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OPTIMAL DESIGN OF SKELETAL STRUCTURES UNDER ELASTIC AND PLASTIC DESIGN CRITERIA

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This study presents a sequential linear programming approach for the optimal design of skeletal structures satisfying simultaneously both elastic and plastic design criteria at the service and ultimate load levels, respectively. The paper also examines the effective range of the ultimate load constraint by determining the upper and lower bounds of the effective load factor. The design method develops a minimum weight design focused on the merit of plastic design by a linear programming. Three examples of truss and frame are designed to illustrate the features and scope of application of the approach.

1. INTRODUCTION

In the past studies on the optimal design of skeletal structures, two design methods have been developed separately. That is, one is the optimal elastic design[1] which considers the constraints on the elastic stresses and deformations at the service load level, and the other is the optimal plastic design[2] which considers the ultimate load constraint ensuring adequate safety against collapse.

Recently from the viewpoint of the earthquake resistant design, it has become important to develop the design method satisfying simultaneously both elastic and plastic design criteria. As a typical example, the earthquake resistant design method of buildings in Japan is listed. But there is no study on such an optimal design method except one.

Grierson and Schmit[3] has proposed the synthesis under service and ultimate performance constraints. However, the study is primarily focused on the optimal elastic design and, therefore, the ultimate load factor has not been well approximated in the effective range of plastic design criterion. In other words, the merit of plastic design by a linear programming (LP) has not been utilized enough in its formulation. Furthermore, the effectiveness of the ultimate load constraint has not been pointed out yet in its study.

In order to overcome the problems mentioned above, this study develops the optimal design method focused on the optimal plastic design under both elastic and plastic design criteria. Then, this study also

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examines the effective range of ultimate load constraint by determining the upper and lower bounds of the effective load factor.

The design process involves the minimization of total steel weight, subject to satisfactory stress (S) and deformation (D) constraints at the service load level and the ultimate load factor (U) constraint. The former (S and D) constraints are formulated in the first approximation of Taylor expansion by design variable, and the latter (U) constraint is explicitly expressed as a LP formulation based on the static theorem of plastic design. Therefore, a minimum weight design can be formulated by a LP primal or dual problem for each design stage and the problem can be solved by using a SLP technique which is easy to handle.

The 3-bar truss, the 1-story 1-span frame and the 2-story 4-span frame are designed to illustrate the features and scope of application of the approach.

2. DESIGN FORMULATION AND PROCEDURE

(1) Original Problem

A minimum weight design under service and ultimate performance constraints is formulated by combining the optimal elastic design with the optimal plastic design as follows[3]:

$$\text{Object}: V=\rho \boldsymbol{L}^T \boldsymbol{X} \rightarrow \min. \quad (1\cdot a)$$

$$\text{Constraint}: \boldsymbol{\sigma}_a^L \leqq \boldsymbol{\sigma} \leqq \boldsymbol{\sigma}_a^U \quad (1\cdot b)$$

$$\boldsymbol{u}_a^L \leqq \boldsymbol{u} \leqq \boldsymbol{u}_a^U \quad (1\cdot c)$$

$$\alpha_0 \leqq \alpha \quad (1\cdot d)$$

$$\boldsymbol{X}^L \leqq \boldsymbol{X} \leqq \boldsymbol{X}^U \quad (1\cdot e)$$

where Eq.(1·a) defines the minimum total steel weight of the structure; Eqs.(1·b) and (1·c) represent the stress (S) and deformation (D) constraints at the service load level, respectively; Eq. (1·d) expresses the ultimate load factor constraint (U) ; Eq. (1·e) means the minimum element size constraint. $\boldsymbol{\sigma}$, $\boldsymbol{u}$=the stress and deformation vectors at the service load level, respectively ; α=the plastic collapse load factor ; $\boldsymbol{\sigma}_a^U$, $\boldsymbol{\sigma}_a^L$=the upper and lower bounds of the allowable stress vector; $\boldsymbol{u}_a^U$, $\boldsymbol{u}_a^L$=the upper and lower bounds of the allowable deformation vector; α_0=the ultimate design load factor ensuring adequate safety against plastic collapse; $\boldsymbol{X}^U$, $\boldsymbol{X}^L$=the upper and lower bounds of the design variable (element size) vector; ρ=the weight per unit volume ($\rho=1$ is used in this study).

(2) Formulation of SLP Primal Problem

Noting the characteristics of LP formulation in the plastic design, the ultimate load constraint of Eq. (1·d) can be expressed as the LP problem based on the static theorem of the plastic design. On the other hand, the service performance constraints of Eqs. (1·b) and (1·c) can be formulated by a first-order Taylor series in the design variable X_i. Therefore, the optimal design under elastic and plastic design criteria can be expressed as the following LP primal problem for each design stage.

Given : ρ, $\boldsymbol{L}$, $\boldsymbol{\sigma}_a^U$, $\boldsymbol{\sigma}_a^L$, $\boldsymbol{u}_a^U$, $\boldsymbol{u}_a^L$, $\boldsymbol{X}^U$, $\boldsymbol{X}^L$, μ, $\boldsymbol{\sigma}^*$, $\nabla\boldsymbol{\sigma}^*$, $\boldsymbol{u}^*$, $\nabla\boldsymbol{u}^*$, $\boldsymbol{X}^*$, $\boldsymbol{C}$, $\boldsymbol{F}$, $\boldsymbol{N}$, $\overline{\boldsymbol{R}}$

Find : primal variable $\boldsymbol{Q}$, $\boldsymbol{X}$, α

dual variable $\boldsymbol{\beta}_S^{\pm}$, $\boldsymbol{\beta}_D^{\pm}$, $\boldsymbol{\beta}_J^{\pm}$, β_U, $\dot{\boldsymbol{u}}$, $\boldsymbol{\lambda}$

$$\text{Object}: V=\rho \boldsymbol{L}^T \boldsymbol{X} \rightarrow \min. \quad (2\cdot a)$$

Constraint :

$$S: \boldsymbol{\sigma}_a^L \leqq \boldsymbol{\sigma}^* + \nabla\boldsymbol{\sigma}^*(\boldsymbol{X}-\boldsymbol{X}^*) \leqq \boldsymbol{\sigma}_a^U \quad (2\cdot b)$$

$$D: \boldsymbol{u}_a^L \leqq \boldsymbol{u}^* + \nabla\boldsymbol{u}^*(\boldsymbol{X}-\boldsymbol{X}^*) \leqq \boldsymbol{u}_a^U \quad (2\cdot c)$$

$$U: \begin{cases} \alpha_0 \leqq \alpha & (2\cdot d) \\ \boldsymbol{C}^T\boldsymbol{Q} - \alpha\boldsymbol{F} = \boldsymbol{0} & (2\cdot e) \\ \boldsymbol{N}^T\boldsymbol{Q} - \overline{\boldsymbol{R}}\boldsymbol{X} \leqq \boldsymbol{0} & (2\cdot f) \end{cases}$$

$$\boldsymbol{X}^L \leqq \boldsymbol{X} \leqq \boldsymbol{X}^U \quad (2\cdot g)$$

$$(1-\mu)\boldsymbol{X}^* \leqq \boldsymbol{X} \leqq (1+\mu)\boldsymbol{X}^* \quad (2\cdot h)$$

where Eqs. (2·b), (2·c) represent the approximation by first-order Taylor series in the design variable X_i of Eqs. (1·b), (1·c), respectively; Eqs. (2·e), (2·f) define the conditions of equilibrium and yield at the ultimate load level, respectively. Eq. (2·h) means the move limit constraint for the SLP technique. $\boldsymbol{C}$=the compatibility matrix in the whole structure; $\boldsymbol{Q}$=the stress resultant vector at ultimate load level; $\boldsymbol{F}$=the external applied load vector; $\boldsymbol{N}$=the exterior unit normal matrix at the yield line; $\overline{\boldsymbol{R}}$=the plastic capacity matrix which is independent of the design variable $\boldsymbol{X}$; μ=the move limit parameter; $\nabla\boldsymbol{\sigma}$, $\nabla\boldsymbol{u}$=the sensitivity coefficient matrix of stress $\boldsymbol{\sigma}$ and deformation $\boldsymbol{u}$ by the design variable X_i ; * =the value at the previous design stage; $\boldsymbol{\beta}_S^+$, $\boldsymbol{\beta}_S^-$=dual variable vectors corresponding to the upper and lower bounds of stress constraint of Eq.(2·b); $\boldsymbol{\beta}_D^+$, $\boldsymbol{\beta}_D^-$=dual variable vectors corresponding to the upper and lower bounds of deformation constraint of Eq. (2·c); $\boldsymbol{\beta}_J^+$, $\boldsymbol{\beta}_J^-$=dual variable vectors corresponding to the upper and lower bounds of Eq.(2·g); β_U=dual variable corresponding to the ultimate load constraint of Eq.(2·d).

(3) Formulation of SLP Dual Problem

In order to enhance the computational efficiency, the primal problem of Eq.(2) can be transformed into a dual problem by applying duality theorem of LP as follows:

Given : ρ, $\boldsymbol{L}$, σ_a^U, σ_a^L, u_a^U, u_a^L, X^U, X^L, μ, $\boldsymbol{\sigma}^*$, $\nabla\boldsymbol{\sigma}^*$, $\boldsymbol{u}^*$, $\nabla\boldsymbol{u}^*$, $\boldsymbol{X}^*$, $\boldsymbol{C}$, $\boldsymbol{F}$, $\boldsymbol{N}$, $\overline{\boldsymbol{R}}$

Find : primal variable $\boldsymbol{\beta}_S^\pm$, $\boldsymbol{\beta}_D^\pm$, $\boldsymbol{\beta}_J^\pm$, β_U, $\dot{\boldsymbol{u}}$, $\dot{\boldsymbol{\lambda}}$

dual variable $\boldsymbol{X}$, $\boldsymbol{Q}$, α

Object:

$$V=(-\boldsymbol{\sigma}_a^U-\nabla\boldsymbol{\sigma}^*\boldsymbol{X}^*+\boldsymbol{\sigma}^*)^T\boldsymbol{\beta}_S^+ +(\boldsymbol{\sigma}_a^L+\nabla\boldsymbol{\sigma}^*\boldsymbol{X}^*-\boldsymbol{\sigma}^*)^T\boldsymbol{\beta}_S^- +(-\boldsymbol{u}_a^U-\nabla\boldsymbol{u}^*\boldsymbol{X}^*+\boldsymbol{u}^*)^T\boldsymbol{\beta}_D^+ +(\boldsymbol{u}_a^L+\nabla\boldsymbol{u}^*\boldsymbol{X}^*-\boldsymbol{u}^*)^T\boldsymbol{\beta}_D^- +(-1-\mu)(\boldsymbol{X}^*)^T\boldsymbol{\beta}_J^+ +(1-\mu)(\boldsymbol{X}^*)^T\boldsymbol{\beta}_J^- +\alpha_0\beta^U \rightarrow \max. \quad (3\cdot a)$$

Constraint :

$$(-\nabla\boldsymbol{\sigma}^*)^T\boldsymbol{\beta}_S^+ +(\nabla\boldsymbol{\sigma}^*)^T\boldsymbol{\beta}_S^- +(-\nabla\boldsymbol{u}^*)^T\boldsymbol{\beta}_D^+ +(\nabla\boldsymbol{u}^*)^T\boldsymbol{\beta}_D^- +\overline{\boldsymbol{R}}\dot{\boldsymbol{\lambda}}-\boldsymbol{\beta}_J^+ +\boldsymbol{\beta}_J^- \leqq \rho\boldsymbol{L} \quad (3\cdot b)$$

$$\boldsymbol{C}\dot{\boldsymbol{u}}-\boldsymbol{N}\dot{\boldsymbol{\lambda}}=\boldsymbol{0} \quad (3\cdot c)$$

$$\beta_U=\boldsymbol{F}^T\dot{\boldsymbol{u}} \quad (3\cdot d)$$

where $\dot{\boldsymbol{u}}$=the nodal displacement rate vector which is dual variable vector corresponding to the equilibrium condition of Eq.(2·e); $\dot{\boldsymbol{\lambda}}$=the plastic multiplier rate vector which is dual variable vector corresponding to the yield condition of Eq.(2·f).

Consequently, the primal or dual design problem can be easily solved by using a SLP technique in which the results of optimal plastic design are adopted as the initial values. The steps of the design procedure are consicely listed in the flow chart as shown in Fig.1.

It is noted that the cross-sectional area is taken as the design variables for the truss structures and the plastic section moduls for the flexural structures, in which the current moment of inertia I_i and cross-sextional area A_i may be estimated for a given design variable X_i. For example, for a wide-flange section[4],

Column : $I_i=(X_i/0.78)^{4/3}$, $A_i=0.80\,I_i^{1/2}$ ············· (4·a)

Girder : $I_i=(X_i/0.58)^{4/3}$, $A_i=0.58\,I_i^{1/2}$ ············· (4·b)

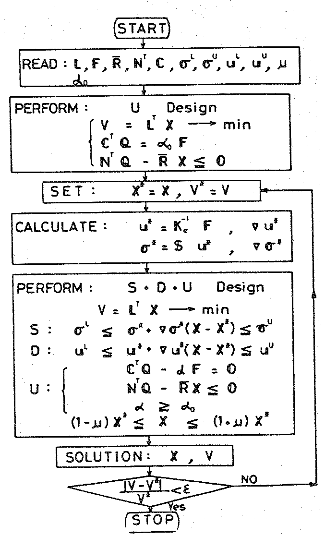

Fig. 1 Design flow chart.

3. EFFECTIVE RANGE OF ULTIMATE LOAD CONSTRAINT

The proposed design method has the ineffective region depending upon the value of the ultimate design load factor α_0. Therefore, it is necessary to determine the effective range of the ultimate load factor ($\alpha_0^L \leqq \alpha_0 \leqq \alpha_0^U$), in which α_0^U, α_0^L are the upper and lower bounds of the effective load factor. Herein, the value of α_0^U means the boundary value between the $S+U$ design (both S and U constraints are active) or the $D+U$ design (D and U constraints are active) and the U design (U constraint is only active, i.e., optimal plastic design). This value can be determined by combining the S or D constraint with the results of the U design as follows: Initially, noting that the U design is proportional to the value of α_0, the design variable $\boldsymbol{X}$ at the arbitrary value α_0 is given by

$$\boldsymbol{X}=\alpha_0\cdot\overline{\boldsymbol{X}} \qquad (5)$$

where $\overline{\boldsymbol{X}}$ is the known design variable found by the U design at the value $\alpha_0=1$.

Therefore, the elastic stress $\boldsymbol{\sigma}$ and the deformation $\boldsymbol{u}$ at the arbitrary value α_0 are found by using Eq.(5) as follows:

$$\boldsymbol{\sigma}=\frac{1}{\alpha_0}\overline{\boldsymbol{\sigma}},\quad \boldsymbol{u}=\frac{1}{\alpha_0}\overline{\boldsymbol{u}} \qquad (6)$$

where $\overline{\boldsymbol{\sigma}}$ and $\overline{\boldsymbol{u}}$ are the stress and the deformation due to the design variable $\overline{\boldsymbol{X}}$, respectively. Substituting Eq.(6) into Eq.(1·b) or (1·c), the upper bound of the effective load factor α_0^U is found by the following equation.

$$\alpha_0^U=\max_{i,\,j}\ (\alpha_{0i},\ \alpha_{0j}) \qquad (7\cdot a)$$

in which

$$\alpha_{0i}=\begin{cases}\overline{\sigma}_i/\sigma_{ai}^U & (\overline{\sigma}_i\geqq 0)\\ \overline{\sigma}_i/\sigma_{ai}^L & (\overline{\sigma}_i<0)\end{cases} \qquad (7\cdot b)$$

$$\alpha_{0j}=\begin{cases}\overline{u}_j/u_{aj}^U & (\overline{u}_j\geqq 0)\\ \overline{u}_j/u_{aj}^L & (\overline{u}_j<0)\end{cases} \qquad (7\cdot c)$$

where $\overline{\sigma}_i$ and $\overline{u}_j$ are the elastic stress at member or section i and deformation at node j for the design variable $\overline{\boldsymbol{X}}$ which is found by performing the optimal plastic design at load level $\alpha_0=1$.

On the other hand, the lower bound α_0^L means the boundary value between the elastic design (S, D, $S+D$ design) and the design satisfying both elastic and plastic constraints ($S+U$, $D+U$, $S+D+U$ design). Therefore, the value of α_0^L is found when the design ultimate load factor α_0 coincides with the analytical collapse load factor α at the optimal elastic design field.

Initially, the plastic capacity is determined by the optimal elastic design as follows:

$$\boldsymbol{R}_e=\boldsymbol{X}_e\sigma_y \qquad (8)$$

where $\boldsymbol{X}_e$=the known design variable found by the optimal elastic design; $\boldsymbol{R}_e$=the plastic capacity vector, e.g., the plastic moment vector; σ_y=the yield stress.

Then, the value of α_0^L can be found by performing the following LP collapse load analysis[5] based on the kinematic theorem.

$$\text{Object}:\ \alpha_0^L=\boldsymbol{R}_e^T\dot{\boldsymbol{\lambda}}\rightarrow\min. \qquad (9\cdot a)$$

$$\text{Constraint}:\ \boldsymbol{C}\dot{\boldsymbol{u}}-\boldsymbol{N}\dot{\boldsymbol{\lambda}}=\boldsymbol{O} \qquad (9\cdot b)$$

$$\boldsymbol{F}^T\dot{\boldsymbol{u}}=1 \qquad (9\cdot c)$$

$$\dot{\boldsymbol{\lambda}}\geqq\boldsymbol{O} \qquad (9\cdot d)$$

where Eq.(9·a) defines the minimum internal work due to the plastic capacity; Eq.(9·b) expresses the mechanism condition; Eq.(9·c) means that the external work due to the loads $\boldsymbol{F}$ is unity and Eq.(9·d) specifies that plastic flow must always involve dissipation of mechanical energy; $\dot{\boldsymbol{\lambda}}$=the vector of the plastic multiplier rates; $\dot{\boldsymbol{u}}$=the vector of the nodal displacement vector.

4. EXAMPLES

(1) Example 1

The 3-bar truss shown in Fig. 2 is to be designed to resist service and ultimate loads.

a) Comparison with the Envelope Design

The proposed method is initially compared with the envelope design as shown in Table 1. The envelope design is defined that each design variable adopts the maximum value among the S design (stress constraint is only active) or D design (deformation constraint is only active) and the U design.

For instance, in the case of Table 1(a), the optimum values of X_1 obtained by the S design and the U

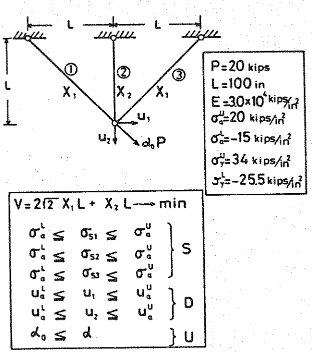

Fig. 2 3-bar truss.

Table 1 Comparison with envelope design.

(a) $\alpha_0=2.0$, $u_a=\infty$

Design \ Variable	X_1 (in²)	X_2 (in²)	V (in³)
S	0.789	0.409	263.9
U	0.672	0.713	261.4
Envelope	0.789	0.713	294.4
S+U	0.721	0.644	268.3 (9.7%)

(b) $\alpha_0=2.0$, $u_a=0.075$ in

Design \ Variable	X_1 (in²)	X_2 (in²)	V (in³)
D	0.889	0.0	251.4
U	0.672	0.713	261.4
Envelope	0.889	0.713	322.7
D+U	0.889	0.411	292.5 (10.3%)

design are 0.789 in² and 0.672 in², respectively, and, therefore, the envelope design adopts the larger value, i.e., 0.789 in². Similarily, as for the value of X_2, the optimum of the U design, 0.713 in², is larger than the one of the S design, 0.409 in², and, as such, the envelope design takes the larger value, i.e., 0.713 in². It is found from Table 1 that the volumes of the $S+U$ design and the $D+U$ design are about 10% less than those of the envelope design, respectively. The proposed approach satisfying simultaneously S and U constraints or D and U constraints are more economical than the envelope design found by performing the optimal elastic design (S or D design) and the optimal plastic design (U design), separately.

b) Effective Range of Ultimate Load Constraint

Fig. 3 shows the relationships between the total volume V or design variable X_i and the ultimate load factor α_0 at the allowable deformation $u_a=\infty$. It is found from Fig. 3 that the results of this approach considering both S and U constraints agree with those of the S design and the U design in the region of $\alpha_0<1.83$ and $\alpha_0>2.08$. In other words, this design method is very effective in the region of $\alpha_0=1.83\sim2.08$ where S and U constraints are simultaneously active. The value of $\alpha_0^L=1.83$ is determined by performing the collapse load analysis of Eq.(9), while the value of $\alpha_0^U=2.08$ is found by using Eqs. (7·a) and (7·b).

It should by noted from Fig. 3 that the optimal values X_1, X_2 found by the S design are quite different from those obtained by the U design and, as such, these values are turned upside down, in the region of the $S+U$ design as the value of α_0 increases. It is also noted from Fig. 3 that the total volume V by this approach is smaller than that by the envelope design.

In order to examine the validity of the effective range in Fig. 3, the relation of dual variables β_S, β_V vs. ultimate load factor α_0 is shown in Fig. 4. It is confirmed from Fig. 4 that the maximum value of α_0 at $\beta_V=0$ and $\beta_S>0$ agrees with $\alpha_0^L=1.83$ in Fig. 3, and the minimum value of $\alpha_0$at $\beta_U>0$ and $\beta_S=0$ coincides with α_0^U

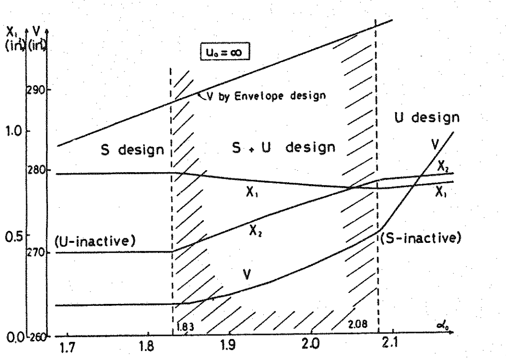

Fig. 3 Total volume V or design variable X_i vs. load factor α_0 (allowable deformation $u_a=\infty$).

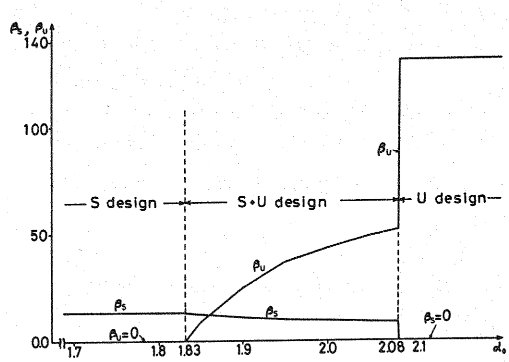

Fig. 4 Dual vairables β_S, β_U vs. load factor α_0.

=2.08 in Fig. 3. Hence, Eqs.(9) and (7) are very useful to determine the effective range of the ultimate load factor.

Similarly, Fig. 5 shows the relation of total vlulume V vs. ultimate load factor α_0 at the allowable deformation u_a=0.075 inch. It should be noted that the effective range of the $D+U$ design is α_0=1.73~2.64 and the design variable X_2 changes places with X_1 in its region.

Then, Fig. 6 represents the relationships between V and u_a as the parameter α_0. It is noted that the total volume V increases as the value of α_0 becomes larger and the value of u_a becomes smaller.

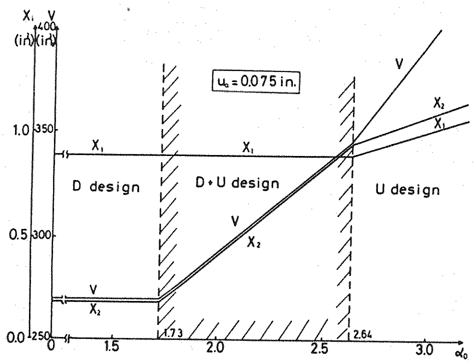

Fig. 5 Total volume V or design variable X_i vs. load factor α_0 (allowable deformation u_a=0.075 inch).

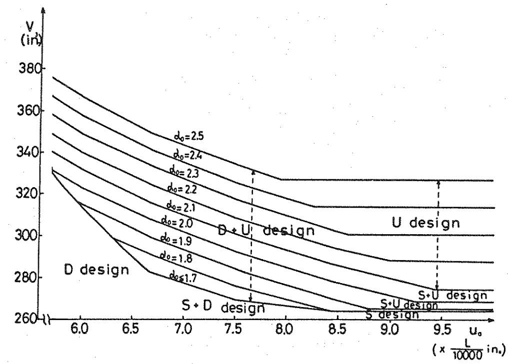

Fig. 6 Total volume V vs. allowable deformation u_a as the parameter α_0.

Finally, Fig. 7 points out the effective range of the $S+U$ and the $D+U$ designs depending upon the values of α_0 and u_a. It is found from Figs.6 and 7 that the region of the $S+U$ and the $D+U$ designs are very wide and the true optimal solutions in its region can not be obtained by performing the usual optimal elastic or plastic design method alone.

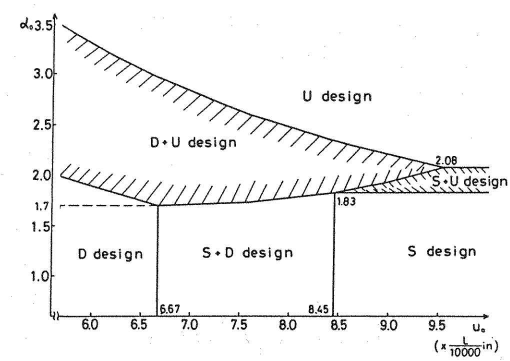

Fig. 7 Effective region of ultimate load factor constraint.

(2) Example 2

The 1-story, 1-span frame as shown in Fig. 8 is to be designed to illustrate a simple flexural structure.

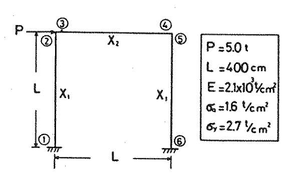

Fig. 8 1-story 1-span frame.

Fig. 9 represents the relationships between the total volume V or design variable X_i and the load factor α_0 at the constant allowable deformation u_a=2.5 cm. It is found from Fig. 9 that the four types of design region are classified within the range of α_0=1.7~1.9 at u_a=2.5 cm whose values are special case in Fig. 10.

Fig. 10 shows the effective regions of the $D+U$ and the $S+U$ design where exist in the narrow band in this structure.

(3) Example 3

The 2-story 4-span frame as shown in Fig. 11 is designed in order to illustrate the application of the approach to the complex frame. Herein, the D constraint is taken as the horizontal displacement at the second story.

Figs. 12 and 13 show the similar graphs as Figs. 9 and 10 in the case of the 1-story 1-span frame,

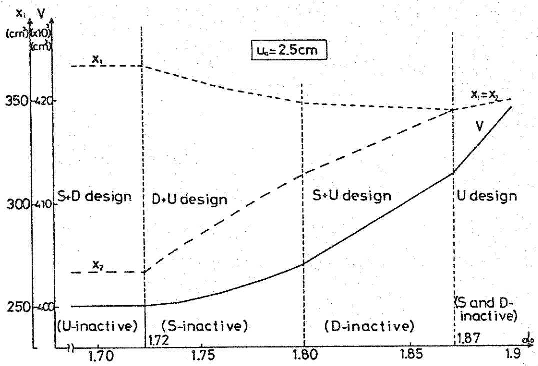

Fig. 9 Total volume V or design variable X_i vs. load factor α_0 (allowable deformation u_a=2.5cm).

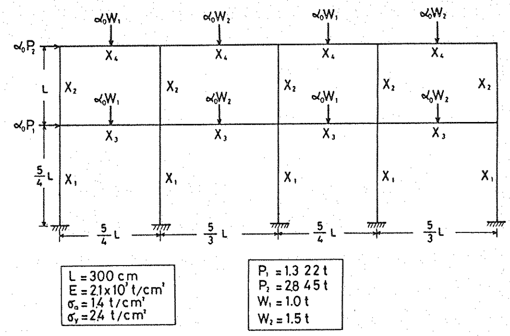

Fig. 11 2-story 4-span frame.

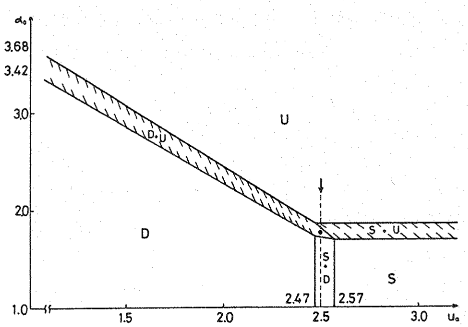

Fig. 10 Effective region of ultimate load factor constraint.

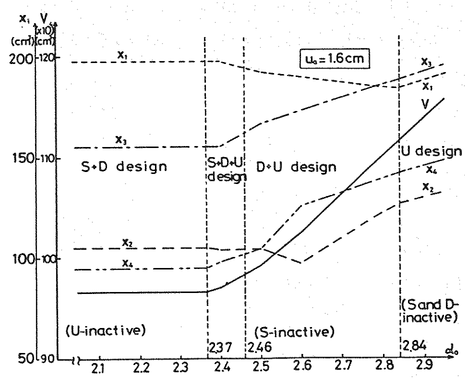

Fig. 12 Total volume V or design variable X_i vs. load factor α_0 (allowable deformation u_a=1.6cm).

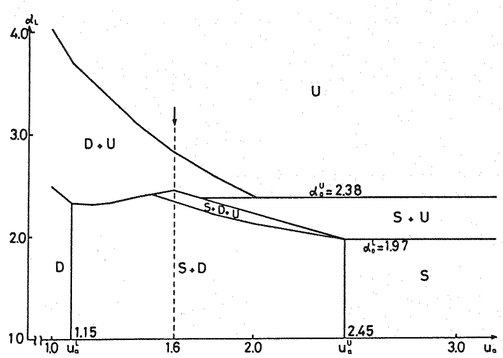

Fig. 13 Effective region of ultimate load factor constraint.

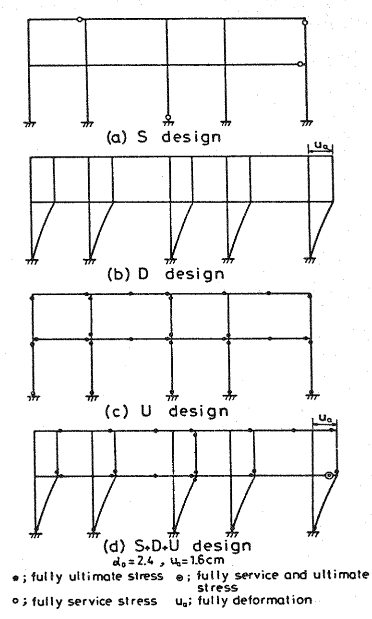

Fig. 14 Location of active constraints (α_0=2.4 and u_a=1.6cm).

respectively. It is found from Figs. 12 and 13 that four types of design region are identified by the value of α_0= 2.1~2.9 and u_a=1.6 cm. The existance of the $S+U+D$ design (S, U, D constraints are all active) is quite different from the case of simple frame.

Fig. 14 shows the location of the active constraints in each design method. It is recognized from Fig. 14 that the $S+U+D$ design is welll combined with the S, D and U

designs by using the proposed design method.

5. CONCLUSIONS

The following conclusions are drawn from this study.

(1) The proposed design method makes the best use of the merit of plastic design based on the LP approach and, as such, the method can be solved as the conventional SLP primal or dual problem.

(2) It is found that the region satisfying simultaneously elastic and plastic design constraints exists in the optimal design space. Only the proposed design method can obtain the correct solutions and it is very valid in its region.

(3) It is confirmed that the upper and lower bounds of effective load factor proposed by Eqs. (7) and (9) agree with the values obtained by using dual variables.

(4) It is clarified that the solutions obtained in this method is more economical and rational than those of the envelope design method.

(5) The method adopts the results of optimal plastic design as the initial values, and, therefore, it is not necessary to be worried about the initial values for SLP approach to the large-scale structure.

(6) With but minor revision, the method may be applied taking plastic deformation constraint at the ultimate load level into account.

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REFERENCES

1) e.g., Kirsch, U.: Optimum Structural Design, McGraw-Hill, pp.277～289, 1981.
2) e.g., Ishikawa, N. et al.: A Study on the Optimal Plastic Design of Space Frames, Proc. of JSCE, No.279, pp. 45～59, Nov., 1978.
3) Grierson, D.E. and Schmit, L.A.: Synthesis under Service and Ultimate Performance Constraints, Computers and Structures, Vol.15, No.14, pp.405～417, 1982.
4) Nakamura, Y. and Little, N.: Plastic Design Method for Multi-Story Plane-Frames with Deflection Constraints, MIT Report 68～12, March, 1968.
5) Grierson, D.E.: Collapse Load Analysis, Engineering Plasticity by Mathemaical Programming, edited by Cohn and Maier, Pergamon Press, pp.73～106, 1979.

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