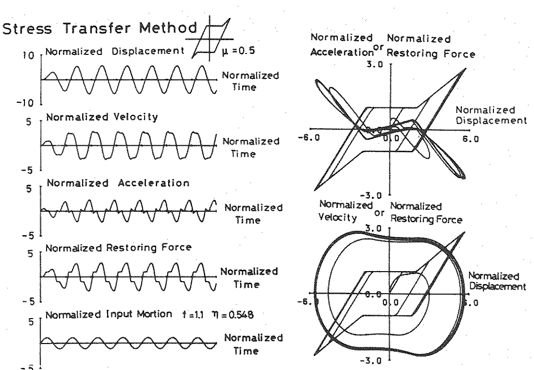


Fig. 11 Effects of Restoring Force With Slip (Lower Frequency Range).

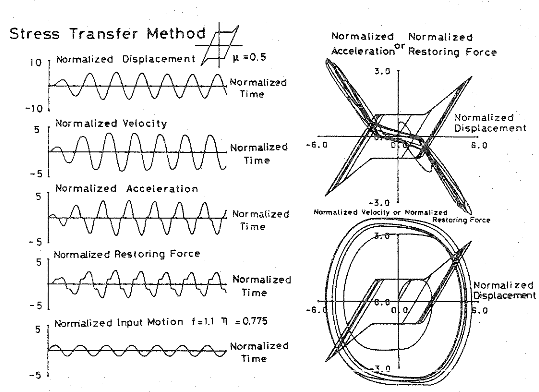


Fig. 12 Effects of Restoring Force With Slip (Higher Frequency Range).

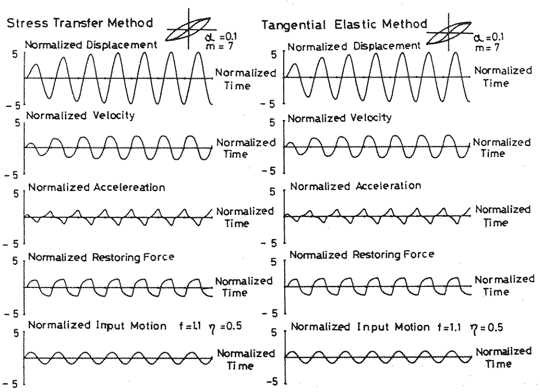


Fig. 13 Response Waves Calculated by Both Methods for Non-Linear Analysis.

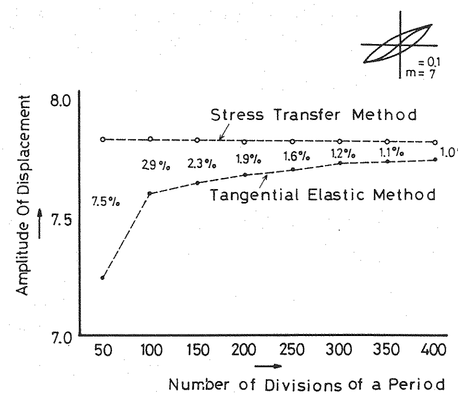


Fig. 14 Effects of Time Interval on Accuracy of Calculation.

determined only with the area bounded by hysteresis loop and the term of external disturbance.

(2) The loss energy balance between original hysteresis loop and equivalent linear system holds accurately in any frequency range out of resonant point if the values of h_{eq} is sufficiently small.

(3) Concerning 4 sorts of Tri-Linear models which show various restoring force characteristics K-B solutions and the equivalent linear solutions have been newly derived for steady state responses. Besides, concerning Bi-Linear model and R-O model the equivalent linear solutions have been newly derived too.

(4) The equivalent linear solutions linearized with secant moduli of hysteresis loops are very good approximate solutions for exact ones as far as concerning steady state responses.

(5) Above equivalent linear solutions under earthquake excitations approximate considerably well the exact ones in acceleration responses, especially good in near about maximum response, however, the discrepancy of them from exact ones becomes large in the parts of small responses.

(6) In displacement responses of above equivalent linear solutions under earthquake excitations, the values of maximum responses are sometimes underestimated This causes of applying 2/3 of maximum amplitude in constitutive equations. 0.5 is proposed in this paper.

(7) Concerning the formulation of hysteresis law any drift appearing in responses due to numerical error can be removed by specifying the rule that new hysteretic curve should start always from a skelton curve. And the effect of configuration of slip type in hysteresis curve on acceleration comes out from such mechanism that surplus part of external force which can not be supported by restoring force changes into response acceleration in reversed direction so that corresponding response acceleration is transformed.

(8) With sufficiently small size of time mesh solutions by incremental strain method coincide almost perfectly with those by stress transfer method, but much more operating time is needed in incremental strain method. Permitting error of several per cent, however, it becomes less relatively.

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