OPTIMUM CROSS SECTIONAL SHAPES OF STEEL COMPRESSION MEMBERS WITH LOCAL BUCKLING

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Strength evaluation and optimality of steel compression members are examined focusing on the interactive behavior between overall and local failures. The results of optimization indicate that optimal and efficient design is basically obtained in the region where the local buckling does not occur before yielding of component plates, even if the occurrence of local buckling is allowed for design. This implies that restricting the local buckling by specifying the maximum width thickness ratios may be worth while again to consider to accommodate in design specifications, particularly for ordinary civil engineering structures. The practical importance of allowing the occurrence of local buckling may appear only in the designs of large scale and/or specialty-oriented steel structures and components.

1. INTRODUCTION

Design for steel compression members can be made by either allowing or restricting the possible occurrence of the local buckling of component plates. So far it has been common practice to determine cross sectional shapes of compression members through restricting the local buckling by specifying the maximum width thickness ratios applicable for design. This seems because the design procedure needs to be simple and/or, even if desired, rational design methods for the interactive behavior of columns with local buckling have not been found at present.

However there are opininos, as hinted by Usami and Fukumoto[1] especially for very high strength steel columns, that efficient design only be obtained by allowing the possible local buckling of component plates and thus enlarging the freedom for the determination of cross sectional shapes with arbitrary selections of width thickness ratios. With this consideration, some of specifications have adopted design procedures which allow the local buckling. However, rational theoretical basis has not been given for the interaction formulae available, and moreover it still remains to be resolved in a wide range of practical applications whether allowing local buckling may lead to efficient design or not.

This paper firstly discusses the interactive design formula of the Japanese specification for the design of steel highway bridges[2], referred to as the JRA specification, and compares, for reference, with that of

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the appendix C of the AISC specification[3] which is equivalent to that of the AISI specification[4]. Secondly and rather important in this paper, optimality for the design of compression members is presented in order to facilitate efficient design, based both on the original JRA and the AISC interactive formulae with some revised variations. Steel columns examined here include box, H and stiffened box shapes with a variety of steel grades. Structural optimization technique is applied to examine whether allowing local buckling may lead to efficient design or not. Load maximization method[5] is used to obtain optimum solutions in which maximum load carrying capacity under constant weight of materials gives the optimum configuration of structures.

2. OPTIMIZATION BY MAXIMUM LOAD DESIGN

In general, a structure is designed so as to satisfy the conditions as

$$D_j(P, X, Y) \leq C_j(P, X, Y) \qquad (j=1, 2, \cdots n) \cdots\cdots\cdots (1)$$

in which D_j and C_j are the structural response called design function and the corresponding limiting value called code function respectively, and subscript j indicates the incident of concern for design such as stresses and deflections, where both of D_j and C_j can be functions of load P, geometrical configurations $X \equiv \{x_1, \cdots x_m\}$ with m degrees of freedom, and material property Y. Introducing the analysis function as $S_j \equiv D_j/P$ and defining the incident capacity function as $P_j \equiv C_j/S_j$, Eq. (1) is transformed into

$$P \leq P_j(P, X, Y) \qquad (j=1, 2, \cdots n) \cdots\cdots\cdots (2)$$

Noting that linear analysis is used for common practice to evaluate D_j and most of the design codes stipulate C_j irrespective of applied load P, the incident capacity function P_j is assumed the function only of geometrical configurations X without detriment to the practical consequence, when material property is fixed as constant. Design is considered most efficient when the geometrical configuration is determined so as to attain the maximum load under the constant weight of materials. This is mathematically expressed as

$$P_{\max} = \operatorname*{Max}_X \{\operatorname*{Min}_j P_j(X)\} \qquad (j=1, 2, \cdots n) \cdots\cdots\cdots (3)$$

subject to $W(X) = \text{const.}$

in which $P_{\max}$ is the maximum of applicable load P, and W is the total weight or volume of materials used. By selecting total weight or its equivalent as one of the geometry X with the remaining geometry denoted by $\overline{X}$, Eq. (3) is reduced to the unconstrained maximization as

$$P_{\max} = \operatorname*{Max}_{\overline{X}} \{\operatorname*{Min}_j P_j(\overline{X})\} \qquad (j=1, 2, \cdots n) \cdots\cdots\cdots (4)$$

in which the number of the degrees of freedom is decreased by one to $\overline{X} \equiv (x_1, \cdots x_{m-1})$.

3. INTERACTIVE DESIGN FORMULAE

Consider a box shaped column as shown in **Fig. 1**. Utilizing the effective width concept, the Ultimate strength P_u of the column is given symbolically as

$$P_u = \alpha \frac{\pi^2 E I_e}{l^2} = \alpha \frac{\pi^2 E I}{l^2} \frac{I_e}{I} = P_{uc} \frac{I_e}{I} \cdots\cdots\cdots (5)$$

in which $P_{uc} \equiv \alpha \pi^2 E I/l^2$ is the ultimate strength of the column disregarding the local buckling with the Young's modulus, the length of the column, and the buckling coefficient reflecting all the influences other than the local buckling denoted respectively by E ($= 2.06 \times 10^5$ MPa), l and α. The original and the reduced effective moments of inertia denoted respectively by I and I_e are given approximately for a boxed shape as

$$I = b^2 A_f/2, \quad I_e = b^2 A_{fe}/2 \cdots\cdots\cdots (6 \cdot a, b)$$

in which b, A_f and A_{fe} indicate the depth of the web, the original and the effective areas of the flange respectively as shown in **Fig. 2**. Taking
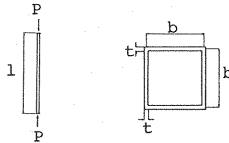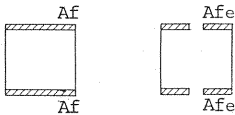

Fig. 1   A Box Shaped Column.

Fig. 2   An Effective Cross Section.

the ratio of Eqs. (6) as

$$\frac{I_e}{I}=\frac{A_{fe}}{A_f}=\frac{A_{fe}\cdot\sigma_y}{A_f\cdot\sigma_y}=\frac{P_{uf}}{P_{yf}}=\frac{\sigma_{ul}}{\sigma_y} \quad\text{···}(7)$$

with σ_y, P_{yf}, P_{uf} and σ_{ul} indicating the yield stress of material, the yield strength and the ultimate strength and stress of the flange portion respectively, Eq. (5) consequently leads to

$$P_u=P_{uc}\frac{\sigma_{ul}}{\sigma_y} \quad\text{···}(8)$$

By dividing the both sides of Eq. (8) by the original cross sectional area A of the whole section, the ultimate strength in terms of stresses interacting with the local buckling of component plates can finally be expressed as

$$\sigma_u=\frac{P_u}{A}=\frac{P_{uc}\sigma_{ul}}{A\sigma_y}=\sigma_{uc}\frac{\sigma_{ul}}{\sigma_y} \quad\text{···}(9)$$

in which σ_{uc} denotes the ultimate stress of the column disregarding the occurrence of the local buckling.

The mutually independent overall and local ultimate strengths of the column as introduced by σ_{uc} and σ_{ul} in Eqs. (7) and (9) are expressed, in general, as

$$\sigma_{uc}=\sigma_y f(\lambda_c),\quad \sigma_{ul}=\sigma_y g(\lambda_l) \quad\text{··}(10\cdot\text{a, b})$$

in which

$$\lambda_c\equiv\frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\frac{l_e}{r},\quad \lambda_l\equiv\frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}\cdot\frac{12(1-\nu^2)}{k}}\frac{b}{t} \quad\text{····································}(11\cdot\text{a, b})$$

called the normalized slenderness and width thickness ratios respectively with the effective length of the column and the radius of gyration of the original section denoted respectively by l_e and r for λ_c, and the width and thickness of component plate, the buckling coefficient of plate, and the Poisson's ratio respectively designated by b, t, k and $\nu=0.3$ for λ_l.

The JRA specification adopts Eq. (9) now denoted by σ_{uJ} for the design of steel compression members, which can be transformed into

$$\sigma_{uJ}=\sigma_y f(\lambda_c)g(\lambda_l) \quad\text{···}(12)$$

using Eqs. (10). The interactive formula adopted by the AISC specification denoted here by σ_{uA} has been derived simply by replacing the yield stress of material in the overall strength formula by the local strength as

$$\sigma_{uA}=[\sigma_{uc}]_{\sigma_y=\sigma_{ul}} \quad\text{···}(13)$$

which, for the convenience of comparison, can be transformed into

$$\sigma_{uA}=\sigma_y f(\lambda_c')g(\lambda_l)=\sigma_y f(\sqrt{g(\lambda_l)}\,\lambda_c)g(\lambda_l) \quad\text{·····································}(14)$$

in which

$$\lambda_c'\equiv\frac{1}{\pi}\sqrt{\frac{\sigma_{ul}}{E}}\frac{l_e}{r}=\sqrt{g(\lambda_l)}\,\lambda_c \quad\text{···}(15)$$

Noting that the values of $f(\lambda_c)$ and $g(\lambda_l)$ are not greater than unity, and decrease with the increase of λ_c and λ_l respectively as typically shown in **Fig. 3**, the following inequality as

$$\sigma_{uA}/\sigma_{uJ}\geq1 \quad\text{··}(16)$$

is always satisfied.

Meanwhile, if the overall and local failures of compression members be assumed non-interactive, the ultimate strength would be given as

Fig. 3 Local and Overall Strengths.

$$\sigma_{uM}=\text{Min}\{\sigma_{uc}(\lambda_c),\ \sigma_{ul}(\lambda_l)\} \quad\text{····································}(17)$$

which may not indicate the actual behavior but shall be used for reference in this paper.
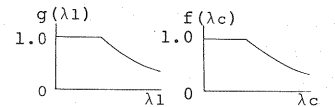
4. OPTIMALITY FOR COMPRESSION MEMBERS

Optimization for steel compression members were performed, focusing mainly on whether allowing the

local buckling of component plates will bring the benefits for efficient design or not. The box, H shaped and stiffened box columns are considered in this paper. The load maximization procedure as expressed by Eq. (4) is used to obtain the optimal configurations. The compression members of concern are assumed simply supported at both ends. The yeild stress of material is taken as 235 MPa corresponding to SS 41 Steel. The results are given for JRA and AISC interaction formulae as well as non-interactive formula of Eq. (17) for comparision. The design formula used for the optimization of compression members is taken from the design provisions of the JRA specification as

$$\frac{\sigma_{uc}}{\sigma_y} = f(\lambda_c) = \begin{cases} 1.0 & (0 < \lambda_c \leq 0.2) \\ 1.0 - 0.545(\lambda_c - 0.2) & (0.2 < \lambda_c \leq 1.0) \\ 1/(0.773 + \lambda_c^2) & (\lambda_c > 1.0) \end{cases} \cdots\cdots (18)$$

for the part of overall ultimate strength of Eq. (10·a) throughout this paper.

(1) Box shaped columns

Consider a box shaped square column with width b and thickness t as shown in **Fig. 1**. The buckling coefficient k for the plate component in Eq. (11·b) is taken as 4.0. The design formula for the local ultimate strength of Eq. (10·b) is given also from the design provisions of JRA specification as

$$\frac{\sigma_{ul}}{\sigma_y} = g(\lambda_l) = \begin{cases} 1.0 & (0 < \lambda_l \leq 0.7) \\ 0.49/\lambda_l^2 & (\lambda_l > 0.7) \end{cases} \cdots\cdots\cdots (19)$$

The adoption of Eq. (19) to reflect the influence of local buckling on the interactive strength is considered a considerably safer estimate of the true strength for design purpose, because Eq. (19) is simply the buckling strength of plate components, and thus may account for a decrease of the flexural stiffness, but not for the possible reserved post-buckling strength.

Defining the geometrical and material properties nondimensionalized as

$$x \equiv b/t, \qquad R \equiv l^2/A = l^2/4\,bt$$
$$g_y \equiv \sqrt{E/\sigma_y}, \qquad \overline{P} \equiv P/\sigma_y l^2 \cdots\cdots (20 \cdot a\sim d)$$

the load carrying capacity in Eq. (4) is expressed as

$$\overline{P}_j = \overline{P}_j(x, g_y, R) \qquad (j=1) \cdots\cdots (21)$$

Note that the number of the incident of concern is only one, that is, $n=1$ for j. Assuming that the length of column be prescribed for design, the constraint of the constant volume becomes identical to the value of R being constant. Thus, noting that g_y is constant due to the material being given, the only one variable x is subject to optimization, namely $\overline{X} = \{x\}$ in Eq. (4).

Fig. 4 shows the result of optimization, depicting the relation between the maximum load $\overline{P}_{\max}$ and the parameter R combined with **Fig. 5** indicating the optimal width thickness ratio which maximizes the load. The respective ultimate strengths $f(\lambda_c)$ and $g(\lambda_l)$ at optimum are shown in **Fig. 6**. It should be noted that **Fig. 6** indicates neither σ_{uJ} of Eq. (12) nor σ_{uA} of Eq. (14), but only implies notional ultimate strength determined from the optimal configurations.

As a matter of fact, it is seen from **Figs. 4~6** that the use of the non-interactive formula produces the highest maximum load at the concurrent overall and local failures with varying optimum width thickness ratios. On the other hand, the use of the interactive formulae of the JRA and AISC specifications does not give the optimum at the concurrent failures as clear in **Fig. 6**. It is interesting to note for the JRA formula from **Figs. 5** and **6** that the optimum width thickness ratios are found at the maximum ratios which do not allow the occurrence of local buckling before yielding, as given by the equality of $\lambda_l = 0.7$ in Eq. (19). Although the AISC formula may give the optimum in the region reflecting the reduction due to the local buckling for larger R which corresponds to a slender column, the maximum load shows a very little difference from the case of the JRA formula, and it is also worth while to mention that the ultimate column strength seems only to exist in the range of $f(\lambda_c) < 0.45$, when allowing local buckling may give the different consequence compared with restricting it from the view of optimum design.
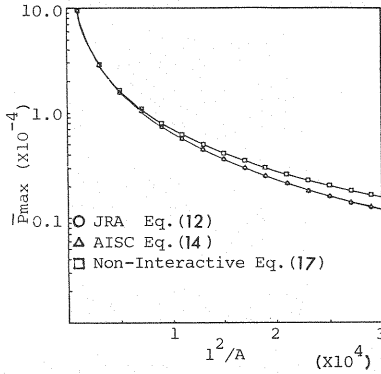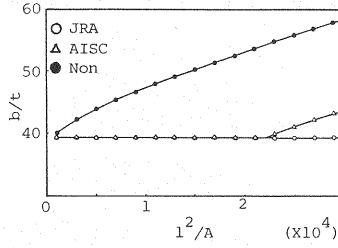
Fig. 4 $\overline{P}_{\max}$ for a Box Column.
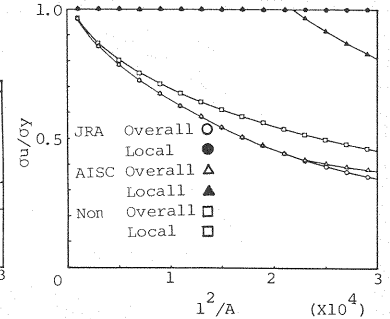

Fig. 5 Optimal b/t for a Box Column.

Fig. 6 σ_{uc}/σ_y and σ_{ul}/σ_y at optimum for a Box Column.

(2) H shaped columns

As the second example of practical sections, consider a symmetrical H shaped column with flange width and thickness denoted respectively by b and t_f, and web depth and thickness denoted respectively by h and t_w, as shown in Fig. 7. Two independent states of local ultimate strength with the same design formula as Eq. (19) denoted now by σ_{ulf} and σ_{ulw} can be introduced for a flange and a web of the H shaped column as given respectively by

$$\sigma_{ulf}=\sigma_y g(\lambda_f), \quad \sigma_{ulw}=\sigma_y g(\lambda_w) \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (22\cdot\text{a, b})$$

in which λ_f and λ_w are defined in Eq. (11·b) by replacing the width thickness ratio b/t to $b/2\,t_f$ and h/t_w with the buckling coefficient k substituted by 0.43 and 4.0 respectively. The local ultimate strength in contrast to the overall strength for H shaped columns now is defined by

$$\sigma_{ul}=\text{Min}\,\{\sigma_{ulf}(\lambda_f), \quad \sigma_{ulw}(\lambda_w)\} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (23)$$

Similar as for box columns, the nondimensionalized geometrical properties are introduced for H shaped columns as

$$x_1\equiv b/t_f, \quad x_2\equiv h/t_w, \quad x_3\equiv A_w/A_{cf}=h\,t_w/b\,t_f \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (24\cdot\text{a}\sim\text{c})$$

with the same definitions for R, g_y and $\overline{P}$ as in Eqs. (20·b~d). A_w and A_{cf} in Eq. (24·c) denote the areas of web and single flange respectively. Using Eqs. (24), the load carrying capacity in Eq. (4) is expressed by

$$\overline{P}_j=\overline{P}_j(x_1, x_2, x_3, g_y, R) \quad (j=1)\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (25)$$

in which the three variables are subject to optimization, namely $\overline{X}=\{x_1, x_2, x_3\}$.

Fig. 7 A H Shaped Column.

The results of optimization are presented similar as for box columns. Fig. 8 shows the maximum load $\overline{P}_{\max}$ vs. R relations with Fig. 9 (a) ~ (c) indicating the optimal width thickness ratios for flanges and web, and the optimal area ratio A_w/A_{cf} which maximize the load. The respective ultimate strengths $f(\lambda_c)$ and $g(\lambda_l)$ at optimum are shown in Fig. 10, in which it is remembered that the optimal configurations have given the same local ultimate strength for flanges and webs, irrespective of the design formulae of Eqs. (12), (14) and (17) applied. The optimal characteristics for H shaped columns are found similar as the results for box shaped columns, only exhibiting for the case of the AISC formula a little shift which expands the region where allowing the local buckling gives different optimum configurations from those obtained by restricting it. However, it should also be reminded that, even for this case, the maximum
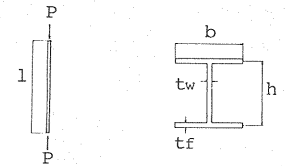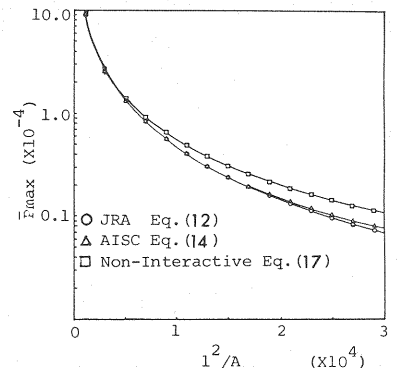

Fig. 8 $\overline{P}_{\max}$ for a H Column.

(a) Optimal b/t_f (b) Optimal h/t_w (c) Optimal A_w/A_{cf}
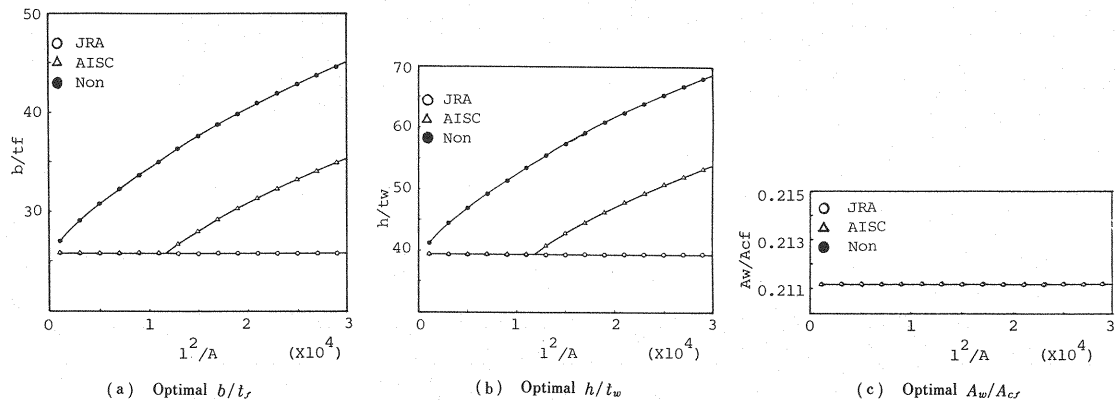
Fig. 9 Optimal Shapes for a H Column.

load does not increase much from that obtained by the use of the JRA formula, and moreover for the region of concern the independent ultimate column strength of $f(\lambda_c)$ reaches only up to 0.45, which is almost the same magnitude as for box columns.

(3) Stiffened box shaped columns

As the third example of practical sections, consider a stiffned box column as shown in **Fig. 11**. Two independent states of local ultimate strengths of component plate denoted by σ_{uls} and σ_{ul} can be introduced for stiffened panel and stiffner respectively. The design formula used for the stiffened panel is given from the JRA specification as

$$\frac{\sigma_{uls}}{\sigma_y} = g(\lambda_{ls}) = \begin{cases} 1.0 & (0 < \lambda_{ls} \leq 0.5) \\ 1.5 - \lambda_{ls} & (0.5 < \lambda_{ls} \leq 1.0) \\ 0.5/\lambda_{ls}^2 & (\lambda_{ls} > 1.0) \end{cases} \quad \cdots\cdots\cdots\cdots\cdots\cdots (26 \cdot a)$$

in which

$$\lambda_{ls} = \frac{1}{\pi} \sqrt{\frac{\sigma_y}{E} \cdot \frac{12(1-\nu^2)}{k_s}} \frac{b}{t} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (26 \cdot b)$$

The buckling coefficient k_s appeared in Eq. (26·b) is given by

$$k_s = \text{Min}\{k_F, \ k_R\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (27)$$

representing a possible local failure mode of the stiffened panel, in which

$$k_F = \begin{cases} \dfrac{(1+\alpha^2)^2 + n\gamma}{\alpha^2(1+n\delta)} & (\alpha < \sqrt[4]{1+n\delta}) \\ \dfrac{2(1+\sqrt{1+n\gamma})}{1+n\delta} & (\alpha > \sqrt[4]{1+n\delta}) \end{cases} \quad \cdots\cdots\cdots\cdots\cdots\cdots (28 \cdot a)$$

and

$$k_R = 4\,n^2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (28 \cdot b)$$

in which α, δ and γ are aspect ratio of the plate component, area ratio and rigidity ratio of single sided rectangular stiffener respectively given by

$$\alpha = \frac{a}{b} \qquad \delta = \frac{b_s t_s}{bt} \qquad \gamma = \frac{4(1-\nu^2)b_s^3 t_s}{bt^3} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (29 \cdot a \sim c)$$

using the geometries shown in **Fig. 11**, and n is the number of panels divided by stiffeners. The design formula applicable for the stiffener itself is the same as Eq. (19) with b/t and k substituted by b_s/t_s and 0.43 respectively.

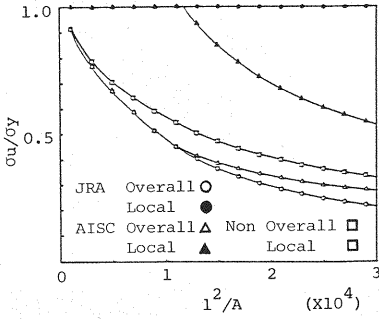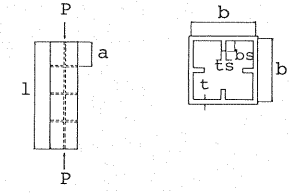With the adoption of the design formulae presented above, the local ultimate strength of the column is given by

Fig. 10 σ_{uc}/σ_y and σ_{ul}/σ_y at Optimum for a H Column.

Fig. 11   A Stiffened Box Column.
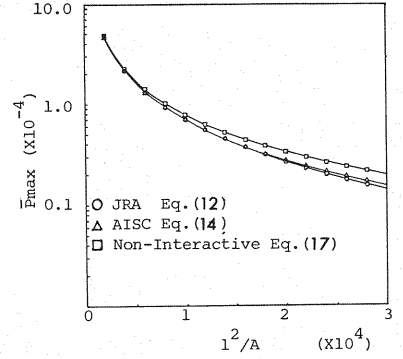
Fig. 12 $\overline{P}_{max}$ for a Stiffened Box Column.

(a) Optimal b/t (b) Optimal b_s/t_s (c) Optimal δ
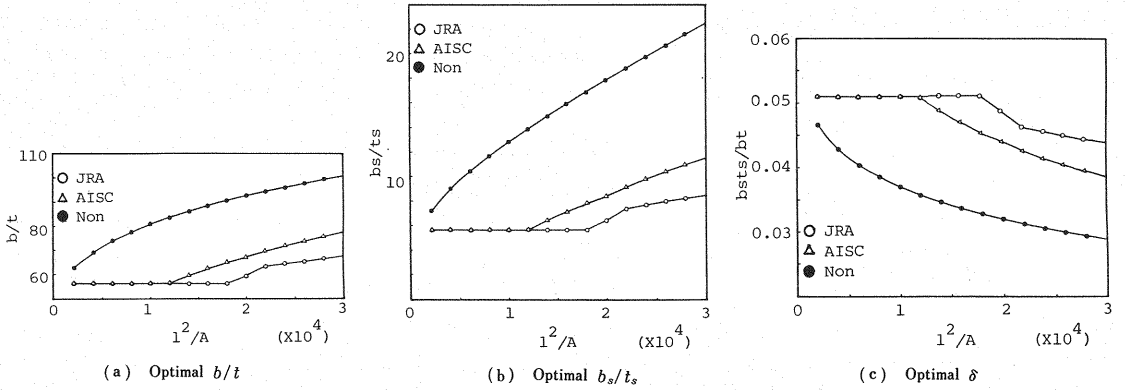
Fig. 13   Optimal Shapes for a Stiffened Box Column.

$$\sigma_{ul} = \text{Min}\{\sigma_{uls}, \ \sigma_{ul}\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (30)$$

Introducing the following nondimentionalized parameters as

$$x_1 \equiv b/t, \ \ x_2 \equiv b_s/t_s, \ \ x_3 \equiv \delta = b_s t_s/bt \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (31)$$

the load carrying capacity in Eq. (4) can be expressed as

$$\overline{P}_j = \overline{P}_j(x_1, \ x_2, \ x_3, \ g_y, \ R, \ n, \ \alpha) \ \ \ \ (j=1) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (32)$$

in a similar way as for box and H shaped columns, in which the three variables x_1, x_2 and x_3 are subject to optimization with n, α, g_y and R as parameters.

The results of optimization with a particular case of $n=2$ and $\alpha=1$ are presented in the same manner as for the previous two examples. Fig. 12 shows the maximum load $\overline{P}_{max}$ vs. R relations while Fig. 13 indicates the changes of optimal geometries of respective width thickness ratios b/t, b_s/t_s and area ratio δ with respect to R. Shown in Fig. 14 are the respective ultimate strengths $f(\lambda_c)$, $g(\lambda_{ls})$ and $g(\lambda_l)$ at optimum, in which it is remembered that the optimal geometries have given the same local ultimate strength for stiffened panels and stiffeners with k_F identical to k_R in Eq. (28). Although the general characteristics of optimum configurations are similar to the previous examples, the range to exhibit the advantage of allowing the occurrence of local buckling tends to be enlarged, particularly for the case of the JRA interactive formula. It is noted from Fig. 14, however, that the maximum of the independent ultimate column strength of $f(\lambda_c)$ in this range is only 0.57 and 0.64 for the JRA and AISC interactive formulae respectively. The column slenderness corresponding to those values may not be said impractical, different from the results of box and H shaped columns, but can be regarded to constitute only a fraction of practical members.
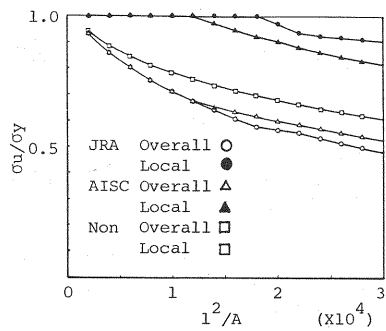
(4)   Comparative studies

Fig. 14 σ_{uc}/σ_y and σ_{ul}/σ_y at Optimum for a Stiffened Box Column.

Table 1   Critical values of $\sigma_{uc}/\sigma_y = f(\lambda_c)$ for the advantage of local buckling.

		Eq.(19) or Eq.(26a)		Eq(33)	
		JRA	AISC	JRA	AISC
Box	SS41	*	0.42	*	0.55
	SM53	*	0.43	*	0.56
H	SS41	*	0.43	*	0.56
	SM53	*	0.44	*	0.57
Stiffened Box	SS41	0.56	0.64	0.56	0.67
	SM53	0.57	0.67	0.57	0.68

Remarks σ_y= 235 MPa for SS41 and σ_y= 352 MPa for SM53

The results of optimization presented above are considered only true for the interactive formulae and the respective overall and local ultimate strengths employed and the yield stress of material used. In order to examine the effects and sensitivity of the premise cited above, two additional cases are considered to obtain the consequences of optimization. The one is to employ another evaluation of the design formula for local ultimate strength as given by

$$\frac{\sigma_{ul}}{\sigma_y} = g(\lambda_l) = \frac{0.7}{\lambda_l} \qquad (\lambda_l > 0.7) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (33\cdot a)$$

instead of Eq. (19), and

$$\frac{\sigma_{uls}}{\sigma_y} = g(\lambda_{ls}) = \frac{0.5}{\lambda_{ls}} \qquad (\lambda_{ls} > 1.0) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (33\cdot b)$$

instead of Eq. (26·a), both of which are employed here by the reason that a possible reserved post-buckling strength of component plates may be incorporated effectively in the evaluation of the interactive formulae used. The other is the use of different grades of steel, in which the yeild stress of 352 MPa corresponding to SM 53 steel is employed here for comparison.

Computation is made in a similar way as before. The most concerned for the results is a possible shift of the critical point of column slenderness beyond which the advantage of allowing the occurrence of local buckling is expected for the design of steel columns. As has been discussed in the previous cases, the critical points of concern are given in Table 1 in terms of the independent ultimate column strength of $\sigma_{uc}/\sigma_y = f(\lambda_c)$, not in terms of the direct expressions of column slenderness λ_c. This means that if a column is proportioned with its characteristic strength more than the value given in the Table, the optimum shape is found only in the range without local buckling. Table 1 includes the summary of the previous cases of Eq. (19) or (26·a) with σ_y=235 MPa for the convenience of comparision, in which asterisk (*) indicates that the optimum solutions are found to exist in the range where the occurrence of local buckling is not expected up to the column slenderness l^2/A equal to 30 000.

From the Table, it is observed that an alternate use of Eq. (33·a) incorporating post-buckling strength for box and H shaped columns tends to enlarge the range where the advantage to allow local buckling is expected for the case of the AISC interactive formula, but the use of higher yeild strength steel does not so remarkably. As for stiffened box columns it is noted that those alternative uses do not change the consequence so much compared with the previous results for Eq. (26·a) with σ_y=235 MPa, since the optimum solutions have been obtained in the range of $0 < \lambda_{ls} \leq 1.0$, where alternative use of Eq. (33·b) does not cause any effect.

In most civil engineering structures, steel columns tend to be designed in stocky and intermediate ranges of slenderness, where the independent ultimate column strength $f(\lambda_c)$ rarely falls below the value of 0.6. Considering this practical situations combined with the summerized results of Table 1, it is said to be exception rather than a general rule that allowing the occurrence of local buckling gives the advantages for

practical design. It should be noted, however, that the above advantages tend to be experienced more for higher evaluation of the interactive formula and local ultimate strength, for higher grades of steel, and for stiffened box columns, although the consequence is not so remarkable within the content examined in this paper.

5. CONCLUDING REMARKS

Conceptual rationale is explained for the interactive design formula between the overall and local buckling of steel compression members adopted by the JRA specification, although the design formula itself was originally given by intuitive basis as a safer approximation. With this JRA interactive formula compared for reference with the AISC interactive formula, the optimization using the load maximization technique is performed in order to make clear of the optimal configurations of steel compression members, when the occurrence of local buckling is allowed with the use of the interactive formulae.

When the interaction is ignored, although it is theoretically meaningless, the optimal configurations are obtained when the concurrent failures occur. On the other hand, when the interactive formulae are applied, whichever from the JRA and the AISC specifications, the optimal configurations are found at the geometry in which the respective overall and local ultimate strengths differ from each other. It is interesting to note from the results of optimization that optimal and efficient design is basically obtained in the region where the local buckling does not occur before yielding of component plates, even if the occurrence of local buckling is allowed for design. This implies that restricting the local buckling by specifying the maximum width thickness ratios may be worth while again to consider to accommodate in design specifications, particularly for ordinary civil engineering structures.

It is noted, however, that the advantage to allow local buckling tends to be observed more for higher evaluation both of the interactive formula and local ultimate strength, for higher grades of steel, and for stiffened box columns, although the consequence is not so remarkable within the contents examined in this paper. It is said henceforth that the practical importance of allowing the occurrence of local buckling may only appear in the designs of large scale and/or specialty-oriented steel structures and components.

6. ACKNOWLEDGEMENT

REFERENCES

1) Usami, T. and Fukumoto, Y. : Local and Overall Buckling of Welded Box Columns, Journal of the Structural Division, ASCE, Vol. 108, No. ST 3, pp. 525~542, March, 1982.
2) JRA : Specification for the Design of Highway Bridges, Part 2, Steel Bridges, Japanese Road Association, Feb. 1980.
3) AISC : Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, American Institute of Steel Construction, Nov. 1978.
4) AISI : Specification for the Design of Cold-Formed Steel Structural Members, American Iron and Steel Institute, 1968.
5) Hasegawa, A., Hall, W. B. and Lind, N. C. : Maximum Load Design of Cold Formed Steel Channel Beams, Journal of the Structural Division, ASCE, Vol. 104, No. ST 7, pp. 1093~1107, July 1978.

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