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OVERALL LATERAL BUCKLING OF THROUGH PLATE GIRDER BRIDGES

By Fumihito ITOH and Kuniei NOGAMI***

This paper developes the strict theory on the lateral buckling of a through plate girder bridge, and simple approximate formulae for critical loads have been proposed. The validity of this theory has been examined by comparing with some experimental results, and the elastic lateral buckling loads and the load carrying capacities for the illustrative practical bridge models have been calculated.

The results have shown that the approximate formulae for critical values of symmetric and asymmetric elastic buckling under uniformly distributed load have to be useful, and that the load carrying capacity for long spans would depend on asymmetric overall lateral buckling. The symmetric buckling will be not so significant.

1. INTRODUCTION

Dealing with lateral buckling in a through plate girder bridge, we shall encounter three problems. The first problem is old one against lateral buckling as a main girder iteslf, which occurs between adjacent points connected with floor beam, and may be considered as local one in this context.

The second problem is simultaneous lateral buckling of both main girders due to insufficient bending rigidity and strength of floor beams or knee bracings and to the absence of upper lateral bracings, as in a pony truss. Japanese specifications for railway bridge design[1] include regulations against such buckling, and some experimental studies to certify those regulations have been done by Tajima *et al.* [2].

The third problem is lateral buckling of a bridge as a whole. To treat such buckling, the lateral bracings have been usually replaced by their equivalent thin plate and the whole bridge has been regarded as a solid beam[1] [3] [4]. However, the lateral bracings must be subjected to very large shearing deformation compared with any plate. This submits some questions how to determine the moment of inertia around the vertical axis. A two girder system tied by floor beams without lateral bracing has been analysed rather strictly by Komatsu *et al.* [3]. However, the result will be not applicable to the one with trussed lateral bracing, as it is.

On the other hand, it has been proposed by Nishino et al. to correct conventional formula for buckling moment of a beam, taking deformation appeares before buckling into consideration[5]. The proposed expression clarifies a fact that no lateral buckling will occurs under exteral moment around the weak axis, and yet holds practical simplicity. It must be desirable, if possible, such a simplified expression to estimate buckling limit of through plate girder bridge having similar nature of above proposal can be made.

* Member of JSCE, Dr. Eng., Professor, Department of Civil Engineering, Tokyo Metropolitan University
** Member of JSCE, Research associate, Department of Civil Engineering, Tokyo Metropolitan University
(2-1-1, Fukazawa, Setagaya-Ku, Tokyo)

This paper aims at obtaining strict solutions of above second and third problem and get practically simplified expressions for critical loads, excluding intuitive replacement.

2. ASSUMPTIONS

Principal assumptions which are adopted in this study have been summarized as follows :

A 1. Through plate girder bridge is straight and is composed of two main girders, floor beams and lower lateral bracings.

A 2. Cross sectional dimensions of two main girders are similar to each other and have doubly symmetrical open section.

A 3. Floor beams are arranged with a certain intervals, and have only bending rigidity about holizontal axis.

A 4. Lateral bracings are assembled crosswise, forming so-called doubly Warren truss. The bracing themselves have only the axial rigidity.

A 5. Distance between two main girders remaines unchanged at the level of lateral bracings.

A 6. All members of the bridge system are perfectly elastic and obey to the elementaly theory of beams.

3. NOTATION

X,Y,Z : Local right-handed Cartesian coordinates.
u,v,w : Displacements in x,y and z direction.
u_s,v_s : Displacements of the shear center in x and y direction.
w_c : Axial displacement of the neutral axis of a girder.
θ,ω_0 : Torsion angle and warping function.
V_M,V_L,V_F : Strain energy of the bridge system (main girders, lateral bracings and floor beams)
W : Potential loss of the external loads.
L : Total span.
B : Central distance between main girders.
λ : Panel length between the floor beams.
A : Cross sectional area of a main girder.
I_x,I_y : Moment-of-inertia of the main girder, around x and y axis.
I_ω : Sectorial moment-of-inertia of the main girder.
t,b : Thickness and width of the flange of the main girder.
t_w,h_w : Thickness and depth of web of the main girder.
A_L,L_L : Cross sectional area and length of lateral bracing member.
H_L : Vertical distance to lateral bracing from main girder centroid.
I_F : Moment-of-inerita of floor beam.
H_F : Vertical distance to floor beam from main girder.
E,G : Modulus of elasticity and elastic shear modulus.
K : St. Venant's torsion constant.
σ_{cr} : Lateral buckling strength.
σ_e : Elastic lateral buckling stress.
σ_Y : Yield stress.
$(\ )'$: Differentiation with respect to z.

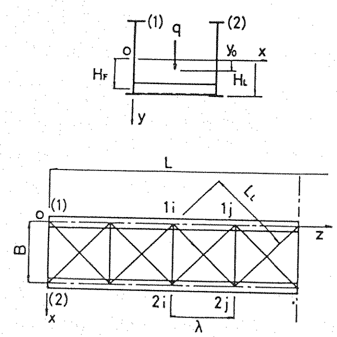


Fig. 1 Coordinate system and model of the through plate girder bridge.

4. DISPLACEMENTS AND STRAINS

A right-handed coordinates having origin at the centroid of an end cross section of main girder as shown in Fig. 1 have been used. The shear center coincides with the origin due to the assumption (A 2). So, displacements of the main girder can be given by

$$\left.\begin{array}{l} u=u_s-y\cdot\theta-x\cdot\theta^2/2 \\ v=v_s+x\cdot\theta-y\cdot\theta^2/2 \\ w=w_c-x(u'_s+v'_s\theta)-y(v'_s-u'_s\theta)+\omega_0\theta' \end{array}\right\} \quad (1)$$

We distinct these displacements with left-hand subscript for each of two main girder as ${}_1u$, ${}_2u$, … according to necessity. Now, it must be more convenient that these are transformed into another global denotations which may be simplified for the total bridge system. Then the followings are newly introduced :

$$\left.\begin{array}{ll} {}_1u=u^0+u^*, & {}_2u=-u^0+u^* \\ {}_1v=v^0+v^*, & {}_2v=v^0-v^* \\ {}_1w=w^0+w^*, & {}_2w=w^0-w^* \\ {}_1\theta=\theta^0+\theta^*, & {}_2\theta=-\theta^0+\theta^* \end{array}\right\} \quad (2)$$

in which the superscript (^{0}) refers to symmetric with respect to the vertical central plane of the bridge perpendicular to the x axis. Similarly the superscript (*) implies the asymmetric displacement.

The axial strain of the main girder is give by

$$\varepsilon_z=w'+(u'^2+v'^2)/2 \quad (3)$$

as well known. This equation contains the 2nd order terms of the derivative of displacement. Therefore the 3rd order terms in the potential energy equation and the 2nd order terms in the derived equilibrium equations must be significant. Further, the shear strain by St. Venant's torsion has to normally taken into account.

5. TOTAL POTENTIAL ENERGY

Total potential energy Π of the bridge can be expressed as the sum of the strain energies of individual member and the potential loss of external loads. That is

$$\Pi=V_M+V_L+V_F+W \quad (4)$$

On the other hand, rewriting Eq. (4) with the global notations of displacement, the total potential energy can be classified into two groups, one does not contains any component of asymmetric displacement and the other. Moreover, the two respective groups can be devided also into two, one contains no term of u^0_s, θ^0 and the other. Then the total potential energy may be expressed as

$$\Pi=\Pi^0_1(v^0_s,\ w^0_c)+\Pi^0_2(u^0_s,\ v^0_s,\ w^0_c,\ \theta^0)+\Pi^*_1(v^0_s,\ w^0_c,\ u^*_s,\ w^*_c,\ \theta^*)$$
$$+\Pi^*_2(u^0_s,\ v^0_s,\ w^0_c,\ \theta^0,\ u^*_s,\ v^*_s,\ w^*_c,\ \theta^*) \quad (5)$$

If the external loads distribute symmetrically about bridge axis, and if sectional deformation can be ignored in the range of small deflection, then the lst term of right side in the above equation may be regarded as the component corresponding to the deformation before buckling. Another terms correspond to post-buckling deformations. Second term does to symmetric, and the 3rd does to asymmetric one. The 4th term corresponds to interaction of symmetric and asymmetric buckling, but this term may be ignored in this study because the influence must be small. Really, this term contains no first or second order term of any displacement and their derivatives.

(1) Main girders

The strain energy V_M of two parallel main girders can be now written as

$$V_M=\frac{1}{2}\sum_{k=1}^{2}\int_0^L[EA\{{}_kw'^2_c+{}_kw'_c({}_ku'^2_s+{}_kv'^2_s)\}+EI_{y,k}u''^2_s+EI_{x,k}v''^2_s+EI_{\omega,k}\theta''^2+GK_k\theta'^2$$
$$+2E(I_y-I_x)\,{}_ku''_s\,{}_kv''_s\,{}_k\theta+E(I_y+I_x)\,{}_kw'_c\,{}_k\theta'^2]dz \quad (6)$$

with displacements of each girder.

The strain energy can be rewritten with global notations Eq. (2). But, since the global neutral axis of total system exists at lower position than the one of individual main girder, it will be desirable that this fact is taken into considerations in the treatment of the deformational behavior before buckling. We introduce therefore the new symbol of axial displacement w_g at a point which is on the symmetrical plane and at a distance H_g below the centroid of main girder. From Eq. (1), this axial displacement w_g is defined as

$$w_g=w_c-H_g(v'_s-u'_s\theta) \quad (7)$$

in which

$$H_g = H_L A_T/(A+A_T), \quad A_T = A_L \lambda^3/L_L^3 \quad \cdots\cdots (8)$$

and for the symmetric deformation we shall use w_g^0 instead w_c^0 hereafter. Thus the strain energies of both main girder becomes

$$\left.\begin{aligned}
V_{M1}^0 &= \int_0^L [EA\{w_g^{0\prime 2} + H_g^2 v_s^{0\prime\prime 2} + 2H_g w_g^{0\prime} v_s^{0\prime\prime} + (w_g^{0\prime} + H_g v_s^{0\prime\prime}) v_s^{0\prime 2}\} + EI_x v_s^{0\prime\prime 2}]dz \\
V_{M2}^0 &= \int_0^L [EA\{-2H_g w_g^{0\prime}(u_s^{0\prime\prime}\theta^0 + u_s^{0\prime}\theta^{0\prime}) - 2H_g^2 v_s^{0\prime\prime}(u_s^{0\prime\prime}\theta^0 + u_s^{0\prime}\theta^{0\prime}) + (w_g^{0\prime} + H_g v_s^{0\prime\prime}) u_s^{0\prime 2}\} \\
&\quad + EI_y u_s^{0\prime\prime 2} + EI_\omega \theta^{0\prime\prime 2} + GK\theta^{0\prime 2} + 2E(I_y - I_x) u_s^{0\prime\prime} v_s^{0\prime\prime}\theta^0 + E(I_x + I_y)(w_g^{0\prime} + H_g v_s^{0\prime\prime})\theta^{0\prime 2}]dz \\
V_{M1}^* &= \int_0^L [EA\{w_c^{*\prime 2} - 2H_g w_g^{0\prime}(u_s^{*\prime\prime}\theta^* + u_s^{*\prime}\theta^{*\prime}) - 2H_g^2 v_s^{*\prime\prime}(u_s^{*\prime\prime}\theta^* + u_s^{*\prime}\theta^{*\prime}) + (w_g^{0\prime} + H_g v_s^{0\prime\prime})(u_s^{*\prime 2} + v_s^{*\prime 2}) \\
&\quad + 2w_c^{*\prime} v_s^{0\prime} v_s^{*\prime}\} + EI_y u_s^{*\prime\prime 2} + EI_x v_s^{*\prime\prime 2} + EI_\omega \theta^{*\prime\prime 2} + GK\theta^{*\prime 2} + 2E(I_y - I_x)\theta^* u_s^{*\prime\prime} v_s^{0\prime\prime} \\
&\quad + E(I_x + I_y)(w_g^{0\prime} + H_g v_s^{0\prime\prime})\theta^{*\prime 2}]dz
\end{aligned}\right\} \cdots\cdots (9)$$

(2) Lateral bracings

The lateral system is assumed that can be regarded as an ideal truss. Consider now the panel $(i\text{-}j)$ shown in Fig. 1. If ε_L is the axial strain of a member $(1i\text{-}2j)$ determined by the displacements $({}_1u_i, {}_1v_i, {}_1w_i, {}_1\theta_i)$, $({}_2u_j, {}_2v_j, {}_2w_j, {}_2\theta_j)$ at the panel points. And similarly $\overline{\varepsilon}_L$ is that of member $(1j\text{-}2i)$, then the respective relations between the strains and displacement are given by

$$\left.\begin{aligned}
\varepsilon_L &= \Delta w(\lambda/L_L) + \Delta u(B/L_L) + \Delta w^2(B/L_L)^2/2 + \Delta u^2(\lambda/L_L)^2/2 + \Delta v^2/2 - \Delta w\,\Delta u(\lambda B/L_L^2)\cdots \\
\overline{\varepsilon}_L &= \Delta\overline{w}(\lambda/L_L) - \Delta\overline{u}(B/L_L) + \Delta\overline{w}^2(B/L_L)^2/2 + \Delta\overline{u}^2(\lambda/L_L)^2/2 + \Delta\overline{v}^2/2 + \Delta\overline{w}\,\Delta\overline{u}(\lambda B/L_L^2)\cdots
\end{aligned}\right\} \cdots\cdots (10)$$

where

$$\left.\begin{aligned}
\Delta w &= ({}_2w_j - {}_1w_i)/L_L, & \Delta\overline{w} &= ({}_1w_j - {}_2w_i)/L_L \\
\Delta u &= ({}_2u_j - {}_1u_i)/L_L, & \Delta\overline{u} &= ({}_1u_j - {}_2u_i)/L_L \\
\Delta v &= ({}_2v_j - {}_1v_i)/L_L, & \Delta\overline{v} &= ({}_1v_j - {}_2v_i)/L_L
\end{aligned}\right\} \cdots\cdots (11)$$

The strain energy V_{Lij}^e of this lateral panel is thus written as

$$V_{Lij}^e = EA_L L_L(\varepsilon_L^2 + \overline{\varepsilon}_L^2)_{ij}/2 \cdots\cdots (12)$$

We may introduce now the approximations

$$\left.\begin{aligned}
w_j^0 - w_i^0 &\fallingdotseq \int_{zi}^{zj} w^{0\prime}\,dz, & w_j^* + w_i^* &\fallingdotseq 2\int_{zi}^{zj} w^*\,dz/\lambda \\
u_j^* - u_i^* &\fallingdotseq \int_{zi}^{zj} u^{*\prime}\,dz, & u_j^0 + u_i^0 &\fallingdotseq 2\int_{zi}^{zj} u^0\,dz/\lambda \\
v_j^0 - v_i^0 &\fallingdotseq \int_{zi}^{zj} v^{0\prime}\,dz, & v_j^* + v_i^* &\fallingdotseq 2\int_{zi}^{zj} v^*\,dz/\lambda
\end{aligned}\right\} \cdots\cdots (13)$$

and hence the strain energy of the total lateral bracings can be transformed to an integrated type of formula.

Besides, by the assumption (A 5), the relation

$$u^0 = u_s^0 - H_L\theta^0 = 0 \cdots\cdots (14)$$

holds on the plane of lateral system.

Finally we can write

$$\left.\begin{aligned}
V_{L1}^0 &= \int_0^L EA_T\delta^0\{\delta^0 + v_s^{0\prime 2} + (B/L_L)^2\delta^{02}\}dz \\
V_{L2}^0 &= \int_0^L 2EA_T H_L H_g\delta^0(\theta^0\theta^{0\prime\prime} + \theta^{0\prime 2})dz \\
V_{L1}^* &= \int_0^L [EA_s\gamma^*(\gamma^* + 4v_s^* v_s^{0\prime}/B) + EA_T\delta^0\{2(H_L - H_g)(u_s^{*\prime\prime}\theta^* + u_s^{*\prime}\theta^{*\prime}) \\
&\quad - (1 - 3B^2/L_L^2)(u_s^{*\prime} - H_L\theta^{*\prime})^2 + 12(B/L_L)^2(w_c^*/B - H_L v_s^{*\prime}/B)(u_s^{*\prime} - H_L\theta^{*\prime}) + (B/\lambda)^2(2v_s^*/B)^2 \\
&\quad - (2B/\lambda)^2(w_c^*/B - H_L v_s^{*\prime}/B)(u_s^{*\prime} - H_L\theta^{*\prime}) + 12(B^2/\lambda L_L)^2(w_c^*/B - H_L v_s^{*\prime}/B)^2\}]dz
\end{aligned}\right\} \cdots\cdots (15)$$

where

$$A_S = A_L\lambda B^2/L_L^3, \quad \delta^0 = w_g^{0\prime} + (H_g - H_L)v_s^{0\prime\prime}, \quad \gamma^* = 2w_c^*/B - 2H_L v_s^{*\prime}/B - u_s^{*\prime} + H_L\theta^{*\prime} \cdots\cdots (16)$$

in which δ^0 and γ^* denote the elongation and the shearing strain respectively in the plane of lateral system but containing only the first order term of the displacements.

(3) Floor beams

The bending moment at any point x of a floor beam $(1i\text{-}2i)$ can be written as

$$M=-\frac{EI_F}{B}\left[6\left(\frac{{}_1v_i}{B}-\frac{{}_2v_i}{B}\right)\left(\frac{2}{B}x-1\right)+2\,{}_1\theta_i\left(\frac{3}{B}x-2\right)+2\,{}_2\theta_i\left(\frac{3}{B}x-1\right)\right] \quad (17)$$

Then the strain energy V^e_{Fi} of a beam may be given by

$$V^e_{Fi}=\int_0^B M^2/(2EI_F)dx \quad (18)$$

Let us now transform Eq. (17) with Eq. (2) and substitute it into Eq. (18), then we obtain

$$V^e_{Fi}=\frac{2EI_F}{B^3}[12\,v_i^*(v_i^*+B\theta_i^*)+B^2(\theta_i^{02}+3\theta_i^{*2})] \quad (19)$$

in which the asymmetric deflection v_i^* is

$$v_i^*=v_{si}^*-H_F\,\theta_i^0\,\theta_i^* \quad (20)$$

from Eq. (1) and Eq. (2). Therefore substituting this relation into Eq. (19) and rewriting it in the integrated form using approximation similar Eq. (13), we can obtain

$$\left.\begin{aligned}&V^0_{F_1}=0\\&V^0_{F_2}=\{2EI_F/(B\lambda)\}\int_0^L\theta^{02}\,dz\\&V^*_{F_1}=\{2EI_F/(B^3\lambda)\}\int_0^L[12\,v_s^*(v_s^*+B\theta^*)+3B^2\,\theta^{*2}]dz,\end{aligned}\right\} \quad (21)$$

as the strain energy of the total floor beams.

(4) External load

It may be assumed that the uniformly distributed external load acts downward through the longitudinal bridge axis and at the ordinate y_0 as shown in Fig. 1.

Assuming for simplification that the deformation of the floor beam under the load can be neglected, and using Eq. (1) and Eq. (2), displacement v_p of the applied point can be represented as

$$v_p=({}_1v+{}_2v)/2=v_s^0-y_0(\theta^{02}+\theta^{*2})/2 \quad (22)$$

Then we can obtain

$$\left.\begin{aligned}&W_1^0=-\int_0^L q_y\,v_s^0\,dz\\&W_2^0=\int_0^L q_y\,y_0\,\theta^{02}\,dz/2\\&W_1^*=\int_0^L q_y\,y_0\,\theta^{*2}\,dz/2\end{aligned}\right\} \quad (23)$$

as the potential loss of the external load.

5. DEFORMATION BEFORE BUCKLING

If the load and the total system are both symmetric with respective to the longitudinal bridge axis, all of asymmetric displacements and their derivatives must not appear. Moreover, from the above mentioned assumption of no deformation of floor beams under loading, the cross sectional shape of the bridge must holds, that is $u_s^0=\theta^0=0$. Then the total potential energy Π_1^0 before buckling becomes

$$\Pi_1^0=V^0_{M1}+V^0_{L1}+W_1^0=\int_0^L[E(A+A_T)(w_g^{0\prime 2}+w_g^{0\prime}\,v_s^{0\prime 2})+EI_x^0\,v_s^{0\prime\prime 2}+EA_T(B/L_L)^2\,\delta^{03}]dz \quad (24)$$

where

$$I_x^0=I_x+H_g\,H_L\,A \quad (25)$$

If the small displacement theory can be applicable to the calculation of the deformation before buckling, the third order terms of displacement in the integrand of Eq. (24) may be neglected. Then in this case the equilibrium equations derived from the first variation of Eq. (24) becomes

$$2E(A+A_T)w_g^{0\prime\prime}=0 \quad (26)$$

$$2EI_x^0 v_s^{0''''}=q_y \quad \cdots (27)$$

If we may consider that $2(A+A_T)$ and $2I_x^0$ denote the cross sectional area and the moment of inertia of total system respectively, these equations are perfectly coincide with ones of elementary beam theory. The equivalent component of cross sectional area A_T implies the effect of lateral system acts as a part of lower flanges. Second term in the right side of Eq. (25) denotes similarly the equivalent component of inertia born by the lateral system.

Moreover, in case of a simply supported girder bridge, w_g^0 must vanish. This shows that the neutral axis of the global system exists below the centroid of main girder by distance H_g.

6. CALCULATION OF BUCKLING LOADS

(1) Symmetrical buckling

In this case, the term Π_2^0 for the symmetrical deformation appears in Eq. (5). Now, summing up V_{M2}^0, V_{L2}^0, V_{F2}^0 and W_2^0 and rearranging them, we have

$$\Pi_2^0=\int_0^L [EJ_{\omega y}\theta^{0''2}+GK\theta^{0'2}+EJ_1 H_g v_s^{0''}\theta^{0'2}-2EJ_2 H_L v_s^{0''}\theta^0\theta^{0''}+\{2EI_F/(B\lambda)+q_y y_0/2\}\theta^{02}]dz \quad \cdots (28)$$

where

$$J_{\omega y}=I_\omega+H_L^2 I_y,\quad J_1=I_x+I_y-H_L^2 A,\quad J_2=I_x-I_y+H_L H_g A \quad \cdots (29)$$

From the first variation of equation (28), the equilibrium equation can be derived as

$$2EJ_{\omega y}\theta^{0''''}-2GK\theta^{0''}-2E(J_1 H_g+2J_2 H_L)[v_s^{0''}\theta^{0'}]'+\{4EI_F/(B\lambda)-2EJ_2 H_L v_s^{0''''}+q_y y_0\}\theta^0=0 \quad \cdots (30)$$

Thus the solution of Eq. (30) must be determined using v_s^0 which satisfies Eq. (27). In case of simple supported bridge, torsion angle θ^0 can be represented in the form of trigonometric series

$$\theta^0=\sum_{i=1} d_i^0 \sin(i\pi z/L) \quad \cdots (31)$$

Using Galerkin's method, the equation to obtain eigenvalues q_{ycr} can be derived as

$$[2EJ_{\omega y}(i\pi/L)^4+2GK(i\pi/L)^2+4EI_F/(B\lambda)-\{(J_1 H_g+2J_2 H_L)(1+i^2\pi^2/6)/(2I_x^0)+J_2 H_L/I_x^0-y_0\}q_y]d_i^0$$
$$-2(J_1 H_g+2J_2 H_L)/I_x^0 \sum_{j\neq i} ij(i^2-3j^2)/(i^2-j^2)^2\cdot d_j^0=0 \qquad (j\pm i:\text{even}) \quad \cdots (32)$$

for the ith order term of the above series (31)

The eigenvalues must be obtained by solving the above equations, but the minimum one corresponds not necessarily to the first order. If we neglect the second interaction term and H_g in Eq. (32), the equations become independent each other, and eigenvalue $q_{ycr,i}$ for the ith order is approximately

$$q_{ycr,i}\doteqdot\frac{2EJ_{\omega y}(i\pi/L)^4+2GK(i\pi/L)^2+4EI_F/(B\lambda)}{J_2 H_L(2+i^2\pi^2/6)/I_x^0-y_0} \quad \cdots (33)$$

The quantity of the denominator of this equation divided by $(i\pi)^2$, if $(i\pi)$ is large, approaches to the constant value $J_2 H_L/(6I_x^0)$. In such case, the order of series which results minimum eigenvalue can be approximately given by

$$i=(L/\pi)\sqrt[4]{2I_F/(B\lambda J_{\omega y})} \quad \cdots (34)$$

If eigenvalues are estimated with some equations in Eq. (32) corresponding to the orders adjacent i obtained from Eq. (34), we can find the minimum symmetrical lateral buckling load accurately. For the practical purpose, result from Eq. (33) using i obtained by Eq. (34) may be sufficient.

(2) Asymmetric overall lateral buckling

In this article, only the asymmetrical energy term Π_1^* appears in the total potential Π.

Since the equation simply obtained by summing up V_{M1}^*, V_{L1}^*, V_{F1}^* and W_1^* becomes too complicated, assumption that the cross section of total system remains may also be adopted in this case. This means that sufficient rigidity of floor beams can be assumed, and then the relation

$$v_s^*=-\theta^* B/2 \quad \cdots (35)$$

hold. Then V_{F1}^* vanishes. Hereafter, we look upon this state as the overall lateral buckling.

Besides, the cross sectional area of lateral bracings are usually sufficiently smaller than that of main girders of a bridge. Then, assumming that (A_L/A), (A_T/A) and (A_s/A) have same degree of the quantities with the first order term of the displacements, the quantities multiplied these values by the third order terms of displacement can be neglected.

Finally we can express Π_1^* in the following form using Eq. (27) :

$$\Pi_1^*=\int_0^L[EAw_c^{*\prime 2}+EI_y u_s^{*\prime\prime 2}+EJ_{\omega x}\theta^{*\prime\prime 2}+GK\theta^{*\prime 2}+EA_s\gamma^{*2}$$
$$-EAB\,w_c^{*\prime}\theta^{*\prime}v_s^{0\prime}+2E(I_y-I_x)u_s^{*\prime\prime}\theta^* v_s^{0\prime\prime}+2EI_x^0 y_0 v_s^{0\prime\prime\prime\prime}\theta^{*2}/2]dz \quad \cdots\cdots (36)$$

where

$$J_{\omega x}=I_\omega+B^2\cdot I_x/4 \quad \cdots\cdots (37)$$

and from Eq. (35) the shearing strain γ_* becomes

$$\gamma^*=2w_c^*/B-u_s^{*\prime}+2H_L\theta^{*\prime} \quad \cdots\cdots (38)$$

From the first variation of Eq. (36) the equilibrium equations can be obtained as

$$\left.\begin{aligned}&-[2EAw_c^{*\prime}-EAB\,v_s^{0\prime}\theta^{*\prime}]'+2EA_s(2/B)\gamma^*=0\\&[2EI_y u_s^{*\prime\prime}+2E(I_y-I_x)v_s^{0\prime\prime}\theta^*]''+2EA_s\gamma^{*\prime}=0\\&[2EJ_{\omega x}\theta^{*\prime\prime}]''-[2GK\theta^{*\prime}+4EA_s\gamma^*-EAB\,v_s^{0\prime}w_c^{*\prime}]'+2E(I_y-I_x)v_s^{0\prime\prime}u_s^{*\prime\prime}+2EI_x^0 y_0 v_s^{0\prime\prime\prime\prime}\theta^*=0\end{aligned}\right\} \quad \cdots\cdots (39)$$

Generally the displacement v_s^0 can be solved for any loading pattern by Eq. (27) In all cases we can expand this solution in the Fourier series. If the bridge have been simply supported,

$$v_s^0=q_y^0\sum_i b_i^0\sin(i\pi z/L) \quad \cdots\cdots (40)$$

and then, u_s^*, w_c^* and θ^* can be express in the form

$$\left.\begin{aligned}u_s^*&=\sum_i a_i^*\sin(i\pi z/L)\\w_c^*&=\sum_i c_i^*\cos(i\pi z/L)\\\theta^*&=\sum_i d_i^*\sin(i\pi z/L)\end{aligned}\right\} \quad \cdots\cdots (41)$$

Substituting them into Eq. (39) multiplying by $\cos(i\pi z/L)$ or $\sin(i\pi z/L)$ and integrating with respect to z the equilibrium equations become

$$\left.\begin{aligned}&\left\{EA\left(\frac{i\pi}{L}\right)^2+\frac{4EA_s}{B^2}\right\}c_i^*-\frac{2EA_s}{B}\left(\frac{i\pi}{L}\right)a_i^*+\frac{4EA_sH_L}{B}\left(\frac{i\pi}{L}\right)d_i^*\\&\qquad+q_y^0 EAB\left(\frac{i^2\pi^2}{L^3}\right)\sum_j d_j^*\sum_k\frac{2jk(i^2-j^2-k^2)}{i^4+j^4+k^4-2i^2j^2-2j^2k^2-2k^2i^2}b_k^0=0\\&-\frac{2EA_s}{B}\left(\frac{i\pi}{L}\right)c_i^*+\left\{EI_y\left(\frac{i\pi}{L}\right)^4+EA_s\left(\frac{i\pi}{L}\right)^2\right\}a_i^*-2EA_sH_L\left(\frac{i\pi}{L}\right)^2d_i^*\\&\qquad-q_y^0E(I_y-I_x)\left(\frac{i^3\pi^3}{L^4}\right)\sum_j d_j^*\sum_k\frac{8jk^3}{i^4+j^4+k^4-2i^2j^2-2j^2k^2-2k^2i^2}b_k^0=0\\&\frac{4EA_sH_L}{B}\left(\frac{i\pi}{L}\right)c_i^*-2EA_sH_L\left(\frac{i\pi}{L}\right)^2a_i^*+\left\{EJ_{\omega x}\left(\frac{i\pi}{L}\right)^4+GK\left(\frac{i\pi}{L}\right)^2+4EA_sH_L^2\left(\frac{i\pi}{L}\right)^2\right\}d_i^*\\&\qquad+q_y^0\Bigg[EAB\left(\frac{i\pi^2}{L^3}\right)\sum_j c_j^*\sum_k\frac{2j^2k(j^2-i^2-k^2)}{i^4+j^4+k^4-2i^2j^2-2j^2k^2-2k^2i^2}b_k^0\\&\qquad-E(I_y-I_x)\left(\frac{i\pi^3}{L^4}\right)\sum_j a_j^*\sum_k\frac{8j^3k^3}{i^4+j^4+k^4-2i^2j^2-2j^2k^2-2k^2i^2}b_k^0\\&\qquad-EI_x^0y_0\left(\frac{i\pi^3}{L^4}\right)\sum_j d_j^*\sum_k\frac{8jk^5}{i^4+j^4+k^4-2i^2j^2-2j^2k^2-2k^2i^2}b_k^0\Bigg]=0\end{aligned}\right\} \quad \cdots\cdots (42)$$

in which the summation appeared in the coefficients of q_y^0 must be carried out only for the combinations that $(i\pm j\pm k)$ equals to odd number. Thus if the load is symmetrical respective to the span center, v_s^0 of Eq. (40) has only the odd order terms, and this case it is possible that the odd and even order modes of buckling can be calculated independently each other.

The critical load may be obtained by the eigen-equation which makes the determinant of the coefficients of Eq. (42) zero. The result is just the overall lateral buckling load.

(3) Approximate solution under uniformly distributed loading

When the bridge carries a uniformly distributed load, it is more convenient to integrate v_s^0 directly rather than to expand it in sinusoidal series. If the solution using only the first order mode can be regarded as to have sufficient accuracy, it must be advisable to give an approximate formula of solution.

For the above purpose, by elimination of Eq. (39) and taking end conditions into consideration, we can obtain first

$$w_c^{*\prime}=\frac{B}{2}\left[-\frac{4I_y}{AB^2}u_s^{*\prime\prime}-\frac{4(I_y-I_x)}{AB^2}v_s^{0\prime\prime}\theta^*+v_s^{0\prime}\theta^{*\prime}\right] \qquad (43)$$

and substituting this relation into Eqs. (38) and (39), we can eliminate w_c^*. Then the equilibrium equations can be reduced to

$$\left.\begin{aligned}&EI_y u_s^{*\prime\prime\prime\prime}+E(I_y-I_x)(v_s^{0\prime\prime}\theta^*)^{\prime\prime}-EA_s\{1+4I_y/(AB^2)\}u_s^{*\prime\prime}\\&\qquad+2EA_sH_L\theta^{*\prime\prime}+EA_sv_s^{0\prime}\theta^{*\prime}-\{4EA_s(I_y-I_x)/(AB^2)\}v_s^{0\prime\prime}\theta^*=0\\&EJ_{\omega x}\theta^{*\prime\prime\prime\prime}-GK\theta^{*\prime\prime}+2E(I_y-I_x)H_L(v_s^{0\prime\prime}\theta^*)^{\prime\prime}+EI_x^0y_0v_s^{0\prime\prime\prime\prime}\theta^*\\&\qquad+2EI_yH_Lu_s^{*\prime\prime\prime\prime}-EI_yv_s^{0\prime}u_s^{*\prime\prime\prime}-EI_xv_s^{0\prime\prime}u_s^{*\prime\prime}=0\end{aligned}\right\} \qquad (44)$$

if the third order terms can be omitted.

Substituting the solution in case of uniform loading into v_s^0 in Eq. (44) and taking only the first term of u_s^*, θ^* in Eq. (41), we obtain the eigen-equation including the terms up to the second order of deflection before buckling. Solving this equation and expressing the eigenvalue by maximum bending moment M_{cr}, the result can be given by

$$M_{cr}=\frac{3\pi^4EI_y^*}{2(3+\pi^2)L^2\left(1-\frac{I_y^*}{2I_x^*}\right)}\cdot\left[-\beta^*\pm\sqrt{\beta^{*2}+\left(1-\frac{I_y^*}{2I_x^*}\right)\cdot\frac{EI_\omega^*+GK^*L^2/\pi^2}{EI_y^*}}\right] \qquad (45)$$

since $(I_y/I_x\ll1)$ and $(2I_y\ll I_x^*)$. In above expression, I_x^*, $\cdots$, K^* denote the cross sectional parameters of total bridge system, and are defined as

$$\left.\begin{aligned}&I_x^*=2I_x,\ \ I_y^*=2A(B/2)^2/[1+A(B/2)^2/\{A_s(L/\pi)^2\}]+2I_y,\ \ I_\omega^*=2J_{\omega x}\\&K^*=2K,\ \ \beta^*=2H_L-y_0\cdot6/(3+\pi^2)\end{aligned}\right\} \qquad (46)$$

7. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Since the method of analysis introduced here have adopted some approximation particularly on overall lateral buckling, their validity should be examined by comparing the theoretical values with experimental results. This paper avails itself of experiments on elastic lateral buckling carried out by Japanese National Railways[6) 7)], and Tokyo Metropolitan University[8)].

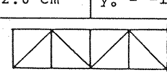
The numeric values of the cross sectional properties necessary to the analysis are shown in Table 1, and the used material had been SS 41 in all cases. A concentrated load P was applied to centre of the span and a loading appatatus had been put on upper flanges of both main girder as to push them vertically. Floor beams had been arranged in a certain interval, and lateral bracings had been welded directly to lower flanges of both main girder.

As the number of deflection modes, allowing for only the symmetrical one, theoretical results obtained with only one term in Eq. (40) and Eq. (41) have been compared with those with three terms (i=1, 3, 5).

Fig. 2 and Fig. 3 show comparisons of measurements by JNR and TMU with the above theoretical results. As shown in these figures, although the theoretical values allowing for the self-weight of tested model are somewhat lower than the measurements, but we can find a very good coincidence of them.

As known in Fig. 2 and Fig. 3, there can be found some differences in theoretical results between the case with only one term

Table 1 Dimensions and cross sectional properties of the experimental bridges.

	J. N. R.	T. M. U.
Main girder	I-159x20x2.3x4.5	]-150x16.8x1.2
Lateral bracing	■-8x8	L-5x5x1.2
Floor beam	|-40x2.3	|-30x1.2
Skeleton	B = 8.0 cm λ = 16.0 cm H_L = 7.5 cm H_F = 5.5 cm y_0 = -12.0 cm	B = 4.88 cm λ = 7.5 cm H_L = 7.38 cm H_F = 5.88 cm y_0 = -11.0 cm
		

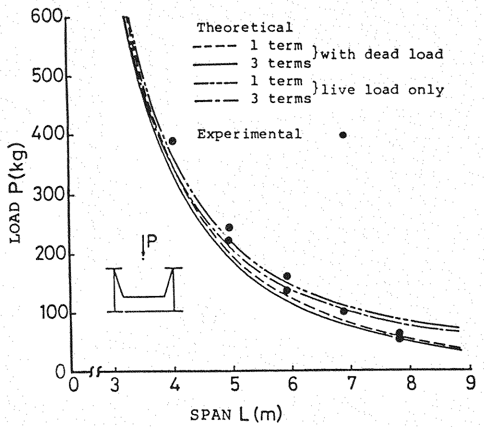


Fig. 2 Comparison of the experimental values (Japanese National Railways) with the theoretical curves for the overall lateral buckling.

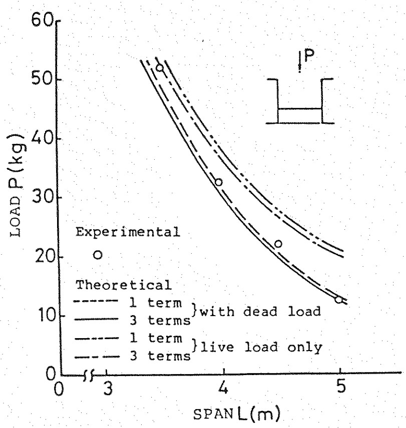


Fig. 3 Comparison of the experimental values (Tokyo Metropolitan University) with the theoretical curves for the overall lateral buckling.

and with three terms of mode, but variations originating from repetition of experiment are almost the same as the above differences. Hence, the analysis taking only one term would be said sufficient for the practical requirements even against the concentrated loading.

8. ILLUSTRATIVE ANALYSIS OF MORE PRACTICAL MODELS

In the experimental models investigated in previous sections, the distance between main girders had been made to be extremely narrow to make the lateral elastic buckling occur, and they had not to represent practical bridge behaviors in such sense. It may be desirable to try to estimate behaviors of more practical models.

There exist few actual through plate girder railway bridges exceeding 40 m span, and it was difficult to gather and analyse sufficient number of actual bridges. Then a single line through plate girder bridge designed by JNR having 32 m span had been picked up as standard practical model, and had been modified to another spans as "practical models" used in this paper. In the modifications, the central distance between main girders, the panel length between floor beams, and the cross sectional dimension of lateral bracings and floor beams had been not changed, and only the main girder's cross section had been determined by optimum design. The used material was always SM 50. The width-thickness ratio of a web plate had been fixed to 200, assuming the web having one holizontal stiffener. The width-thickness ratio of flange plates had been supporsed as 6 in all cases, where the thickness included that of a cover plate. Fig. 4 shows the dimensions and properties of the practical models introduced herein

Fig. 5 shows the relations between span and elastic buckling load determined by applying our method of analysis to the above mensioned practical models. The black dots for symmetrical buckling in the figure refer to estimated results by approximate equation (33) with the number of order i given by Eq. (34). These points are well agreeded with the curve (two dots chain line) determined by Eq. (32) taking three terms ($i-1$, i, $i+1$), without significant differences.

The orders of buckling mode appeared in these calculations were 5 to 8 in the range of span shown in Fig. 4. The order becomes lower as the span does shorter, and the half wave length of the mode becomes also shorter. In the range of considerably small L/B, resulted half wave length had been as short as or shorter than the panel length. Hence the validity of simplification replacing summation of the effect of lateral system by integration must be questionable in such special cases.

Concerning asymmetrical buckling, Eq. (40) and Eq. (41) have been used. The buckling loads estimated only one term of mode in Eq. (41) and that with three ones have been compared with each other. In both cases v_s^0 have been expressed using three terms. Few practical differences can be found between them, though the latter provides

a) Main girder

L cm	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2	128.0
M tm	185	657	1380	2336	3515	4907	6506	8037	10304	12496
b cm	20.0	27.0	35.0	42.0	47.0	52.0	57.0	62.0	67.0	72.0
t_F cm	3.3	4.4	5.8	6.7	7.7	8.6	9.4	10.2	11.0	11.9
h_w cm	143.0	218.0	280.0	333.0	381.0	426.0	468.0	508.0	546.0	582.0
t_w cm	0.9	1.1	1.4	1.7	1.9	2.1	2.3	2.5	2.7	2.9

b) Lateral bracing $A_L = 17.0\ cm^2$

c) Floor beam $I_F = 125750\ cm^2$

$H_g = 0.296\ cm(L=128m) \sim 0.936\ cm(L=12.8m)$

$A_S = 6.54\ cm^2$, $A_T = 3.46\ cm^2$

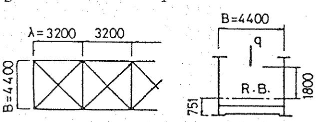


Fig. 4 Dimensions and properties of the calculated practical models.

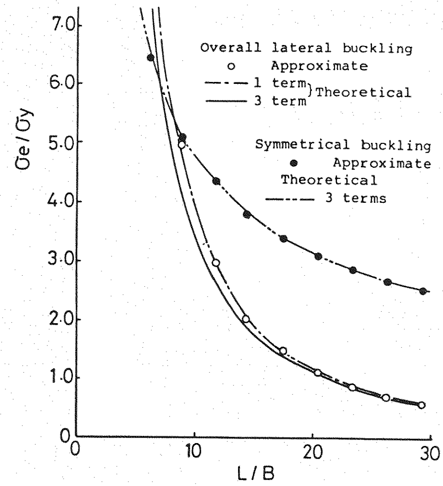


Fig. 5 Comparison of the symmetrical buckling curve with the overall lateral buckling ones under uniformly distributed loading.

somewhat safeside solutions. The small white circles refer to an approximate solutions obtained by Eq. (45). The approximation may be considered as to provide considerably good accuracy. Then the formula may be sufficiently useful in practical purpose.

In general, the symmetrical buckling load is higher than the asymmetrical buckling load except for short spans.

The buckling curves in Fig. 5 appears in the range far exceeding the yield point. Because this means that the actual bridge will buckles in the inelastic range, the above results for elastic buckling may be not realistic as it is. Therefore, analysis of actual bridge must be required to review expressions for the total potential increments, taking into considerations the effects of initial imperfection, residual stress distribution by welding, and nonlinear property of materials. In this paper however, an approximate load carrying capacities have been roughly estimated from the above elastic buckling curves.

Many experimental data in the past show the fact that the relation between theoretical elastic buckling curve and inelastic experimental strength of beams are rather similar to that of columns[9]. Then as a rough approximation, if it may be permitted to assume that the relations are perfectly similar, the load carrying capacities of practical models against lateral buckling can be estimated from elastic results using such relation.

In our study, the relation between elastic and inelastic strength had been adopted from the comments on Japanese Specifications for Highway Bridges[10]. Fig. 6 shows illustrative results, in which the load carrying capacity curve determined by local buckling moment formula

$$M_{cr} = \frac{\pi}{L}\sqrt{\frac{EI_y\,GK}{(1-I_y/I_x)}\left(1+\frac{\pi^2 EI_\omega}{L^2\,GK}\right)} \quad \cdots\cdots\cdots\cdots\cdots\cdots (47)$$

proposed by Nishino *et al.*[5] have been added.

The lowest value among the three curves shown in Fig. 6 must represents the load carrying capacity of the system. As is clear in Fig. 6, local buckling between panel point predominates in the shorter span range, while the overall lateral buckling does in the longer span. For the practical models used in our study, the boundary exists about $L/B=10.5$ ($L=46$ m).

Although this illustrative calculation tells that symmetrical buckling will never occurs in any case, we can not deny that such buckling could occurs due to want of

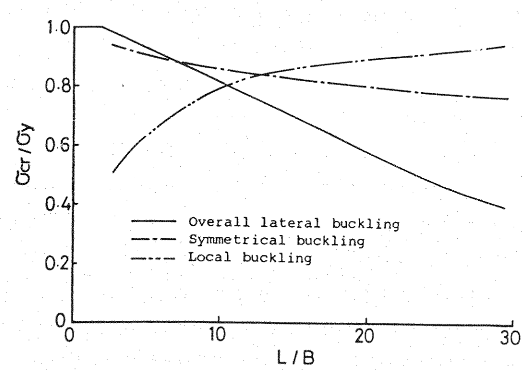


Fig. 6 Buckling strength curves of the practical bridge models.

floor beam rigidity, possibly in the vicinity of above boundary.

9. CONCLUSIONS

The results can be summerized as follows :

(1) The general method has been derived successfully which may be used to determine the symmetrical and asymmetrical lateral buckling load of an open floor type through plate girder bridge.

(2) The approximate formulae for critical value and mode of symmetrical buckling under uniformly distributed load have been obtained.

(3) The approximate formula has been obtained which provides overall asymmetrical lateral buckling load under the uniformly distributed condition. The cross sectional properties of total system can be defined clearly.

(4) The analytical results have coincided well with measurements on elastic overall lateral buckling.

(5) The application of the above method to the practical bridge models has led to a suggestion that the load carrying capacity for long spans would depend largely on overall lateral buckling.

(6) Illustrative calculations for the practical models have suggested that symmetrical buckling would rarely be realized.

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