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STRESS FLOW OF MEMBRANE EFFECT IN POINT-LOADED REINFORCED CONCRETE SLABS

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The present work is concerned with the membrane action in reinforced concrete slabs subjected to point-loading after cracking. Much attention is focused on the characteristic stress flow of membrane effect caused by point-loading. In a finite element crack analysis, the crack patterns are well simulated and the associated principal directions of membrane forces are calculated. Plotting of both these figures for each load step reveals the strong dependence of the stress flow of membrane forces on the corresponding crack pattern.

1. INTRODUCTION

It is well known that a reinforced concrete slab behaves quite differently before and after cracking. This is true, especially when the RC slab is in such a way supported that the inplane load-carrying capacity can be utilized. An experimental study of the nonlinear response of RC slabs is very costly and requires much time and labour. Consequently, a parameter study is not possible in the laboratory. On the other hand, the theoretical crack analysis by using a computer has been well established and the post-cracking behavior of RC salbs can be predicted with fair accuracy today (reference 1)). For this reason, numerical experiments are more expedient to predict the complex behavior of RC slabs under arbitrary loading conditions. The reproducibility of a previous slab test too is a strong point of the numerical approaches.

The present study aims to investigate the mechanical system of membrane action in cracked RC slabs subjected to point loads. It is demonstrated that the characteristic stress flow of membrane effect can be predicted to some extend by using the observed crack distribution on the slab surfaces.

2. MEMBRANE EFFECT

The membrane action of edge-constrained RC slabs has been known over a long period of time. However, this load-carrying capacity of RC slabs after cracking has not yet been well studied theoretically and even experimentally. The analytical and experimental research reported in 8) is in this respect worthy of special mention. The mechanical behavior of dome action was investigated for instance by F. Fujii in 2), 3), 4), 5) and 6).

For the practical utilization of the enhanced slab strength, the informations on the characteristic

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membrane-bending response of cracked RC slabs are necessary and a reasonable design formula must be established.

The first step to this goal has been made in the references 5) and 6) where the crack patterns were used to predict the principal directions of membrane forces. It was concluded that there is a close correlation between the principal directions of moments, the crack patterns observed on the slab surfaces and also the stress trajectories of membrane action. To generalize this result obtained in the previous papers 5) and 6) by the author, other cases of RC slabs must be examined. In the present study, point-loaded RC slabs are considered, because the point-loading causes a localized singular crack distribution that is quite different from the crack figures for uniform loading.

The crack model used here has been presented in detail elsewhere 2),3) and 4). Therefore, it is unnecessary to repeat the theory of the model. The basic idea of the crack model is only to replace the actual cracked slab with an equivalent plate model stiffened eccentrically. The assumptions made in the analysis is outlined as follows: For concrete, a linear stress-strain relationship is employed in the elastic range before cracking. Cracking can be checked by comparing the tensile stress in concrete to its tensile strength. No tension stiffening after cracking is considered and any compressive failure of concrete is excluded. Reinforcing steel bars are assumed to be ideal-plastic after yielding. No slip between concrete and steel will occur in the analysis. Other usual assumptions in plate theory are also valid in the formulation of the crack model.

The following presentation is limited to the results obtained and its discussion only.

3. EXAMPLES

Rectangular slab

Figure 1 describes a rectangular slab with the side ratio 3/2. For this RC slab, two loading cases are considered: one point load P in the slab center and two point loads $(P/2+P/2)$ on the horizontal center line: The slab properties shown in Fig. 1 are used for the two loading cases commonly. The obtained results for the different cases are shown in Fig. 2 and in Fig. 3 respectively.

In Fig. 2, the cracking starting on the lower side causes immediately a very clear stress flow like

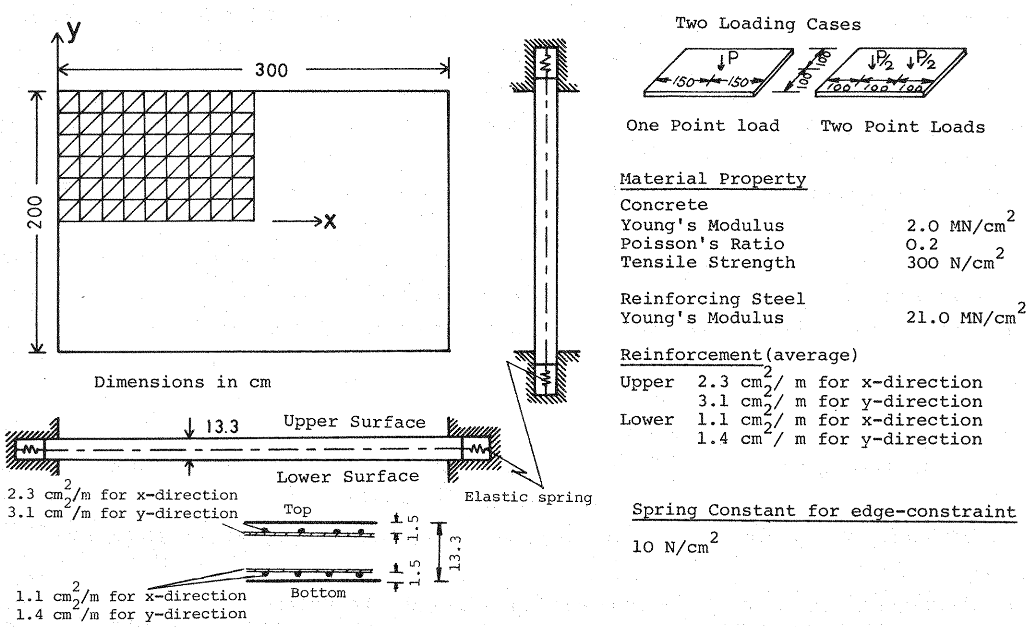

Fig.1 Rectangular Slab.

concentric circles around the slab center, as Fig. 2(a) demonstrates. The crack patterns for $P=200$ KN in Fig. 2(b) show that the crack lines form elliptical figures on the top surface. The corresponding stress trajectories of membrane action in the RC slab include the crack figures on the slab surface in it.

The rectangular slab subjected to two point loads $(P/2+P/2)$ shows before cracking no considerable membrane forces. However, the unsymmetrically reinforced slab section combines the bending and the membrane behavior of the slab. This interaction yields the membrane stresses theoretically already in the uncracked slab. Once the positive cracking appears near the point loads on the bottom surface, the membrane stresses show a flow in Fig. 3(a) like in a magnetic field around two magnetic poles. For the increased loading $P=150$ KN and 200 KN, cracks develop along the fixed slab edges on the top side

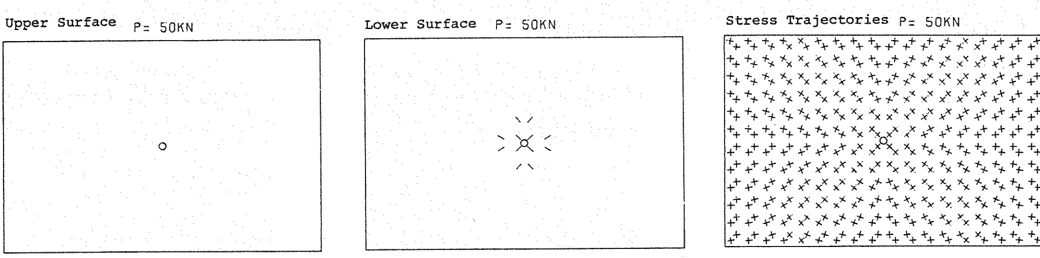

Fig.2 (a) Crack Pattern and Stress Trajectories for P=50 KN.

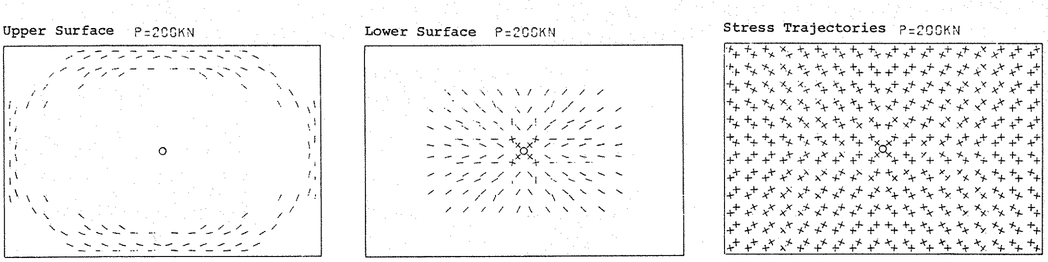

Fig.2 (b) Crack Pattern and Stress Trajectories for P=200 KN.

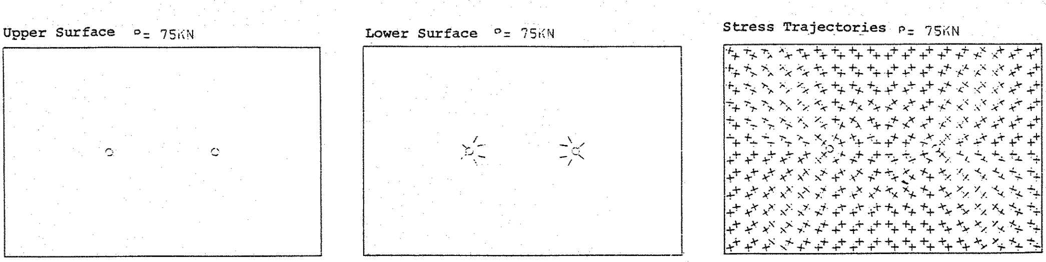

Fig.3 (a) Crack Pattern and Stress Trajectories for P=75 KN.

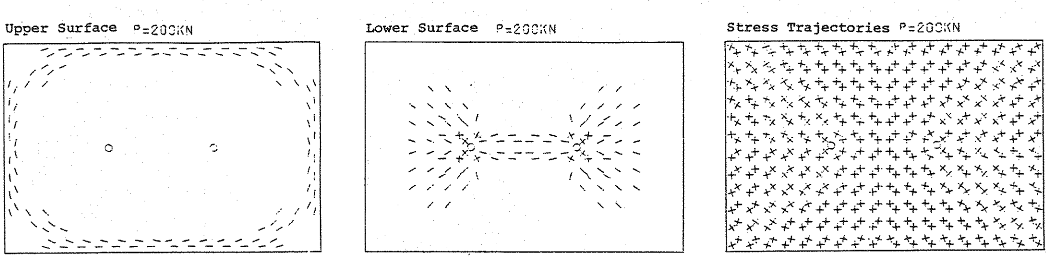

Fig.3 (b) Crack Pattern and Stress Trajectories for P=200 KN.

and extend to the corners on the bottom side. In the last step $P=200$ KN, the crack patterns on the top and bottom surfaces can be seen superposed in the stress trajectories.

4. CONCLUSIONS

For two point-loaded RC slabs, the compressive membrane action depends on the associated crack pattern more clearly than for the slabs under uniform load reported in the previous papers 5) and 6). It can be concluded that the cracking of concrete causes a change in location of centroid axis in slab section and this eccentricity initiates a strong coupling effect between membrane and bending action. As consequence, the membrane forces will be produced especially in direction perpendicular to crack lines, so that the cracked RC slab can utilize its membrane capacity too for transverse loading. This hypothesis allows to use the observed crack figures on slab surface to obtain the informations on the membrane stress field in the slab. It is also signified to that the results in the present study might be useful for developing practical design formulae.

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