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THREE-DIMESIONAL SEISMIC ANALYSIS FOR SOIL-FOUNDATION-SUPERSTRUCTURE BASED ON DYNAMIC SUBSTRUCTURE METHOD

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This paper presents an efficient 3-dimensional sesimic analysis for a soil-foundation-superstructure system. The dynamic substructure method is used to advantage in formulating the whole system as an integral of the far-field, the near field, the foundation and the superstructure. An axisymmetric modeling is taken for the soil and foundation in order to account for its 3-demensional body with use of the Fourier harmonics expansion for response in the circumferential direction. The interface or the interbody partitioning is applied between foundation and surrounding soils, depending on the type of the foundation concerned. The general 3-dimensional analysis is carried out for the superstructure for its complex geometry. The coupled motion between the soil-foundation system and the superstructure is formulated from the component modes method to attain a drastic reduction of the degrees of freedom for frequency response analysis. Case studies are given as numerical examples.

1. INTRODUCTION

The finite element approach is widely used for the dynamic soil-structure interaction (SSI) analysis, for it can easily accomodate the complex boundary geometry. The direct solution method whcih deals with a complete soil-structure system under the base input situation has been extensively taken for the practical application. Some related computer software package are available on the comercial basis ; for instance, FLUSH[1] or its extended versions. However, the direct method is hardly preferable in case that sturctures for analysis are large and complex since the total degrees of freedom become tremendously large. The computer time and the cost increases accordingly. To overcome this problem, one promising idea is to extend the concept of the so-called impedance function[2] in the continuum approach for the SSI problem to the finite element formulation[3]. This means splitting up the SSI analysis into two steps ; first computing the soil impedance functions and then solving the structural motion which is coupled with these elements for the properly evaluated driving forces through the kinematic interaction between foundation and soil.

Further, in the process of substructuring for the coupled soil and foundation system, two alternative approaches are conceivable ; one is the interface substructuring[4] which specifies the common interface nodes on the foundation face, and the other is the interbody substructuring[5),6)], which makes use of the principle of superposition on material properties and takes the common interbody nodes even within the foundation body.

The more sophistication is to take the three-fold substructuring of soil, foundation and superstructure. For an effective and efficient solution method one may adopt the fixed base superstructural modes to couple with the soil or soil-foundation impedance[7)~9)]. The SSI formulation is thus to use the release modes, besides the above normal modes, according to the degrees of freedom at the foundation.

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The reality of the SSI problem is essentially 3-dimensional. In case of a space structure subjected to seismic input of arbitray direction, the 3-dimensional analysis is definetely imperative. The soil impedance to be incorporated into such analysis should be accounted for accordingly. The truly 3-dimensional analysis for the whole domain, however, may loose the feasibility for its solution cost due to the tremendously large degrees of freedom to handle with. An economical but effective modeling is to assume an axisymmetric nature for the soil-foundation region as proposed herein while to take the general 3-dimensional system for the superstructure, which makes a pseudo-3-dimensional analysis for the complete SSI system. This approach is implemented into the computer code SUBSSIP-A 3 D[8),9)].

2. NEAR FIELD

The dynamics of the soils in the vicinity of foundation, when subjected to seismic input, is essentially described as a force boundary problem. The finite element discretization for the bounded soil domain, with use of the fictituous energy transmitting element[10)], gives the governing equaition of steady state motion of frequency ω, as

$$(\boldsymbol{K}+\underline{i}\omega\boldsymbol{C}-\omega^2\boldsymbol{M})\boldsymbol{U}=\boldsymbol{P}_{sub}-\boldsymbol{P}_b \quad (1)$$

in which $\boldsymbol{U}$ denotes the nodal displacement vector, $\boldsymbol{K}$, $\boldsymbol{C}$, and $\boldsymbol{M}$ indicate the stiffness, damping and mass matrices, respectively; $\boldsymbol{P}_{sub}$ signifies the force vector due to the superstructural inertia, $\boldsymbol{P}_b$ is the boundary force vector at soils, and $\underline{i}$ is an imaginary unit.

The wave field in SSI is given by the superposition of the free field wave and the radiating wave due to the presence of the strucrture. The displacement is then expressed as

$$\boldsymbol{U}=\boldsymbol{U}^*+\boldsymbol{U}^r \quad (2)$$

in which the asterisk denotes the free field response and the supersprict r refers to the additional response due to the radiation wave. For the plane body wave input at the base of the SSI system, one can get

$$\boldsymbol{P}_b=\boldsymbol{K}^*\boldsymbol{U}^*+\boldsymbol{R}(\boldsymbol{U}-\boldsymbol{U}^*) \quad (3)$$

in which $\boldsymbol{K}^*$ is the 1-dimensional free field stiffness matrix, $\boldsymbol{R}$ is the transmitting boundary matrix to reproduce the far field nature.

The substructure analysis yields the soil impedance and the effective seismic force at the common nodes where soil and foundation are separated. Either the interface modeling or the interbody modeling is considered both of which can be treated by the same formulation. Fig. 1 shows the concept in a 2-dimensional way for convenience. Herein, a compact expression is used for Eq. (1) by introducing the dynamic stiffness matrix $\boldsymbol{D}=\boldsymbol{K}+\underline{i}\omega\boldsymbol{C}-\omega^2\boldsymbol{M}$, whose partitioned form is

$$\begin{bmatrix} \boldsymbol{D}_{ii} & \boldsymbol{D}_{is} & \boldsymbol{D}_{ig} \\ \boldsymbol{D}_{si} & \boldsymbol{D}_{ss} & \boldsymbol{D}_{sg} \\ \boldsymbol{D}_{gi} & \boldsymbol{D}_{gs} & \boldsymbol{D}_{gg} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U}_i \\ \boldsymbol{U}_s \\ \boldsymbol{U}_g \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{P}_i \\ \boldsymbol{P}_b \\ \boldsymbol{P}_g \end{Bmatrix} \quad (4)$$

in which the subscript i refers to the soil nodes in common with the foundation, s to other soil nodes including the side boundary nodes b but excluding the base boundary nodes g. The reduced governing equation with respect to the common nodes i only is derived through the condensation process, as

$$\boldsymbol{X}_i\boldsymbol{U}_i=\boldsymbol{P}_i-\boldsymbol{P}_i^0 \quad (5)$$

in which the matrix $\boldsymbol{X}_i$ defines the impedance and $\boldsymbol{P}_i^0$ the effective force through the kinematic interaction, whose formal expres-

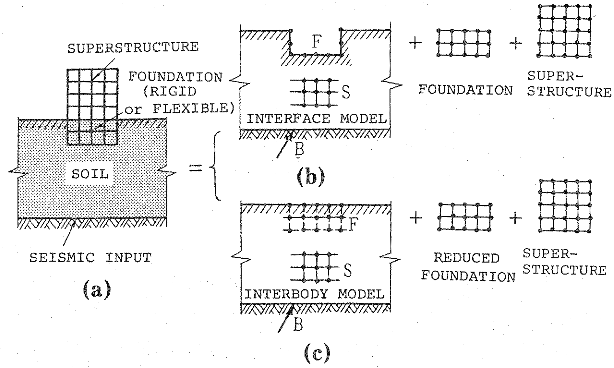


Fig. 1 Substructuring for SSI System[6)]

sions are given by

$$X_i=D_{ii}-[D_{is}\,D_{ig}]\begin{bmatrix} D_{ss} & D_{sg} \\ D_{gs} & D_{gg} \end{bmatrix}^{-1}\begin{bmatrix} D_{si} \\ D_{gi} \end{bmatrix} \cdots\cdots\cdots (6) \qquad P_i^0=[D_{is}\,D_{ig}]\begin{bmatrix} D_{ss} & D_{sg} \\ D_{gs} & D_{gg} \end{bmatrix}^{-1}\begin{Bmatrix} P_b \\ P_g \end{Bmatrix} \cdots\cdots (7)$$

Note in Eq. (5) that the force vectore P_i^0 is nothing but the fixing force for the common nodes immovable. Furthermore, in case that the soil impedance matrix X_i is already known, the effective force is given by changing the sign of the product of this and the free field motion, i.e.,

$$P_i^0=-X_iU_i^* \cdots\cdots (8)$$

The above concept is applied to the axisymmetric soil medium to get the pseudo-3-dimensional solution with use of the Fourier harmonics expansion for response in cylindrical coordinates (see Fig. 2). The displacement is thus expressed as

$$\begin{Bmatrix} U_r(r,\theta,z) \\ U_z(r,\theta,z) \\ U_\theta(r,\theta,z) \end{Bmatrix}=\sum_{n=0}^{\infty}\begin{bmatrix} \cos\widehat{n\theta} & & \\ & \cos\widehat{n\theta} & \\ & & -\sin\widehat{n\theta} \end{bmatrix}\begin{Bmatrix} U_r(r,z) \\ U_z(r,z) \\ U_\theta(r,z) \end{Bmatrix}_n^s$$

$$+\sum_{n=0}^{\infty}\begin{bmatrix} \sin\widehat{n\theta} & & \\ & \sin\widehat{n\theta} & \\ & & \cos\widehat{n\theta} \end{bmatrix}\begin{Bmatrix} U_r(r,z) \\ U_z(r,z) \\ U_\theta(r,z) \end{Bmatrix}_n^a \cdots\cdots (9)$$

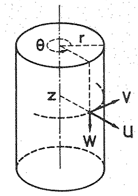


Fig. 2 Cylindrical Coordinates

or in a brief form

$$U(r,\theta,z)=\sum_{n=0}^{\infty}H_n^s(\theta)\,U_n^s(r,z)+\sum_{n=0}^{\infty}H_n^a(\theta)\,U_n^a(r,z) \cdots\cdots (9)'$$

in which the first summation takes over the symmetric harmonics and the second summation over the antisymmetric harmonics. The mimus sign in the symmetric sine-terms leads an identical stiffness matrix for both the symmetric and antisymmetric harmonics. The details about the succeeding finite element formulation is given in Appendix A.

For the plane body wave assumption impinging vertically at the base rock level, the components on the cartesian reference (U_{gx}^*, U_{gy}^*, U_{gz}^*) are equivalently expressed by the Fourier harmonic amplitudes as

$$\begin{Bmatrix} U_r^* \\ U_z^* \\ U_\theta^* \end{Bmatrix}_0^s=\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}U_{gz}^*,\quad \begin{Bmatrix} U_r^* \\ U_z^* \\ U_\theta^* \end{Bmatrix}_1^s=\begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}U_{gx}^*,\quad \begin{Bmatrix} U_r^* \\ U_z^* \\ U_\theta^* \end{Bmatrix}_1^a=\begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}U_{gy}^* \cdots\cdots (10)$$

3. FOUNDATION FORMULATION

(1) Rigid Foundation

A caisson foundation with deep embedment in soils has usually much difference in rigidity from the surrounding soils, whcih makes a rigid body assumption appropriate for the foundation modeling. Since the rigid body motion is uniquely prescribed by the movement of its gravity center, the interface substructuring suits for this analysis.

The soil ring nodes displacements on the interface, when the perfect bond condition is presumed between soil and foundation, must obey the compatibility with the rigid motion. For this requirement, consider first the displacement transformation from the cylindrical to the cartesian coordinates, i.e.,

$$\begin{Bmatrix} U_x \\ U_y \\ U_z \end{Bmatrix}=\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta & 0 & \cos\theta \\ 0 & 1 & 0 \end{bmatrix}\begin{Bmatrix} U_r \\ U_z \\ U_\theta \end{Bmatrix} \cdots\cdots (11)$$

or briefly

$$U(x,y,z)=G(\theta)\,U(r,\theta,z) \cdots\cdots (11)'$$

Substituting the Fourier harmonics expansion for displacement, Eq. (9)′ into Eq. (11), one can get

$$U(x,y,z)=G(\theta)\left(\sum_{n=0}^{1}H_n^s(\theta)\,U_n^s(r,z)+\sum_{n=0}^{1}H_n^a(\theta)\,U_n^a(r,z)\right) \cdots\cdots (12)$$

in which note that the summation takes only the limited Fourier harmonics concerning rigid motion. Namely, the $n=0$ symmetric harmonic represents a vertical motion, the $n=0$ antisymmetric a torsional motion, the $n=1$ symmetric harmonic relates a coupled motion of translation along x-axis ($\theta=0°$) and rotation about y-axis ($\theta=90°$), and the $n=1$ antisymmetric harmonic a coupled motion of translation along y-axis and rotation about x-axis.

The displacement of interface ring nodes is alternatively represented by use of the displacements at the foundation gravity center, U_F on the cartesian reference as

$$U_i(x, y, z)=\sum_{n=0}^{1} S_n^s(r, \theta, z)\, U_F+\sum_{n=0}^{1} S_n^a(r, \theta, z)\, U_F \quad (13)$$

in which the transformation matrices S_n^s and S_n^a, whose detaials are given in Appendix B, should satisfy the termwise correspondence with Eq. (12). As the result, each Fourier harmonic amplitude is expressed in terms of U_F through the specific transformation matrix T_n^s and T_n^a (see Appendix B), so that

$$U_i(x, y, z)=G(\theta)\left(\sum_{n=0}^{1} H_n^s(\theta)T_n^s(r, z)U_F+\sum_{n=0}^{1} H_n^a(\theta)T_n^a(r, z)U_F\right) \quad (14)$$

The corresponding force vector on the cartesian refernce is likewise transformed from the Fourier harmonic amplitudes as

$$P_i(x, y, z)=G(\theta)\left(\sum_{n=0}^{1} H_n^s(\theta)P_{i,n}^s(r, z)+\sum_{n=0}^{1} H_n^s(\theta)P_{i,n}^a(r, z)\right) \quad (15)$$

The soil reaction acting on the foundation face is then integrated along the circumference and summed up over interface nodes to give the total force vector $\{P \,\vdots\, M\}_F^T=\{P_xP_yP_z \,\vdots\, M_xM_yM_z\}_F$ with respect to the gravity center, as

$$\begin{Bmatrix} P \\ M \end{Bmatrix}_F=\sum_{i}^{\text{Interface Ring Nodes}}\begin{Bmatrix} P \\ M \end{Bmatrix}_i=\sum_{i}^{IRN}\sum_{n=0}^{1}\left(\int_0^{2\pi} S_n^{s^T}(r, \theta, z_i)G(\theta)H_n^s(\theta)d\theta\cdot P_{i,n}^s + \int_0^{2\pi} S_n^{a^T}(r, \theta, z_i)G(\theta)H_n^a(\theta)d\theta\cdot P_{i,n}^a\right) \quad (16)$$

Executing the integration results in

$$\begin{Bmatrix} P \\ M \end{Bmatrix}_F=\sum_{i}^{IRN}\sum_{n=0}^{1} \alpha_n(T_n^{s^T}(r, z_i)P_{i,n}^s+T_n^{a^T}(r, z_i)P_{i,n}^a) \quad (17)$$

in which $\alpha_n=2\pi$ for $n=0$ and $\alpha_n=\pi$ for $n=1$. In Eq. (17), the 1-st term gives the forces due to the vertical motion, the 2-nd term due to the torsional motion, the 3-rd term due to the coupled motion of sway and rocking in the plane perpendicular to the y-axis ($\theta=90°$), and the 4-th term due to the coupled motion of sway and rocking in the plane perpendicular to the x-axis ($\theta=0°$).

Substituting the solution in Eq. (5) for $P_{i,n}$ and in view of Eq. (14), the dynamic equilibrium is obtained as

$$\left(-\omega^2 M_F+\sum_{n=0}^{1}\alpha_n[\tilde{T}_n^s\ \tilde{T}_n^a]^T\begin{bmatrix} X_i^s & \\ & X_i^a\end{bmatrix}\begin{bmatrix}\tilde{T}_n^s \\ \tilde{T}_n^a\end{bmatrix}\right)U_F=\sum_{n=0}^{1}\alpha_n[\tilde{T}_n^s\ \tilde{T}_n^a]\begin{bmatrix} X_i^s & \\ & X_i^a\end{bmatrix}\begin{Bmatrix} U_n^{*s} \\ U_n^{*a}\end{Bmatrix}+P_{sub} \quad (18)$$

in which $\tilde{T}_n$ is the expanded diagonal matrix of T_n as many as ring nodes, and M_F denotes the mass matrix comprising the mass and the mass moment of inertia associated with 6 degrees of freedom of the rigid body foundation. The Eq. (18) is simply expressed as

$$(-\omega^2 M_F+X_F)U_F=X_FU_F^*+P_{sub} \quad (18)'$$

(2) Flexible Foundation

In case that the foundation has an axisymmetric nature, the Fourier harmonic analysis is straightforward with the neighboring soil elements. However, when the foundation is analyzed as a flexible body together with the superstructure on a cartesian reference, the following formulation is suggested.

First, taking the interbody substructuring, one can get the governing equation for the reduced foundation by the soil effect filled back at the portion originally the foundation is embedded, as

$$(D_{F,n}-D_{soil,n})U_{i,n}=P_{i,n}+P_{sub,n} \quad (19)$$

for a unit of radian volume in the circumferential direction of the n-th Fourier harmonic. From the soil analysis, evaluating the soil impedance matrix and the effective seismic input and in view of the continuity condition at the interbody nodes between soil and foundation, one can get

$$(\boldsymbol{D}_{F,n}-\boldsymbol{D}_{soil,n}+\boldsymbol{X}_{i,n})\boldsymbol{U}_{i,n}=\boldsymbol{X}_{i,n}\boldsymbol{U}^{*}_{i,n}+\boldsymbol{P}_{sub,n} \quad (20)$$

Further, in order to evaluate foundation effect to yield an averaged uniform input motion for the superstructure, consider the similar coordinates transformation as in Eqs. (14) and (17) for displacements and forces respectively at every ring node. Then the governing equaition becomes as

$$\sum_{n=0}^{1}\alpha_n(\tilde{\boldsymbol{T}}^{sT}_n\tilde{\boldsymbol{D}}^{s}_n\tilde{\boldsymbol{T}}^{s}_n+\tilde{\boldsymbol{T}}^{aT}_n\tilde{\boldsymbol{D}}^{a}_n\boldsymbol{T}^{a}_n)\boldsymbol{U}_F=\sum_{n=0}^{1}\alpha_n(\tilde{\boldsymbol{T}}^{sT}_n\boldsymbol{X}^{s}_{i,n}\boldsymbol{U}^{*s}_{i,n}+\tilde{\boldsymbol{T}}^{aT}_n\boldsymbol{X}^{a}_{i,n}\boldsymbol{U}^{*a}_{i,n})+\boldsymbol{P}_{sub} \quad (21)$$

in which the notation $\tilde{\boldsymbol{D}}_n=\boldsymbol{D}_{F,n}-\boldsymbol{D}_{soil,n}+\boldsymbol{X}_{i,n}$ is used. The Eq. (21) may be rewritten in the conventional expression for a multi-degree-of-freedom system, as

$$(-\omega^2\boldsymbol{M}_F+\underline{i}\omega\boldsymbol{C}_F+\boldsymbol{K}_F)\boldsymbol{U}_F=\boldsymbol{P}^0_F+\boldsymbol{P}_{sub} \quad (22)$$

by retreaving the reduced mass, damping and stiffness matrices for the foundation and the soil impedance matrix which are obvious from Eq. (21).

4. INTERACTION ANALYSIS

The superstructure may assume any geometry in the 3-dimensional space. The generalized lumped mass modeling is taken here. The associated governing equation of steady state motion for the superstructure can be expressed, when the junction nodes with the foundation is retained, as

$$-\omega^2\begin{bmatrix}\boldsymbol{M}_{aa} & \\ & \boldsymbol{M}_{bb}\end{bmatrix}_{sup}+\underline{i}\omega\begin{bmatrix}\boldsymbol{C}_{aa} & \boldsymbol{C}_{ab}\\ \boldsymbol{C}_{ba} & \boldsymbol{C}_{bb}\end{bmatrix}_{sup}+\begin{bmatrix}\boldsymbol{K}_{aa} & \boldsymbol{K}_{ab}\\ \boldsymbol{K}_{ba} & \boldsymbol{K}_{bb}\end{bmatrix}_{sup}\begin{Bmatrix}\boldsymbol{U}_a\\ \boldsymbol{U}_b\end{Bmatrix}=\begin{Bmatrix}\boldsymbol{O}\\ \boldsymbol{P}_b\end{Bmatrix}_{sup} \quad (23)$$

in which $\boldsymbol{M}_{sup}$, $\boldsymbol{C}_{sup}$, $\boldsymbol{K}_{sup}$, denoting, the mass, damping and stiffness matrices for the superstrucre, are partioned on the basis of the free nodes (off-base nodes) with the suffix a, or the base nodes with the suffix b. Following the component modes synthesis formulation[11], express the total displacement $\boldsymbol{U}_a$ as the sum of the dynamic displacement $\boldsymbol{U}^{dyn}_a$ due to the inertial force for the fixed base condition, and the quasi-static one $\boldsymbol{U}^{stat}_a$ due to the base movement, i. e.,

$$\boldsymbol{U}_a=\boldsymbol{U}^{dyn}_a+\boldsymbol{U}^{stat}_a \quad (24)$$

Furthermore, considering a small amount of damping for the superstructure itself, assume that the classical normal modes decomposition holds for the fixed base condition, which satisfies the following orthogonality :

$$\boldsymbol{V}^T\boldsymbol{M}_{aa}\boldsymbol{V}=\boldsymbol{I},\quad \boldsymbol{V}^T\boldsymbol{C}_{aa}\boldsymbol{V}=\text{diag.}(2\xi_l\omega_l),\quad \boldsymbol{V}^T\boldsymbol{K}_{aa}\boldsymbol{V}=\text{diag.}(\omega_l^2) \quad (25)$$

in which $\boldsymbol{V}$ defines the modal matrix, $\boldsymbol{I}$ the identity matrix, ω_l the l-th modal frequency and ξ_l the damping ratio to its critcal value. Hence, the displacement transformation results in

$$\begin{Bmatrix}\boldsymbol{U}_a\\ \boldsymbol{U}_b\end{Bmatrix}=\begin{bmatrix}\boldsymbol{V} & \boldsymbol{\beta}\\ \boldsymbol{O} & \boldsymbol{I}\end{bmatrix}\begin{Bmatrix}\boldsymbol{q}\\ \boldsymbol{U}_b\end{Bmatrix} \quad (26)$$

in which $\boldsymbol{\beta}$, denoting the displacement influence matrix, is generally computed from the static condensation process for Eq. (23), such that

$$\boldsymbol{\beta}=-\boldsymbol{K}^{-1}_{aa}\boldsymbol{K}_{ab} \quad (27)$$

In case that a lumped mass system is taken for the superstructural modeling and a uniform motion is imposed at this base, the displacement influence matrix results in the master-slave nodes relationship as in Eq. (B. 1) in Appendix B which comprises only the geometry of the respective lumped mass location.

Substituting Eq. (26) into Eq. (23) and in view of Eqs. (27), one can get

$$\left(-\omega^2\begin{bmatrix}\boldsymbol{I} & \boldsymbol{V}^T\boldsymbol{M}_{aa}\boldsymbol{\beta}\\ \text{sym.} & \boldsymbol{M}_{bb}+\boldsymbol{\beta}^T\boldsymbol{M}_{aa}\boldsymbol{\beta}\end{bmatrix}+\underline{i}\omega\begin{bmatrix}\text{diag.}(2\xi_l\omega_l) & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{C}_{bb}+\boldsymbol{\beta}^T\boldsymbol{C}_{aa}\boldsymbol{\beta}\end{bmatrix}\right.$$
$$\left.+\begin{bmatrix}\text{diag.}(\omega_l^2) & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{K}_{bb}+\boldsymbol{K}_{bb}\boldsymbol{\beta}\end{bmatrix}\right)\begin{Bmatrix}\boldsymbol{q}\\ \boldsymbol{U}_b\end{Bmatrix}_{sup}=\begin{Bmatrix}\boldsymbol{O}\\ \boldsymbol{P}_b\end{Bmatrix}_{sup} \quad (28)$$

The coupled motion of the superstructure with the soil-foundartion system is now established by use of the displacement compatibility and the force equilibrium between their junction nodes, which are

$$U_{b,sup}=\gamma U_F \tag{29}$$

$$P_{b,sup}+\gamma^T P_F=0 \tag{30}$$

in which γ is the transformation matrix from the gravity center of the foundation to its junction nodes with the superstructure. Substituting Eqs. (18)′ or Eq. (22) and (28) into Eq. (30) and considering Eq. (29), one can derive the governing equation for the soil-foundation-superstructure system as

$$\begin{bmatrix} \text{diag.}(-\omega^2+\underline{i}2\xi_l\omega_l\omega+\omega_l^2) & -\omega^2 V^T M_{aa}\tilde{\beta} \\ \text{sym.} & -\omega^2(\tilde{\beta}^T M_{aa}\tilde{\beta}+M_F)+\underline{i}\,\omega(C_{bb}+\tilde{\beta}^T C_{aa}\tilde{\beta}+C_F) \\ & +K_{bb}+K_{ba}\tilde{\beta}+K_F+X_F \end{bmatrix}\begin{Bmatrix} q \\ U_F \end{Bmatrix}=\begin{Bmatrix} 0 \\ X_F U_F^* \end{Bmatrix} \tag{31}$$

in which $\tilde{\beta}=\gamma\beta$. Note that the final governing equation is expressed in terms of the fixed base superstructural normal modes together with the physical degrees of freedom of the foundation. In prior to solving Eq. (31), one may truncate the superstructural higher modes of less response contribution factor in order to reduce the total degrees of freedom effectively for the complete SSI system analysis.

5. SEISMIC RESPONSE

Following the process given in Sections 2 through 4, the frequency response functions for a unit input amplitude in the three perpendicular direction on the cartesian reference are computed for nodes of interest in the complete system, i. e.,

$$H(\omega)=\begin{bmatrix} H_{xx}(\omega) & H_{xy}(\omega) & H_{xz}(\omega) \\ H_{yx}(\omega) & H_{yy}(\omega) & H_{yz}(\omega) \\ H_{zx}(\omega) & H_{zy}(\omega) & H_{zz}(\omega) \end{bmatrix} \tag{32}$$

in which the 1-st suffix indicates the response direction while the 2-nd one the input direction. The actual frequency response is obtained by multiplying the Fourier transform of the input seismic wave components (F_x, F_y, F_z) at each frequency.

$$\{Y_x(\omega)\ Y_y(\omega)\ Y_z(\omega)\}^T=H(\omega)\cdot\{\mathrm{F}_x(\omega)\ \mathrm{F}_y(\omega)\ \mathrm{F}_z(\omega)\}^T \tag{33}$$

The response time histories are then converted from these frequency responses through the discrete inverse Fourier transform process. The Fast Fourier Transform algorithm is available for an efficient computation of the discrete Fourier and inverse Fourier transforms.

6. CASE STUDIES

As applications of the present computer code SUBSSIP-A 3 D, the following case studies are executed.

(1) Chimney Structure

A very high chimney foundation (see Fig. 3 (a) for the general view) is first investigated with emphasis on the interaction with the surrounding soil in Table 1. Fig. 3 (b) is the finite element model for soil-foundation system in which the interface substructuring is taken. Figs. 4 indicate the soil impedance functions with respect to the foundation gravity center. In thses figures, comparison is made with the solutions from the Novak's approach[12] in which a plane strain assumption is used, and from the soil reaction coefficient formula of Japan Road Association Specification[13]. One may note that the present solution, being different from other soulutions, shows a strong frequency dependent nature, which might be important for evaluating the foundation response coupled with the surrounding soils.

(2) Blast Furnace Structure

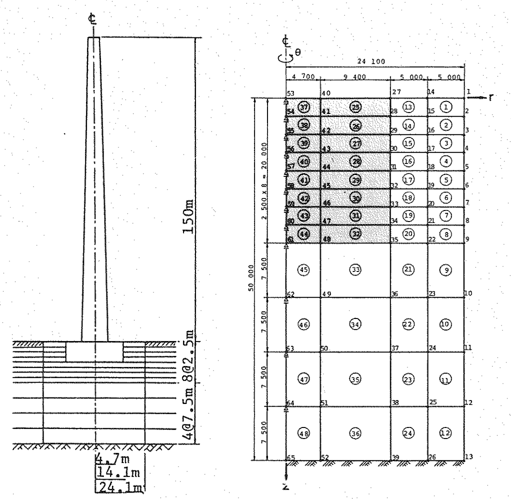
As a second case study, a huge blast furnace structure which makes steel materials (see Fig. 5 (a) for a general view) is analyzed. For the finite element modeling, either a rigid body or a flexible body assumption is made for the caisson foundation for comparison, while a conventional 3-dimensional lumped

Table 1 Soil Profile (Case Study 1)

Layers	Depth (m)	Velocity (m/s)	Poisson Ratio	Weight (tf/m³)	Damping Ratio
1 ∿ 4	2.5	150	1/3	2.0	0.1
5 ∿ 8	2.5	300	1/3	2.0	0.1
9 ∿12	7.5	500	1/3	2.0	0.1

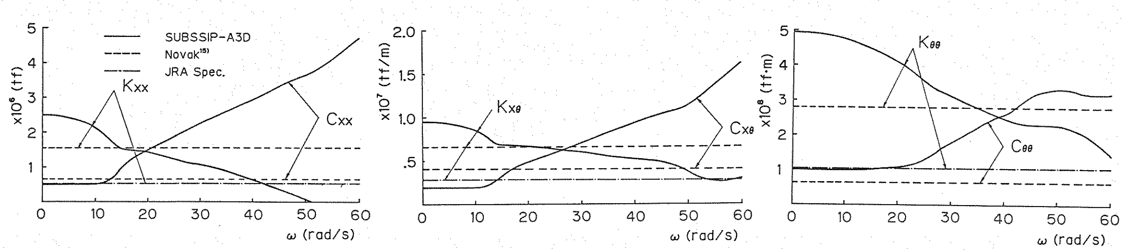
Table 2 Soil Profile (Case Study 2)

NO.	DEPTH (m)	VELOCITY (m/s)	POISSON RATIO	WEIGHT (tf/m³)	DAMPING Ratio
1	6.0	150.0	0.45	1.50	0.10
2	7.0	150.0	0.45	1.50	0.10
3	7.0	150.0	0.45	1.50	0.10
4	4.0	95.0	0.45	1.50	0.10
5	8.0	95.0	0.45	1.50	0.10
6	8.0	220.0	0.45	1.70	0.10
7	10.0	460.0	0.45	2.00	0.10
8	5.6	230.0	0.45	1.70	0.10
9	9.7	480.0	0.45	2.00	0.10
10	9.7	480.0	0.45	2.00	0.10
11	11.0	260.0	0.45	1.80	0.10
12	18.5	440.0	0.45	2.10	0.10



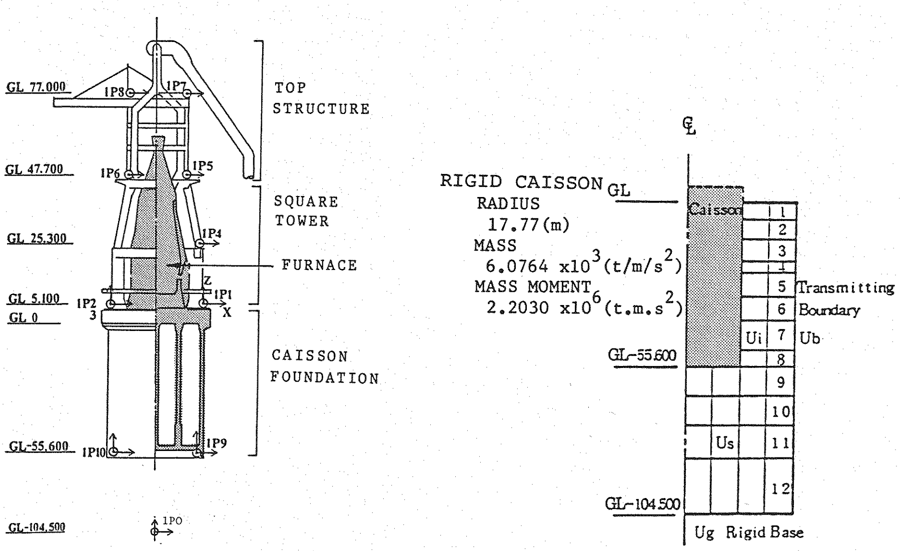
(a) General view (b) FEM Model for Soil-Foundation

Fig. 3 Chimney Structure (Case Study 1)



(a) Translation (b) Interaction (c) Rotation

Fig. 4 Soil Impedance Functions (Case Study 1)



(a) General View (b) FEM Model for Soil-Foundation

Fig. 5 Blast Furnace Structure (Case Study 2)

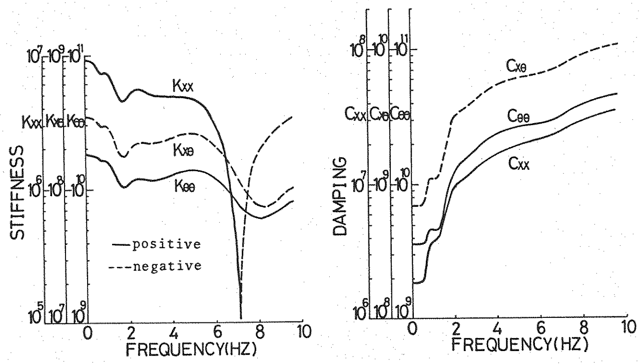


Fig. 6 Soil Inpedance Functions (Case Study 2)

mass modeling is taken for the superstructure[14]. The alternative substructurings are taken depending on the above assumption for the foundation part; the interface or interbody approach. Fig. 5 (b) gives the model for the interface substructuring. An equivalent flexible body, as represented by the solid elements, is considered on the basis of equal mass and mass moment of intertia with the rigid foundation. Figs. 6 depict the soil impedance functions for the rigid foundation modeling. Figs. 7 and 8 show the frequency responses (transmitting functions) at a typical mass location for a unit input at the base level which is

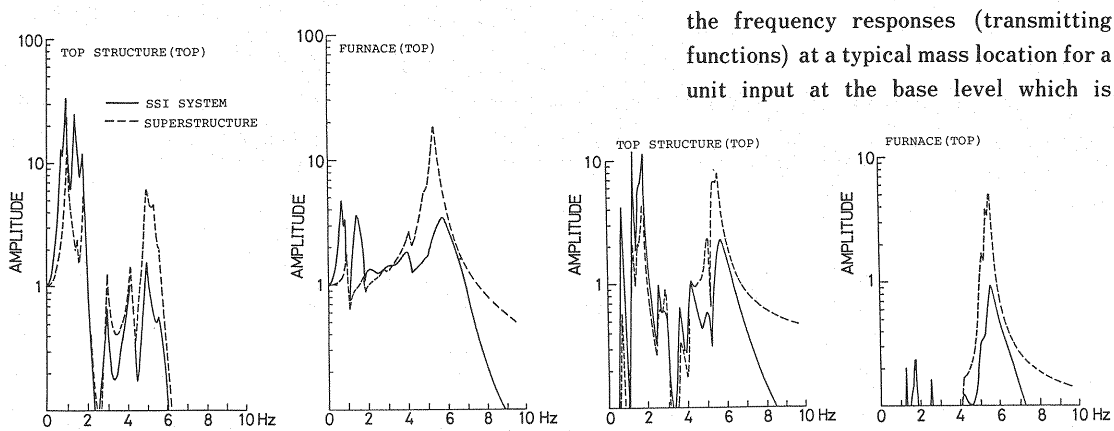


Fig. 7 Frequency Transmitting Functions, X-Direction Input

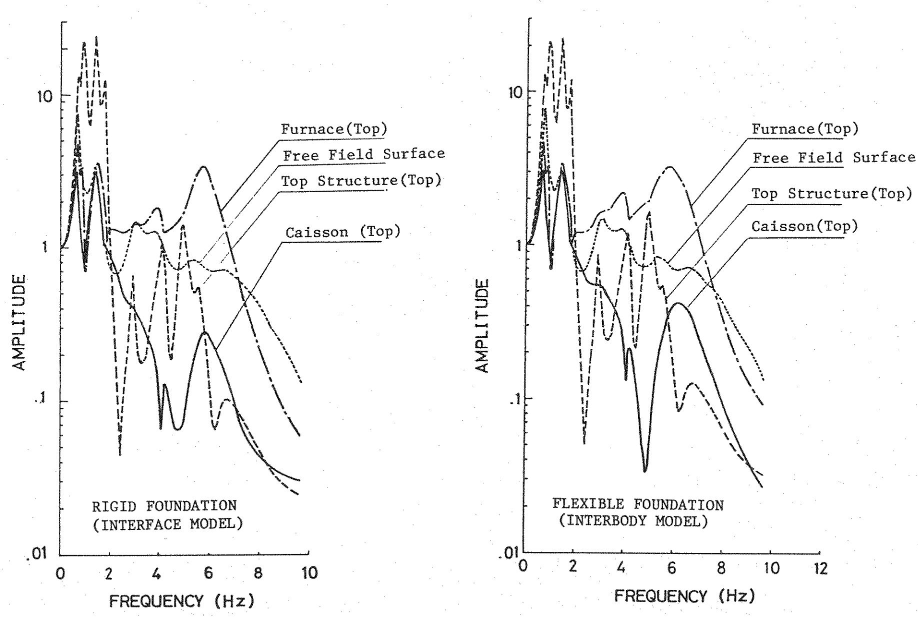


Fig. 8 Frequency Transmitting Functions, X-Direction Input

located at 104 m below the ground surface (the point 1PO in Fig. 5 (a)) in the x-direction. In Figs. 7, the point of investigation is placed on the SSI effect from the comparison with the rigidly supported case at the structural base. Note that the response is greatly changed in the low frequency range to have peaks due to the soil motion as the free field and the soil-structure inertial interaction. In the high frequency range, on the other hand, the response is reduced due to the increase of damping through the SSI. The fact that the response in the perpendicular direction to the input direction is relatively induced emphasizes the importance of the 3-dimensional analysis. In Figs. 8 is geven the response comparison by the foundation modeling as a rigid or a flexibkle body. Almost identical results are attained in the frequency range of interest from the seismic analysis. Also, note that the frequency responses are very indicative of the response features of the respective part of the structure.

7. CONCLUSION

The author has developed an efficient computer code SUBSSIP-A3D for the 3-dimensional SSI seismic analysis. The primary point of this paper is to present its theoretical aspect and the applications to some case studies to get engineering findings regarding the SSI effects.

The unique features of the SUBSSIP-A3D are summarized as follows : (1) The response analysis is based on the substructure method, which makes use of the soil impedance to couple with the superstructural part for the effective input force through the kinematic interaction between soil and foundation concerned. (2) Either the interface or interbody substructuring approach is available depending on the type of the foundation for analysis. The latter formulation makes use of free field response to advantadge in evaluating the effective seismic forces together with the soil impedance matrix. (3) The aximsymmetric modeling is taken for the soil and foundation in order to account for their 3-dimensional body, which facilitates the Fourier harmonics expansion for response in the circumferential direction. The specific appropriate Fourier harmonics are chosen for the type of response concerned. (4) The general 3-dimensional formulation is taken for the superstructure and the normal modes decomposition is presumed for the fixed base condition. Among them, the modes of large participation factors are selectively adopted, in view of the frequency contentents of the seismic input motions, to make a coupled system with the soil-foundation system, which therefore reduces the degrees of freedom drastically in an effective way for the complete system response analysis.

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Appendix A. Axisymmetric FEM Modeling for Soil Medium

The governing equation for steady state motion of frequency ω for a visco-elastic soil medium is given from the virtual work principle by

$$\int_v \delta\epsilon^T \sigma dv = \int_v \delta U^T(-\rho\omega^2 U + b) dv + \int_s \delta U^T t ds \quad \cdots\cdots \text{(A. 1)}$$

in which U denotes the displacement, δU signifies the virtual displacement and $\delta\epsilon$ the corresponding virtual strain all of which satisfy the boundary condition, b defines the body force and t the surface traction, σ the internal stress, and ρ is the density.

The soils in the vicinity of the foundation is modeled as an axisymmetric body that allows the Fourier series expansion in the circumferential direction for describing the behavior. The displacements in the cylindrical frame reference, Eq. (9) are given in a brief form by

$$U(r,\theta,z)=\sum_{n=0}^{\infty}H_n^s(\theta)U_n^s(r,z)+\sum_{n=0}^{\infty}H_n^a(\theta)U_n^a(r,z) \quad \text{(A. 2)}$$

in which the terms U_n^s denote the symmetric harmonic amplitudes while the terms U_n^a the antisymmetric ones. The forces are likewise expanded as

$$P(r,\theta,z)=\sum_{n=0}^{\infty}H_n^s(\theta)P_n^s(r,z)+\sum_{n=0}^{\infty}H_n^a(\theta)P_n^a(r,z) \quad \text{(A. 3)}$$

Substituting Eq. (A. 3) into the strain-displacement relationship in the cylindrical coordinates yields

$$\epsilon=\sum_{n=0}^{\infty}\begin{bmatrix}\cos\widehat{n\theta}B_{1n}\\ \sin\widehat{n\theta}B_{2n}\end{bmatrix}U_n^s+\sum_{n=0}^{\infty}\begin{bmatrix}\sin\widehat{n\theta}B_{1n}\\ \cos\widehat{n\theta}B_{2n}\end{bmatrix}U_n^a \quad \text{(A. 4)}$$

in which

$$B_{1n}=\begin{bmatrix}\frac{\partial}{\partial r} & 0 & 0\\ 0 & \frac{\partial}{\partial z} & 0\\ \frac{1}{r} & 0 & -\frac{n}{r}\\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} & 0\end{bmatrix},\quad B_{2n}=\begin{bmatrix}\frac{n}{r} & 0 & \left(-\frac{1}{r}+\frac{\partial}{\partial r}\right)\\ 0 & \frac{n}{r} & \frac{\partial}{\partial z}\end{bmatrix}$$

The stress components, on the other hand, are given through the constitutive matrix as

$$\sigma=\sum_{n=0}^{\infty}\begin{bmatrix}D_1 & \\ & D_2\end{bmatrix}\begin{bmatrix}\cos\widehat{n\theta}B_{1n}\\ -\sin\widehat{n\theta}B_{2n}\end{bmatrix}U_n^s+\sum_{n=0}^{\infty}\begin{bmatrix}D_1 & \\ & D_2\end{bmatrix}\begin{bmatrix}\sin\widehat{n\theta}B_{1n}\\ \cos\widehat{n\theta}B_{2n}\end{bmatrix}U_n^a \quad \text{(A. 5)}$$

in which

$$D_1=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & \\ \lambda & \lambda+2\mu & \lambda & \\ \lambda & \lambda & \lambda+2\mu & \\ & & & \mu\end{bmatrix}\qquad D_2=\begin{bmatrix}\mu & \\ & \mu\end{bmatrix}$$

and λ and μ define the complex Lame constants. By using Eqs. (A. 4) and (A. 5) in Eq. (A. 1) and integrating termwise along the circumferential direction, one can get

$$\iint\{\alpha_{1n}^s(\delta U_n^{sT}B_{1n}^TD_1B_{1n}U_n^s)+\alpha_{2n}^s(\delta U_n^{sT}B_{2n}^TD_2B_{2n}U_n^s)$$
$$+\alpha_{1n}^a(\delta U_n^{aT}B_{1n}^TD_1B_{1n}U_n^a)+\alpha_{2n}^a(\delta U_n^{aT}B_{2n}^TD_2B_{2n}U_n^a)\}drdz$$
$$=\iint\delta U_n^{sT}\alpha_n^s(\rho\omega^2U_n^s+b_n^s)drdz+\iint\delta U_n^{aT}\alpha_n^a(\rho\omega^2U_n^a+b_n^a)drdz$$
$$+\int_s\delta U_n^{sT}\alpha_n^st_n^sds+\int_s\delta U_n^{aT}\alpha_n^at_n^ads \quad \text{(A. 6)}$$

in which $\alpha_n^s=\mathrm{diag.}(\alpha_{1n}^s\ \alpha_{1n}^s\ \alpha_{2n}^s)$ and $\alpha_n^a=\mathrm{diag.}(\alpha_{1n}^a\ \alpha_{1n}^a\ \alpha_{2n}^a)$, whose elements are

$\alpha_{1n}^s=\alpha_{2n}^a=2\pi,\quad \alpha_{2n}^s=\alpha_{1n}^a=0$ for $n=0$; $\alpha_{1n}^s=\alpha_{2n}^s=\alpha_{2n}^a=\pi$ for $n\neq0$

Note that Eq. (A. 6) reduces the 3-dimensional equation into the 2-dimensional one. Furthermore, the symmetric harmonics and the antisymmetric harmonics are separated.

The dicretization is carried out by assuming a proper shape function $N(r,z)$ for either of harmonics such that

$$U_n(r,z)=N(r,z)\hat{U}_n \quad \text{(A. 7)}$$

in which $\hat{U}_n$ designates the nodal displacements. Following the conventional finite element foumulation, one can get the associated multi-degrees of feedom system whose equation is in geneal expressed as

$$-\omega^2M_n\hat{U}_n+K_n\hat{U}_n=\hat{P}_n \quad \text{(A. 8)}$$

in which M_n and K_n, defining the mass and stiffness matrices, respectively, are computed by

$$M_n=\sum_e\iint\rho N^T\alpha N drdz \quad \text{(A. 9)}$$

$$K_n=\sum_e\{\alpha_{1n}\iint(B_{1n}N)^TD_1(B_{1n}N)drdz+\alpha_{2n}\iint(B_{2n}N)^TD_2(B_{2n}N)drdz\} \quad \text{(A. 10)}$$

and the force vector $\hat{P}_n$ is by

$$\hat{P}_n=\sum_e\left\{\iint N^T\alpha_n b_n d r d z+\int_s N^T\alpha_n t_n ds\right\} \qquad \text{(A. 11)}$$

Appendix B. Transformation Matrices S and T

Among the nodes on a rigid foundation face, a rigid linkage exists with its gravity center like a master-slave nodes relationship so that the displacement vector of the interface nodes i is uniquely determined by that of the gravity center.

$$\begin{Bmatrix}U_x\\U_y\\U_z\end{Bmatrix}_i=\left[\begin{array}{ccc:ccc}1 & & & 0 & z_i & -y_i\\ & 1 & & -z_i & 0 & x_i\\ & & 1 & y_i & -x_i & 0\end{array}\right]\begin{Bmatrix}U_x\\U_y\\U_z\end{Bmatrix}_F \qquad \text{(B. 1)}$$

in which $(x_i,\ y_i,\ z_i)$ is the distance of node i from the gravity center as a origin on a cartesian reference. In order to be consistent in the present modeling for the soil foundation system, Eq. (B. 1) is expressed in a cylindrical coordinates, which results in a combination of displacement modes of

$$\begin{Bmatrix}U_x\\U_y\\U_z\end{Bmatrix}_i=\left(\left[\begin{array}{c:c}\overset{3}{0} & \\ 0 & \\ 1 & \end{array}\right]+\left[\begin{array}{c:c} & \overset{6}{-r\sin\theta}\\ & r\cos\theta\\ & 0\end{array}\right]+\left[\begin{array}{c:c}\overset{1}{1} & \overset{5}{z_i}\\ 0 & 0\\ 0 & -r\cos\theta\end{array}\right]+\left[\begin{array}{c:c}\overset{2}{0} & \overset{4}{0}\\ 1 & -z_i\\ 0 & r\sin\theta\end{array}\right]\right)\begin{Bmatrix}U_x\\U_y\\U_z\end{Bmatrix}_F \quad \text{(B. 2)}$$

in which the 1-st term represents a vertical mode, the 2-nd term the torsional mode, the 3-rd term the coupled motion of translation and rocking in the xz-plane, and the 4-th term also the coupled motion of translation and rocking in the yz-plane. The Eq. (B. 2) then gives the explicit expression for the respective term S_0^s, S_0^a, S_1^s, and S_1^a in Eq. (13) in that order. From the termwise correspondence for the respective modes of motion between Eqs. (12) and (B. 2), the Fourier harmonic expansions end up in the transformatrion matrices, T_n^s and T_n^a in Eq. (14), as

$$T_0^s=\left[\begin{array}{c:c}\overset{3}{0} & \\ 0 & \\ 1 & \end{array}\right]\quad T_0^a=\left[\begin{array}{c:c} & \overset{6}{0}\\ & r\\ & 0\end{array}\right]\quad T_1^s=\left[\begin{array}{c:c}\overset{1}{1} & \overset{5}{z_i}\\ 0 & z_i\\ 0 & -r\end{array}\right]\quad T_1^a=\left[\begin{array}{c:c}\overset{2}{1} & \overset{4}{-z_i}\\ 0 & -z_i\\ 0 & r\end{array}\right] \qquad \text{(B. 3)}$$

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