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ULTIMATE STRENGTH OF TRUSS GIRDER DUE TO FAILURE OF CHORD MEMBER

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The ultimate strength analysis of the Warren type of rigidly jointed truss girder structure, in which the influence of the finite displacements, yielding of material, residual strsses and initial crookedness is taken into consideration, is carried out in this paper. From the analysed results, the generating mechannism of the secondary moments in the chord members, the effect of the secondary moments on the ultimate strength of truss girders, the ultimate strength due to the failure of the chord members which behave as an end-restrained column in the ultimate state are discussed herein. It is shown that trusses have much more strength than that estimated by the end-hinged column strength.

1. INTRODUCTION

The strength of a simple planar truss girder structure is regarded as being governed by the strength of a member if the members are connected by hinges each other. In this case, the strength of the member may be rated by the fundamental end-hinged column strength curves. On the other hand, in the case where the truss joints are connected rigidly, the so-called secondary moments are induced in the members. Thus, the strength of a member should be affected by these secondary moments as well as the end restraint of the adjoining members.

The magnitude of the secondary moment is not constant but varies with the increment of the axial forces as noted by Chu[1] *et al.* The characteristics in finite deformations of a rigidly jointed elastic truss have been discussed in Ref. 2) by the author. Massonnet *et al.* 3) presented a desigh formula based on the equivalent length concept from the ultimate strength analysis of a truss with a small number of panels.

In this paper, the ultimate strength of the rigidly jointed trusses as girder structures is studied with the finite displacements and yielding taken into account. Especially, emphasis is put on the studies of the mechanism of the development of the secondary moments and their effect on the strength of the members. Namely, it is proved here that the deflections of all the chord members due to the secondary moments which are produced at the early loading stage are always caused in the downward direction by the girder action. This behavior of their deflections brings out a strong end-restraining action in the adjoining chord members and gives them additional strength.

2. ANALYSED MODEL

The fundamental configuration of truss girders adopted here for the ultimate strength analysis has similar

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Table 1 Dimensions of chord and diagonal members and their combinations.

Combination I		W_f (cm)	t_f (cm)	W_w (cm)	t_w (cm)	A (cm^2)	I (cm^4)	l/r	λ
Chord member		51.98	1.5	47.25	1.5	297.7	119 022	50.0	0.621
Diagonal member	C	45.68	1.5	23.30	1.5	206.9	19 444	89.9	1.12
	T	42.68	1.5	38.41	3.0	294.5	28 346	88.9	1.10
	C	45.68	1.5	18.73	1.5	193.2	11 814	111.5	1.39
	T	42.68	1.5	31.58	3.0	253.5	15 759	110.6	1.37
	C	45.68	1.5	15.53	1.5	183.6	7 680	134.8	1.67
	T	42.68	1.5	26.89	3.0	225.4	9 743	132.7	1.65

Combination II		W_f	t_f	W_w	t_w	A	I	l/r	λ
Chord member		40.00	1.4	36.00	1.4	212.8	55 052	65.2	0.810
Diagonal member	C	34.10	1.4	24.20	1.4	163.2	15 715	88.8	1.10
	T	31.30	1.4	37.40	2.8	253.3	24 420	88.8	1.10
	C	34.10	1.4	19.20	1.4	149.2	9 214	110.9	1.38
	T	31.30	1.4	30.50	2.8	214.6	13 248	110.9	1.38
	C	34.10	1.4	16.00	1.4	140.3	6 044	132.8	1.65
	T	31.30	1.4	25.80	2.8	188.3	8 021	133.6	1.66

Combination III		W_f	t_f	W_w	t_w	A	I	l/r	λ
Chord member		32.18	1.3	29.25	1.3	159.7	24 944	80.0	0.994
Diagonal member	C	26.94	1.3	24.52	1.3	133.8	12 636	89.7	1.11
	T	24.34	1.3	36.27	2.6	220.2	20 680	90.0	1.12
	C	26.94	1.3	19.40	1.3	120.5	7 319	111.9	1.39
	T	24.34	1.3	29.70	2.6	186.1	11 357	111.6	1.39
	C	26.94	1.3	16.17	1.3	112.1	4 788	133.4	1.66
	T	24.37	1.3	25.08	2.6	162.1	6 840	134.2	1.67

C : compression member, T : tension member

proportions and structural parameters as those used in Ref. 2). The dimensions of each members are varied to elucidate the influence of the rigidity and strength of the members as shown in Table 1, i. e. the slenderness parameter of the chord member is taken as 0.62, 0.81 or 0.99 and that of the diagonal members is about 1.10, 1.38 or 1.66. By combining these members, trusses with 9 different member rigidities are checked here. But, the chord members and compression and tension diagonal members of each truss are uniform over the whold span length. The configuration of the Warren trusses is determined referring to the conventional designs. Two types of truss with either six or five panels illustrated in Fig. 1 are analysed to see the restraint effect of the adjoining chord members. Even in rigidly jointed trusses, each chord member is usually designed according to the hinged-end column strength curve. One reason of this practice is based on the idea that each chord member of a truss is designed such that it is stressed to the critical point of the member. Consequently, each member is regarded as having no redundant capacity to prevent buckling of the adjoining members. In the six-panel truss, the center upper chord member is restained by the adjoining members not stressed to the

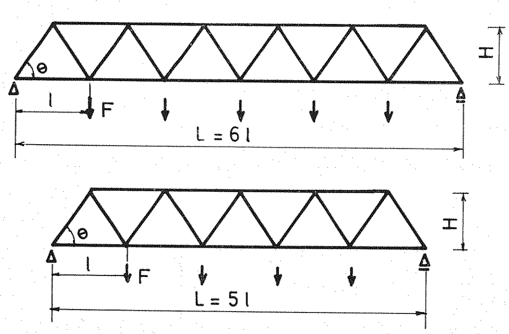

Fig. 1 Configurations of truss girder.

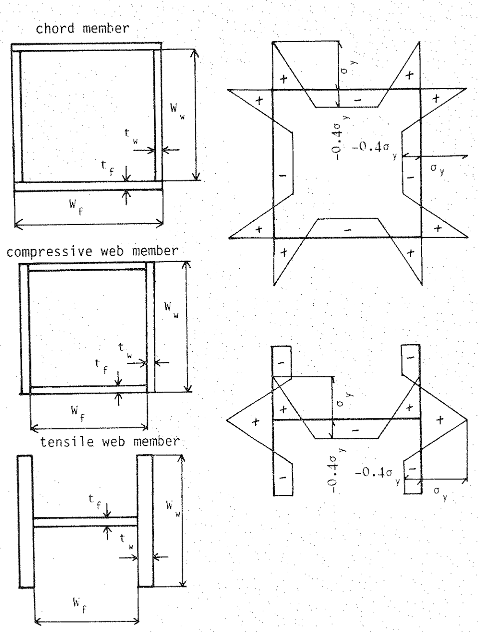

Fig. 2 Cross sectional shapes of members and distribution of residual stresses.

bucking load in analysis. On the other hand, in the five-panel truss, the two central upper chord members are stressed to the equal critical state and are expected to have no capacity to restrain the adjoining chord member each other.

In the analysis, the influenes of the finite displacemnts, residual stresses and yielding are taken into account. The calculation method is almost the same as that abopted in Ref. 4). The yielding point of steel is 238 MN/m^2. The cross sectional form of a chord member and diagonal members, and the residual stress distribution pattern are shown in Fig. 2. Each member is divied into four elements. In each element, numerical integration is carried out by the values at four equally spaced cross sections in it. Each flange and wed plate are divided into 10 segments to check the elastic and plastic state of material.

3. CALCULATED RESUITS

(1) Behavior of a Chord Member

As an example of the behavior of a chord member, the relationship between the axial force and elastic deflection slope and the relationship between the axial force and end moment are shown in Fig. 3. The behavior changes according to the type of truss and relative rigidities of chord members and diagonal members. But, the general tendency is almost same throughout all the analysed cases. Namely, with the increase of the axial force, the deflection slope increases monotononically or with a small amount of irregularity depending on the structural parameters.

But, finally the axial force reaching to the ultimate state, the deflection slope begins to increase rapidly with little or almost no increase in the axial force. In Fig. 3, the axial force is non-dimensionally expressed by the full plastic axial force. While, the end moments (secondary moments) are caused by the deflection of its whole structure from the early stage of loading and increase successively with the increment of axial force. But, the sign of the moment changes from minus to plus at about an half point of the ultimate load. This implies that the chord member is forced to deform by the end rotation due to the deflection of the whole structure at the early stage of loading. But, after the axial force reaches to a certain value of the critical axial force, the member begin to be forced to be deflected by this axial force, and other adjoining members begin to constrain this movement of the member.

This tendency is also similar to the five penels truss as shown in Fig. 3 (b) for an example. This means that the two adjoining chord members deflect to the same direction at the early loading by the reason discussed in the following section and have tendency to buckle to the same direction even in the ultimate state. In this case, the adjoining chord members restrain the buckling deformatons each other. If the two members buckle the opposite directions, the restraining effect may be not expected even in the yielded state. But, in this case, one of the two chord members should jump over the energy barrier and induce the stress reverse. This stress reverse brings out the increment of the flexual rigidity of the member, especially in the yielded state and exerts the restraining effect on the other bucking member.

(2) Initiation of the Secondary Moments

The fundamental mechanism of the initiation of the secondary moments of chord members is explained by Fig. 4. For hinge-jointed trusses, every chord member can keep the initial straight form $\overline{AB}$ and $\overline{BC}$ between the hinges after defiection. But, for rigidly jointed trusses, the chord members must bend by the angle θ as shown in Fig. 4 to get continuity at the joints. According to this behavior, the secondary moments can be appoximately estimated by

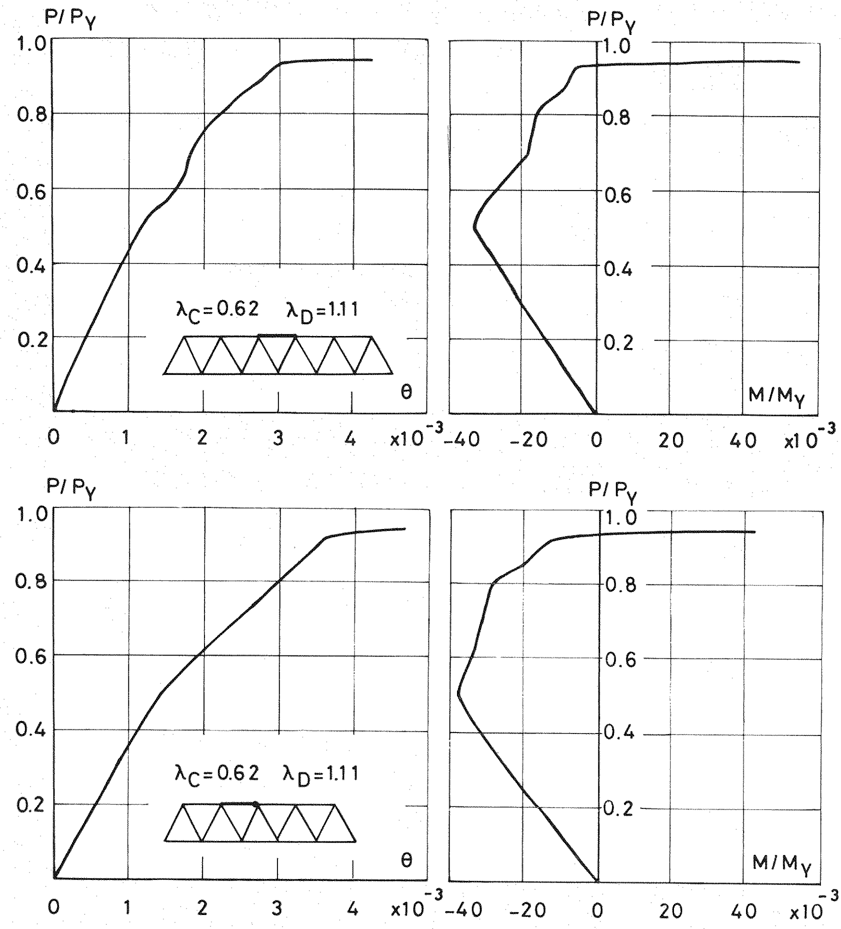

Fig. 3 Relationship between chord member force and end deflectional slope, and the force and secondary moments.

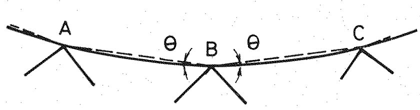

Fig. 4 End rotation of rigidly jointed truss chord members.

the following way. With the assumption of non-extensibility of the diagonal members and application of uniform bending moments, the curvature ϕ of the whole truss girder is given by

$$\phi=\frac{\varepsilon_u+\varepsilon_r}{H}$$

where ε_u and ε_r is the compressive and tensile strain of the upper and lower chords and H is the height of the truss. If equal ε_u and equal cross sectional area are assumed here and the chord member force is denoted by P, the curvature can be rewritten by

$$\phi=\frac{2P}{AEH} \qquad (1)$$

where A is the cross sectional area of the chord members and E is Young's modulus. The bending moment produced in the chord members by this curvature is given by

$$M=EI\ \phi=2IP/AH=r^2\ P/(H/2) \qquad (2)$$

where, I is the moment of inertia and r is the radius of gyration of a chord member.

Expressing in non-dimensional form,

$$M/M_y=(P/P_y)\cdot C/(H/2) \qquad (3)$$

Table 2 Comparison between the secondary moments calculated by the detailed and the approximate analysis.

[t-m]

Type			Truss of six panels			Truss of five panels					
λ_c	λ_o		1.1	1.4	1.7	1.1		1.4		1.7	
	$\bar{M}$		$M_{c,s}$	$M_{c,s}$	$M_{c,s}$	M_c	M_s	M_c	M_s	M_c	M_s
0.62	M	1.067	1.047	1.077	1.101	1.372	1.149	1.367	1.290	1.368	1.401
	M/M		0.98	1.01	1.03	1.29	1.08	1.28	1.21	1.28	1.31
0.81	M	0.449	0.416	0.429	0.439	0.581	0.395	0.568	0.464	0.563	0.516
	$\bar{M}/M$		0.93	0.96	0.98	1.29	0.88	1.27	1.03	1.25	1.15
0.99	M	0.224	0.195	0.202	0.207	0.289	0.163	0.282	0.195	0.276	0.223
	$\bar{M}/M$		0.87	0.90	0.93	1.29	0.73	1.26	0.87	1.23	1.00

$\bar{M}$: secondary moment by Eq. (3), M_c : secondary moment at the center side, M_s : sec. m. at the outerside

where C is the distance from the neutral axis to an extreme fiber. The secondary moments calculated by Eq. (3) are compared with the values obtained by the analysis for $P/P_y=0.1$ in Table 2. It will be seen from the comparison that this approximation yields a fairly good agreement with the detailedly calculated results, especially for the trusses with chord members of small slenderness ratio and diagonal members of large slenderness ratio. Generally speaking, the initial secondary moments become more remarkable with the rigidity of chord members increasing and that of digonal members decreasing. But, this initial secondary moments have no effect on the ultlmate strength of the chord members.

(3) Strength of End Restrained Columns

The behavior of the chord is characterised by the end rotation at the early loading stage and the end restraint to the buckling deformations in the final state of loading. The strength of the members is estimated by that of the end

Fig. 5 Model of restrained chord member.

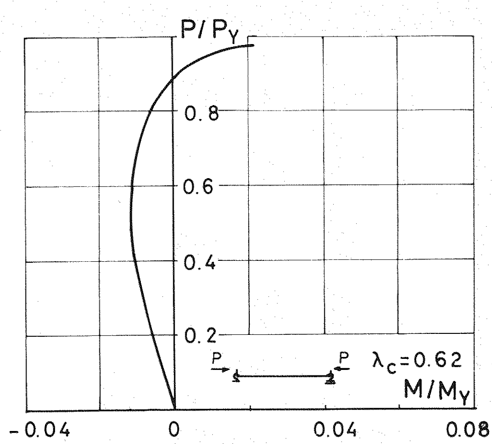

Fig. 6 Calculated result by the simplified model.

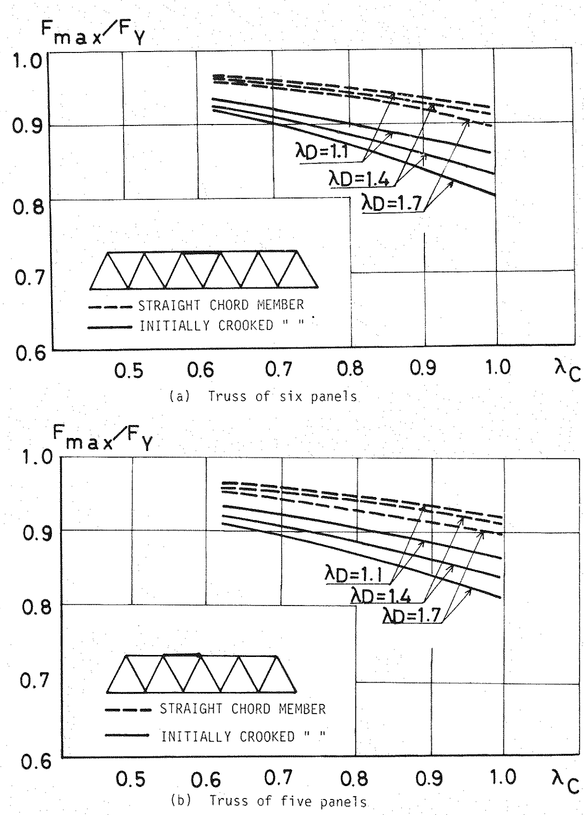

Fig. 7 Load carrying capacity of the trusses.

restrained columns.

From this point of view, the chord members of a truss can be modeled by a column restrained by rotative springs at both ends subjected to an eccentric axial force. The eccentricity of the force is estimated by $e=r^2/(H/2)$ referred to Eq. (3) and the stiffness coefficient of two rotative springs is approximated by $K=3EI/1$ of the bending stiffness of the adjoining chord members. An example of the axial force-end moment curve is shown in Fig. 6. The result indicates that the general behavior of the column is simulated well and the ultimate strength is in good agreement with the value of detailed calculation. But, as expected from the prior assumption, the secondary moment by this model shows lower value than that of the detailed calculation.

(4) Ultimate Strength of Trusses

In Fig. 7, the ultimate load-slenderness parameter relationships are shown. Fig. 7 (a) shows the results for the trusses of 6 panels and Fig. 7 (b) is that of 5 panels. The loads of the ordinate are placed on the every lower nodal points and non-dimensionally expressed by the loads which produce the full plastic axial force in the critical chord members according to the lst order elastic analysis. The slenderness parameters of the abscissa is those of the critical members. The dotted lines show the results for the trusses with straight members and the solid lines are for thos of initially crooked members. The initial crookedness is assumed by a sinusoidal curve with the maximum magnitude at the middle being one-thousandth of the length of the member. This initial curvature directs downwards in the critical chord member and upwards in the adjoining members. In Fig. 8, the column strength curves of the chord members are illustrated, in which P_y is the full plastic axial force of the critical chord members and P is calculated by the non-linear analysis of the whole structure. Compare Fig. 7 with Fig. 8 to find that the difference is not so significant that the ultimate strength of the truss can be estimated fairly well by the strength of a chord of which axial force is calculated by the lst order elastic analysis. The influence of the initial crcokedness is also remarkable as wall as the hinged columns, and more sensitive to the slenderness of the diagonal members than those of the non-crooked members. Judging from Fig. 7 and Fig. 8, the restraint effect of the adjoining member is found to be remarkable. In every case, the strength of the chord members is higher than those of the corresponding hinged column due to tho end restraint especially for the slender chord members.

If this increment of th column strength is expressed by the concept of the equivalent bucking length, the coefficient of the equivalent buckling length become 0.81 for $\lambda_c=0.62$ and 0.75 for $\lambda_c=0.99$ judging from Fig. 8. This value may be taken at least as 0.85 for conventioned truss girder structures considering the slenderness limit for main compression members and practical use of steel materials. Note that the application of this value is limited to truss girder structure and not for trestle structures etc.

The ultimate strength of the Warren type of rigid jointed truss girer structure, in which the influence

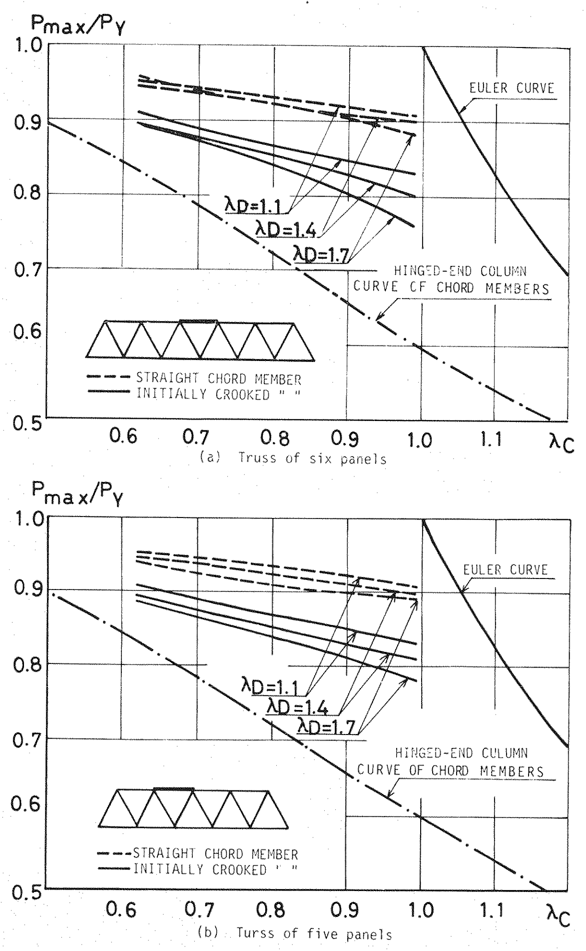

Fig. 8 The maximum strength of the chord chord members.

of the finite displacements, yielding of material, residual stresses and initial crookedness is taken into account is carried out in this paper. From the analysed results, the generating mechanism of the secondary moments in the chord members, the effect of the secondary moments to the ultimate strength of truss girders, the ultimate strength due to the failure of the chord members which behave as an end-restrained column in the ultimate state are discussed herein. It is shown that trusses have much more strength that that estlmated by the hinged hinged column strength.

4. CONCLUSIONS

By the ultimate strength analysis of the Warren type of truss with conventional configurations, the generating mechanism of the secondary moments of trusses, the failure process of the compressive chord members and the effect of the stiffness and the intial crookedness of the members are clarified here. The following conclusions may be drawn from the results ;

1) The initial secondary moments of the chord members of trusses are produced by the flexual deformations of the whole structure.

2) The compressive chord members are subjected to the forced end rotations at the early loading stage, but restrained by the adjoining members in the buckling state.

3) When the strength of trusses is estimated by that of a chord member, the axial force of the chord member can be calculated by the lst order elastic analysis.

4) The strength of compressive chord members of trusses is higher than that of the corresponding hinged column.

5) The influence of the initial crookedness is perceivable also in the chord members of trusses and becomes a little sensitive to the stiffness of the diagonal members.

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