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STATIC EQUILIBRIUM STATE OF RIVER BED COMPOSED OF THREE GRAIN SIZE GROUPS OF SEDIMENT

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SYNOPSIS

The purpose of this study is to investigate the vertical structure of a river bed which is composed of an extremely wide range of sediment grain size in the state of static equilibrium. Several series of experiments were conducted under the conditions which were set systematically. The bed materials were modeled as three grain size groups of sediment: the larger particle of stone or gravel which cannot move at all, the medium particles of gravel which moves as bed load and the finer particle of sand or silt which can move as suspended load. As a result, it was found that a vertical grain sorting occurs clearly in the void of bed which was surrounded by the larger stones. The structure of such bed results from the hiding effect by the larger stones. The characteristics of this structure were evaluated in this paper.

INTRODUCTION

After dam construction, sediment supply passing through the dam to the reach of river just downstream of it decreases drastically. In such an upstream reach of river, the bed is composed of extremely wide range of sediment. Therefore, the armoring of the river bed as well as bed degradation was progressed gradually. In order to create such influence of dam on river beds more moderately and to improve the sediment environment in rivers, some attempts have been carried out recently. One of them is by directly supplying sediment from the gate of a dam or from a bypass tunnel to the downstream reach of a river. In addition, another attempt such as artificial supply was carried out. Sand, which was taken from somewhere else, was placed on the bed just downstream of dam. The sand can be transported by natural river erosion by running water in river. However, strictly speaking, it is not clear how river beds respond to such sediment supply at this stage. Focusing on the river bed of upstream reach, we can observed that there are large rocks or stones exposed on the bed surface, and that they cannot move at all even at this stage of flood. One can easily expect that the mechanism of sediment transport on such a river bed is more complicated than the one in downstream reach which has been studied before by several researchers. As far as the authors know, no study has been conducted yet related to the above mechanism, although the first author attempted more fundamental study (1).

The most important characteristic of the river bed in the above mentioned reach is the structure of bed or the composition of sediment. According to the field observations of actual rivers, grain size distribution of the bed sediment is extremely wide compared to the downstream reach, and there exists the larger stone or gravel which cannot move at all. And the basic structure of the bed is constituted of such larger sediment systematically. And the void space around the larger gravel is filled with finer gravel, sand or silt. We assumed that a river bed is composed of three grain size groups of sediment. In this study, each of these groups is represented by a unique size of particle as follows: (a) the larger particle of stone or gravel which cannot move at all (it is called "L-particle" in this study), (b) the medium particles of gravel which moves as bed load ("M-particle"), and (c) the finer particle of sand or silt which can move as suspended load ("S-particle"). The grain sizes of each particle are defined as D_L, D_M, and D_S in this paper. Among these diameters, there is a remarkable difference by one order ($D_L \gg D_M \gg D_S$).

The objective of our research is to clarify the mechanism of deformation of an actual river bed which was composed of such three grain size groups of sediment. At the first stage of this research, a series of experiments were conducted in a laboratory flume, and the bed structure in a static equilibrium state was investigated experimentally. This may be the first attempt to study such bed deformation process systematically. The process toward a dynamic equilibrium state will be the next step taken by the authors.

To better understand this study, some researches related to this study are reviewed briefly. One of them was the work by Ashida and Fujita (2). They conducted experiments of sediment transport on bed which was composed of two grain sizes of sediment. One was composed of gravel of 3 mm or 4.8mm in diameter, and the other was composed of very fine sand or the silt whose grain size was 0.044-0.074mm or 0.149-0.177mm. The latter sediment was fully-sorted into 6 classes, and each of them was used as the finer sediment whose grain size is assumed to be almost uniform. This finer sediment was transported as suspended load, and was picked-up from the void of the gravels. In this process, the hiding effect of the gravel on the movement of the finer sediment is important. Probabilistic consideration was also made by them, and the erosion rate function and this hiding function were proposed on the basis of experimental results. Okabe et al. (3) and Fukushima et al. (4) also examined the same kind of phenomenon. The influence of big stones of river bed on the movement of finer sediment was investigated in their papers.

SUMMARY OF EXPERIMENTS

An explanation of the experimental apparatus is illustrated in Figure 1. The experiment was conducted in an acrylic closed conduit. The length of the channel is 5.0 m, and the cross section is square with 0.1 m in width and 0.1 m in height. In the reach between 2.5 m and 3.5 m from the upstream end of the channel, the bottom is 0.05 m lower than the one in the other reach of channel. A movable bed was set on this concave bottom with three sizes of sediment which are explained later. The basic frame of the bed is constituted as a series of L-particles as is seen from Photo 1. M-particles and S-particles fill the void space surrounded by L-particles. The surface elevation of the initial bed is equal to the top of the L-particles, and is consistent with the elevation of the fixed bed in both upstream and downstream reach. On the bottom of the fixed bed and on the ceiling of the conduit, a rubber board with dense pyramid-like roughness elements was attached to make the boundary layer fully developed before the flow reached the movable bed. The height of the roughness is 2 mm. The hydraulic conditions of flow in this conduit are controlled only by operating the flow valve. As a preliminary experiment, the flow measurement was conducted by means of a Laser-Doppler anemometer. It was found from this measurement that a fully-developed turbulent shear flow was established at least in this reach. In the main experiment in this study, a measurement of the pressure difference was conducted. A hole was drilled on the side wall at 0.25 m upstream from the upstream end of movable bed reach, and the other was at 0.25 m downstream from the downstream end of it. The distance between them is 1.5 m. The value of the shear stress or tractive force exerted on the surface of the movable bed was evaluated as follows. The total value of the shear stress in cross section was evaluated by using a measured value of energy gradient. Friction coefficient or Manning's roughness coefficient of both sidewalls and wall of the ceiling were estimated as a result of preliminary experiment by means of a Laser-Doppler anemometer. Based on the data, the effect of these walls on the shear stress can be calculated. Also the effective shear stress or the tractive force on the movable bed was evaluated by eliminating the above wall effect from the total shear stress. In order to check up the accuracy of this evaluation of tractive force, a measurement was also conducted to know the critical tractive force of M-particle as bed load. It can be verified that the evaluated value is almost consistent with the one calculated from the Shields diagram.

As was explained above, a movable river bed was modeled by three groups of sediment, and the basic structure or frame of the bed was made of L-particle. An alumina ball of 50 mm in grain diameter D_L (specific weight is 3.98) was used. One can see from Photo 1 that L-particles were densely packed in the concave bottom portion, and they do not move at all under the range of this study. Individual L-particles were arranged at an orthogonal latticed position in the main experiments in this study. A gap among L-particles or void space surrounded by them was filled with the sediment mixture of M-particle and S-particle. A glass bead whose specific weight is 2.5 was used as M-particle, and its diameter D_M is 5 mm. M-particle moves as a bed load. Furthermore silica sand was used as S-particle, whose specific weight is 2.65. It was fully-sorted sediment and its grain size was almost constant of 0.21 mm. S-particle moves as a suspended load. Since there is a difference of the specific weight between a glass bead and silica sand, a question may arise as to how such difference affects the results of this experiment. In order to check this effect up, another series of experiments was conducted as a control experiment by using another size of silica sand whose diameter is distributed in the range between 2 mm and 5 mm as M-particle. Dimensionless critical tractive forces τ^*_{MC} for such silica sand and the glass bead take almost same value.

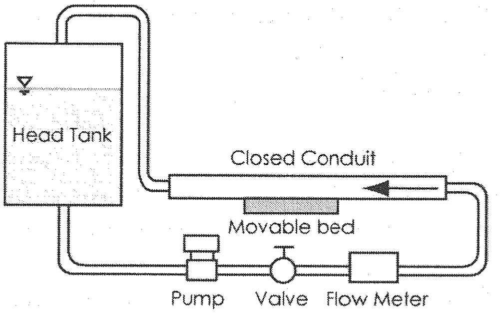

Figure 1. Outline of the experimental system

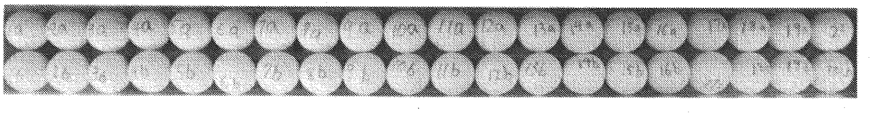

Photo 1. The arrangement of the bed frame (Plane view)

In this study, the parameter R_{PS} is defined as a volumetric mixture ratio of S-particle to the total sum of M- and S-particle in the void space. Sediment composition of bed is characterized by this ratio R_{PS}. In case of R_{PS} = 0, the experiment is named as a series of 'L-M experiment', and the bed is composed of L- and M-particles only. In case of R_{PS}= 1, on the other hand, it is named as a series of 'L-S experiment', and the bed is composed of L- and S-particles only. And the experiments in case of 0 < R_{PS} < 1 is called as 'L-M-S experiment'. In this study, a series of 'L-M-S experiment' was conducted under the condition of R_{PS}= 0.5, 0.6, 0.7, 0.8 and 0.9. 'L-M experiment' and 'L-S experiment' were also conducted. In the initial condition, the state of sediment mixture of M- and S-particles was made uniform in the void space surrounded by L-particles. But in case that R_{PS} is equal to or less than 0.4, it was found that M-particles were partially exposed on the bed surface, and it was impossible to make the state of mixture be uniform in a vertical direction. The value of R_{PS} was set in the above range on the basis of such consideration at the present stage. Frictional velocity is the other important parameter which affects this deformation process of river bed. In this study, a series of experiments was conducted under the different condition of flow discharge, although the discharge was kept constant during each experiment. It was found that the frictional velocity exerted on the bed surface in the static equilibrium state $\tilde{u}^*$ was between 0.06 and 0.11 m/s approximately. This value corresponds to the dimensionless tractive force for M-particle τ_M^* between 0.05 and 0.16, and the dimensionless parameter of $\tilde{u}^*$ divided by a terminal settling velocity w_{oS} of S-particle is between 2 and 4. As a reference, the critical tractive force τ_{MC}^* is about 0.05, and the settling velocity of S-particle w_{oS} is 0.028 m/s. Both values were obtained as a result of another preliminary experiment. In this study, experiments were conducted several times under the same condition to verify the accuracy of the experimental results. Experimental results discussed in later pages were obtained in such a manner.

The measurement procedure of bed structure in case of L-M-S experiment is explained here. After confirming that the bed attained the static equilibrium state, the water flow was stopped without disturbing the state of bed. And then water in the experimental flume was drained. At first, the spatial variation in bed surface elevation was measured. Thus, each individual M-particle was removed by tweezers in M-particle layer, and the elevation of the surface which appeared on the bed was measured in the same manner. Finally, the M- and S-particle which remained there are all sampled, and an analysis was conducted in order to evaluate the value of R_{PS} in the static equilibrium state.

EXPERIMENTAL RESULTS AND DISCUSSION

Static equilibrium state of bed in L-M experiment

First of all, the results in case of L-M experiments are explained here. In this series of experiments, the void space surrounded by L-particles is filled only with M-particles. In Photo 2, one can see the state of such bed. Photo 2(a) was taken through the side wall of the conduit before the experiment, and it corresponds to the state of the initial condition. Photo 2(b), on the other hand, was taken after the experiment, and shows the state in the static equilibrium condition. White spheres are L-particles, and black-colored small particles are M-particles. The focus of this study is the state of bed in the central portion which is away from the side wall. It should be noted that the state in this portion is different from the one shown in these photographs to some extent because of the effect of the side wall.

In the static equilibrium state, M-particles as well as L-particles did not move at all. This means that the tractive force acting on the M-particle is equal to or less than the critical value of it. As for the exposed particles on the bed, one can assume that the frictional velocity $\tilde{u}^*$ or the dimensionless tractive force τ_M^* which is exerted upon it is equal to the critical frictional velocity u_{MC}^* or the dimensionless critical tractive force τ_{MC}^*. The tractive force (or bed shear stress) was evaluated by eliminating the effect of side walls and the ceiling of the conduit.

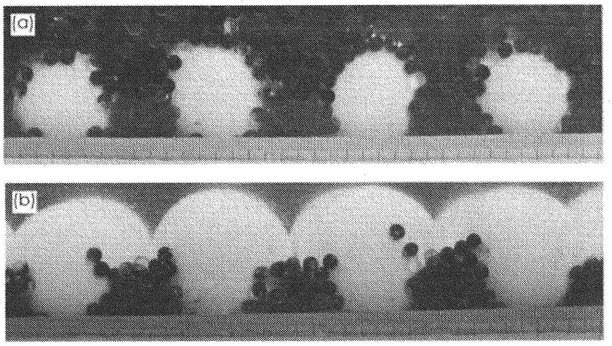

Photo 2. The result of L-M experiment (side view): (a) initial condition, (b) static equilibrium state

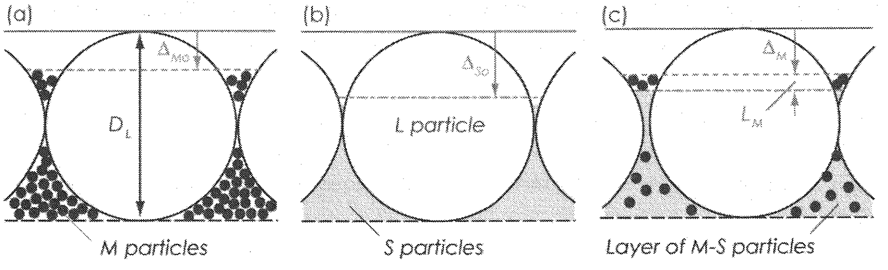

Figure 2. Summary of equilibrium state of bed: (a) L-M experiment, (b) L-S experiment and (c) L-M-S experiment.

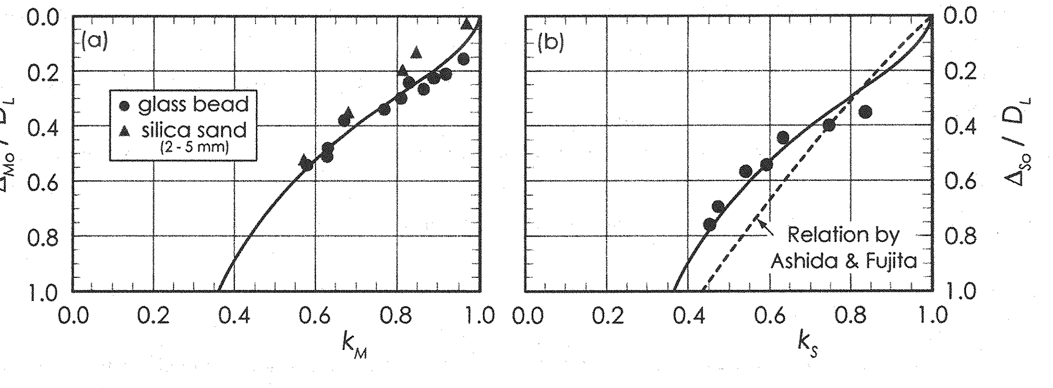

Figure 3. The relation of the cover coefficient

As explained previously, the value $\tilde{u}^*$ or τ_M^* was calculated directly from the value.

Hiding coefficient k_M of M-particle in this case is defined as the ratio of the frictional velocity u_M^*, which is exerted on the top of M-particle exposed on the bed surface, to the frictional velocity $\tilde{u}^*$ on bed. Also, we can assume that a following relation $u_M^* = u_{MC}^*$ is satisfied. This definition is same as that by Ashida and Fujita (2). In Figure 2, summary of the vertical structure of bed is illustrated. In case of L-M experiment, the vertical distance from a top of L-particle to that of M-particle which was exposed on the bed surface is defined as Δ_{Mo} shown in Figure 2(a). Experimental results are shown in Figure 3(a) in this case, and one can see the relation between k_M and Δ_{Mo}/D_L (in which D_L is a diameter of L-particle). Circles in this figure correspond to the results which were obtained in case that glass beads were used as M-particle, and triangles are the results in case that silica sand was used.

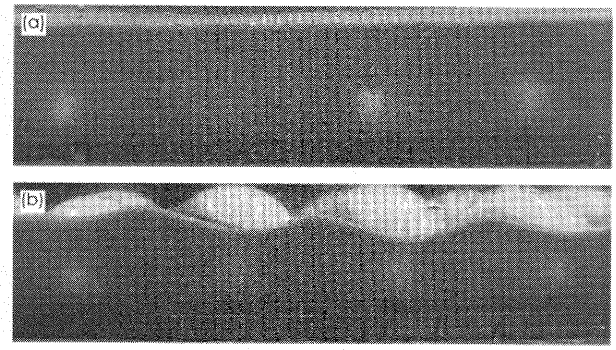

Photo 3. The result of L-S experiment (side view): (a) initial condition, (b) static equilibrium state

One can see that there is no obvious difference between them. The solid line in this figure represents the empirical relation deduced from the experimental results, and it can be formulated by

$$\Delta_{Mo}/D_L = \alpha \times (k_M^{-2} - 1)^{0.5} \tag{1}$$

In this equation, α is proportionality constant, and takes a value of 0.4 in this case. This relation can be transformed into the following expression;

$$\Delta_{Mo}/D_L = \beta_M \times (\tau_M^* - \tau_{MC}^*)^{0.5} \tag{2}$$

In this derivation, β_M is proportional constant (= 1.8 in this case). Following definitions $k_M \equiv u_{MC}^*/\tilde{u}^*$ and $\beta_M \times \tau_{MC}^{*0.5} \equiv \alpha$ were used. One can see from this result that Δ_{Mo} depends upon $\tau_M^* - \tau_{MC}^*$ and is proportional to its square root. This function is similar to a certain respects to the bed load function (for example, Meyer-Peter and Muller's one), although the exponent is different.

For the purpose of reference, the effect of sidewall on the surface elevation of bed is briefly explained. The value of Δ_{Mo} near the side walls is slightly smaller than that in the central portion of conduit, because the velocity was smaller in the former area. However, it can be concluded that the difference is not essential.

Static equilibrium state of bed in L-S experiment

Static equilibrium state is discussed here in case of L-S experiments. In this case, L-particles cannot move at all, but S-particles moves as suspended load. In Photo 3, you can observe the state of such bed before and after the experiment in this case. Photo 3(a) corresponds to the initial condition, and Photo 3(b) corresponds to the static equilibrium condition. White spheres are L-particles, and the dark colored portion corresponds to the S-particles.

The experiments by Ashida and Fujita (2) can be classified into this category, although the sediment grain size of each group in their study was different from the value in this study. In their experiment, the following two groups of sediment were used: one sediment was gravel of 3 mm or 4.8 mm and the other was silt of 0.044-0.074 mm or 0.149-0.177 mm. From the viewpoint of sediment grain size, the gravel was almost same or a little smaller value of M-particle in this study. But the gravel did not move in their experiment, and so the silt moved as suspended load. The initial state of bed was almost the same as this study, and the void space surrounded by gravels were filled with the silt.

The state of bed in the static equilibrium condition is illustrated in Figure 2 (b). The relation between Δ_{So}/D_L

and the hiding coefficient for S-particle k_S is shown in Figure 3 (b). The circles in this figure express the results of this study. The solid line was empirical relation which was deduced from these experimental results. Functional relationship was also derived as follows:

$$\Delta_{So}/D_L = \alpha \times (k_S^{-2} - 1)^{0.5} \tag{3}$$

in which $k_S \equiv u_{SC}^*/\tilde{u}^*$, and u_{SC}^* is the critical frictional velocity for S-particle. As for this value, it was concluded that the value of u_{SC}^* can be approximated to the terminal settling velocity of S-particle w_{oS}. The solid line in Figure 3(b) illustrates the relation of Eq. (3) by substituting w_{oS} for u_{SC}^*. It was also found that the value of α in Eq. (3) takes the same value of 0.4, which is consistent with the value of α in Eq. (1). Thus, Eq. (3) can be transformed into a following expression;

$$\Delta_{So}/D_L = \beta_S \times (\tilde{u}^{*2} - w_{oS}^2)^{0.5} \tag{4}$$

in which $\beta_S \times w_{oS} \equiv \alpha$. The solid lines in Figure 2(a) and (b) are equivalent, and the relation of Eq. (3) is completely equal to Eq. (1). The black dotted lines in Figure 3, on the other hand, represent the relation derived by Ashida and Fujita (2) on the basis of their experiments. These two relations are similar but there is a discrepancy between them. It probably results from the difference of grain size of sediment. The size of gravel in their study is much smaller than that of L-particle in this study.

We found that the hiding coefficient k_ε (= k_M or k_S) satisfies the same relation as a function of Δ_o (= Δ_{Mo} or Δ_{So}). This means that the normalized dimensionless equation is valid without depending upon whether the void space of L-particles is filled with M-particles only or S-particle only. Eq. (1) or (3) can be also transformed into the following equation:

$$k_\varepsilon = \left(1 + \frac{1}{\alpha^2}\left(\frac{\Delta_o}{D_L}\right)^2\right)^{-0.5} \tag{5}$$

This equation expresses some kind of the universal hiding function which is valid in cases of sediment mixture with two grain size groups only. This is one of the most important facts in this study. However, there still remains a question whether or not the constant α keeps about 0.40 if the sediment grain size is different from the value in this study. This will be investigated in future.

Static equilibrium state of bed in L-M-S experiment

Finally, the static state of bed with three grain size groups of sediment is discussed here. In case of L-M or L-S experiment, one can expect the state of bed more easily. In case of bed with more than three groups of sediment, on the other hand, the static state of bed becomes more complicated. Photo 4 illustrates the state in this case. The initial condition can be seen in Photo 4(a), and the static equilibrium state is in Photo 4(b). A summary of the vertical structure of this bed is illustrated in Figure 2(c). It should be noted that sediment sorting can be observed clearly. The characteristics can be summarized as follows:

(1) In the void space surrounded by L-particles, M-particles are exposed on the bed surface, and the layer which is made of M-particle only forms. One can see some black-colored particles which exist on a dark-colored portion. This is referred to as "M-particle layer" in this study. The thickness of the layer is defined as L_M.

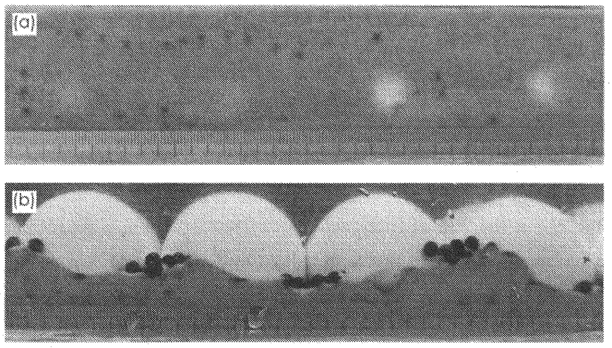

Photo 4. The result of L-M-S experiment (side view): (a) initial condition, (b) static equilibrium state

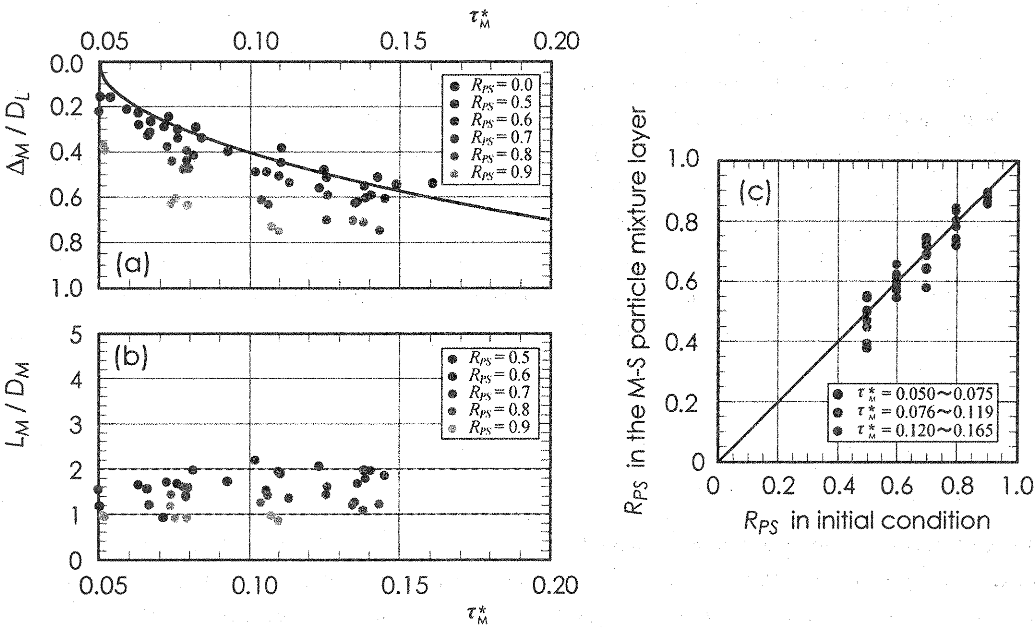

Figure 4. The result of L-M-S experiment

(2) There appears a mixed layer of M- and S-particles just below this layer. The dark-colored portion in Photo 4(b) corresponds to it.

(3) The vertical distance from top of L-particle to the upper surface of M-particle layer is defined as Δ_M in this paper. It is not necessary that Δ_M is equal to Δ_{Mo} which was discussed in the case of L-M experiment.

Experimental results in case of L-M-S experiment are seen in Figure 4. In this figure, R_{PS} was defined to the volume ratio of S-particle to the total sum of M- and S-particle, and is one of the important parameter to be focused on in this case. In Fig. 4(a) or (b), the experimental results were plotted by different symbols according to this parameter. Figure 4(a) shows the relation between Δ_M/D_L and the dimensionless tractive force for M-particle τ^*_M. In case of L-M experiment, following relations are satisfied; $R_{PS} = 0$ and $\Delta_M = \Delta_{Mo}$. The black circles in Fig. 4(a) correspond to the results in case of L-M experiment, and the solid line denotes the same relation as the solid line in Figure 3(a) and Eq. (2). Comparing the results with that in case of L-M experiment, it can be clearly seen that Δ_M is larger than Δ_{Mo}. In case where M-particle moves as bed load and S-particle moves as suspended load, the frictional velocity u_{MC}^* which corresponds to the critical value for M-particle is larger than that for S-particle u_{SC}^*. Also, the value of u_{SC}^* can be approximated as follows; $u_{SC}^* \equiv w_{oS}$. In the process toward the static equilirium state, both M- and S-particles were picked up or eroded, and the frictional velocity exerted on the bed surface among L-particles decreases as time passes. At the moment when the top position of M-particle reaches the elevation which corresponds to Δ_{Mo}, M-particle cannot be picked up any more, but S-particles can be still eroded. Therefore, after this moment, the elevation of top position of M-particle falls down, and Δ_M can be larger than Δ_{Mo}. In this figure, one can see that Δ_M becomes larger as R_{PS} is larger. S-particle exists below the M-particle layer under the double effect of hiding by both L- and M-particle. However, the effect by M-particle becomes smaller in the case where the value of R_{PS} is larger, and the elevation of S-particle is lower in the static equilibrium state. As reference point, the value of Δ_M itself approaches to Δ_{So} as R_{PS} gets closer to unity.

Fig. 4(b) shows the variation in the thickness of M-particles layer L_M against τ_M^*. It can be seen from this figure that the value of L_M is almost a constant value between 1.0 and 2.0, though it is a relatively smaller value in case of R_{PS} = 0.8 or 0.9.

In Fig. 4(c), a comparison was made about the sediment composition in the mixed layer of M- and S-particles which appears below the M-particles layer. The horizontal axis in this figure corresponds to R_{PS} in the initial condition, and vertical axis is the value in static equilibrium state which was measured just after the experiment. As can be seen in this figure, we concluded that the value of R_{PS} is almost constant and does not change from its initial value. This means that the sediment mixing is not active below the bed surface.

CONCLUSION

The purpose of this study has been to understand more deeply the changing process of a river bed which is composed of extremely wide range of sediments. As the first stage, the research on the static equilibrium state was conducted. The bed was composed of three grain size group of the sediment: a larger group of stone (L-particle), a medium one of gravel (M-particle), and a finer one of sand or silt (S-particle). And it was characterized by the volume ratio of S-particle to the total sum of M- and S-particles which fills the void space surrounded by L-particles. Experimental investigation was conducted by changing this parameter R_{PS} as well as the flow velocity or the tractive force. In order to improve the accuracy or reliability of the experimental results, experiments were conducted several times under the same condition. The main conclusions in this study are as follows:

(1) Based on experimental results in case of L-M and L-S experiments, each state of static equilibrium was investigated quantitatively. The hiding effect of L-particles on M- or S-particles was evaluated, and its functional relationship was newly derived.

(2) In case of L-M-S experiment, it was found that the remarkably vertical sorting occurs in the void space surrounded by L-particle. M-particle layer forms in the upper part, and the layer of M-S particle mixture was left in the lower part.

(3) Findings revealed that bed structure in case of L-M-S experiment can be characterized by following three parameters; vertical distance from a top of L-particle to the upper surface of M-particle layer Δ_M, the thickness of M-particle layer L_M and the sediment composition R_{PS} in the layer of M-S mixture.

In this paper, the experimental result in the case where L-particles were arranged in an orthogonal latticed position was explained. However, another series of experiments was conducted under the packed condition of arrangement at cross-stitch position. As a result, it has been confirmed that the result of latter case was almost same as the one in this paper. One can refer to the further discussion in another paper (5).

Further investigation is needed before understanding the mechanism of sediment transport and bed deformation of bed in case of an extremely wide range of sediment size distribution. In case of a static equilibrium state, the effect of each grain size or the combination of them must be examined. The process toward the dynamic equilibrium state of the bed should also be investigated in the future. The authors intend to continue conducting such series of experimental study in order to predict the bed deformation process numerically.

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APPENDIX – NOTATION

The following symbols are used in this paper:

D_L = diameter of L-particle;

k_M = hiding coefficient of M-particle ($\equiv u^*_{MC}/\tilde{u}^*$);

k_S = hiding coefficient of S-particle ($\equiv u^*_{SC}/\tilde{u}^*$);

L_M = thickness of M-particle layer;

R_{PS} = the volume ratio of S-particle to the total sum of M- and S-particles which fills the void space surrounded by L-particles;

w_{oS} = terminal settling velocity of S-particle;

$\tilde{u}^*$ = frictional velocity which is calculated from the shear stress exerted on the bed in static equilibrium state;

u^*_{SC} = critical frictional velocity of S-particle ($= w_{oS}$);

α = constant (= 0.4) in Eq. (1) and (3);

β_M, β_S = coefficient in Eq. (2) and (4);

Δ_{Mo} = a vertical distance from a top of L-particle to that of M-particle which was exposed on the bed surface in case of L-M experiment;

Δ_{M} = a vertical distance from a top of L-particle to the upper surface of M-particle layer in case of L-M-S experiment;

Δ_{So} = a vertical distance from a top of L-particle to that of S-particle which was exposed on the bed surface in case of L-S experiment;

τ_{M}^{*} = dimensionless tractive force evaluated for M-particle; and

τ_{MC}^{*} = critical value of τ_{M}^{*}.

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