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AN EXPERIMENT AND NUMERICAL SIMULATION OF BEDROCK-EROSION CAUSED BY BEDLOAD ON BEDROCK

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SYNOPSIS

Bed degradation has been increasing rapidly in the upper part of the Ishikari River, owing to exposed bedrock erosion caused by bedload on the bedrock. In order to estimate the erosion velocity of bedrock, we performed an experiment by using a circular flume with artificial bedrock made of gypsum. This study also proposes a numerical model which can simulate the bed deformation process of rivers with bedrock to reproduce the bedrock channels induced by bedload. We incorporated the erosion velocity equation of bedrock and a bedload layer model into the plane two-dimensional morphodynamic model. The computational results show that the proposed model can simulate the development of bedrock channels by a concentration of bedload, and channel patterns in the simulation are similar to the channels which are generally observed on bedrock in real rivers. It was found that bed degradation progresses by erosion which is caused by sediment transport itself.

INTRODUCTION

The recent changing of river morphology has caused significant problems from the viewpoint of river engineering. In particular, the exposure of bedrock is an issue in a lot of rivers in Japan, namely, in the upper part of Ishikari River shown in Photo 1, Yubari River and Toyohira River in Hokkaido. The river bed in these rivers was originally covered with the sand gravel. However, many works on rivers dramatically changed the

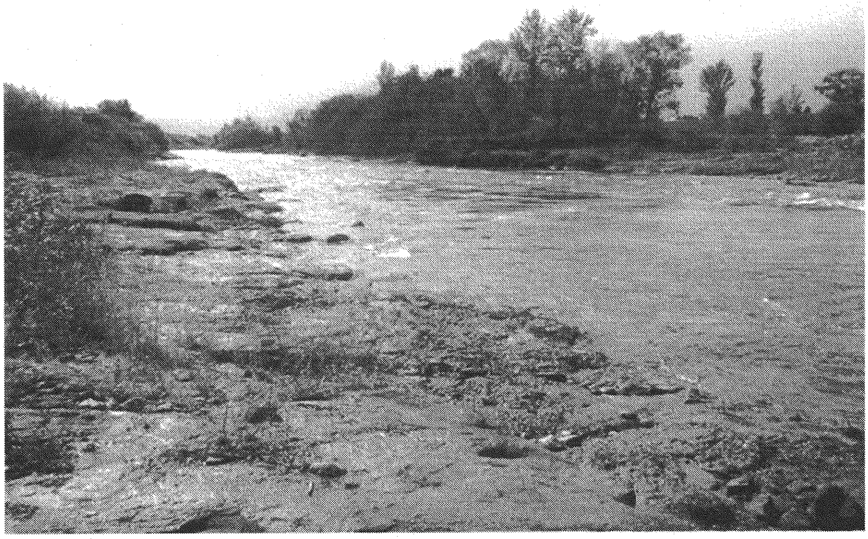

Photo 1 Exposed bedrocks in the upper part of Ishikari River.

river environments. For instance, dam construction and gravel extraction from rivers have changed the budget of sediment in river basins. The increase in bed shear stress on river beds associated with cutoff works of rivers has generated active sediment transport on bedrocks. As a result, the sand gravel flushed out from river bed, and the areas of exposed bedrock increased. Furthermore, significant bedrock degradation has been observed in these rivers, such as in the upper part of Ishikari River (KP164.58～157.28) by Asahikawa Development and Construction Department (1). Their observations have showed that bed degradation on bedrock progresses locally and rapidly in comparison with bedrock degradation in gravel bed rivers. This means that it may cause extensive damage to many river training structures and river environments. Therefore, morphodynamic phenomena on bedrock rivers, namely, the sediment transport and erosion mechanism, are important research topic to understand the bed degradation process of bedrock rivers and also to overcome such kinds of problems.

The bed evolution process on the bedrock is different mechanism from the process in gravel bed rivers. In gravel bed rivers, riverbed changes because of mass balance of sediment associated with sediment transport. Whereas, bed deformation of bedrocks occurs due to the erosion of bedrock induced by various hydraulic conditions, for instance, river flows, sediment transport rate and dry/wetting conditions. The erosion mechanism of bedrocks thus should be examined closely to understand clearly the evolution of bedrock channels and the bed degradation process on rivers which have exposed bedrocks.

It is known that highly cohesive bed or bedrocks can become eroded due to river flows or sediment transported by flow itself. Inoue et al. (1) conducted field experiments focusing on bedrock erosion by sediment transport over it, and investigated the key physical value, i.e., non-dimensional bed shear stress and bedload transport rate, for the erosion of bedrocks. The experimental results indicated that the erosion velocity of bedrocks is closely related to the bedload transport rate, and they derived a simple relation between the erosion of bedrocks and bedload transport rate. This erosion velocity equation of bedrock is a useful empirical formula to analyze the bed degradation of rivers with bedrocks, although a more detailed physical process related to the erosion of bedrocks due to sediment transport should be modeled. However, it is difficult to perform field experiments in all the rivers which have exposed bedrocks for estimating an erosion velocity of bedrocks from the viewpoint of environment and experimental efforts. Therefore, a simple experimental method should be proposed to examine the erosion of bedrocks and dependency of several physical values to the bedrock erosion

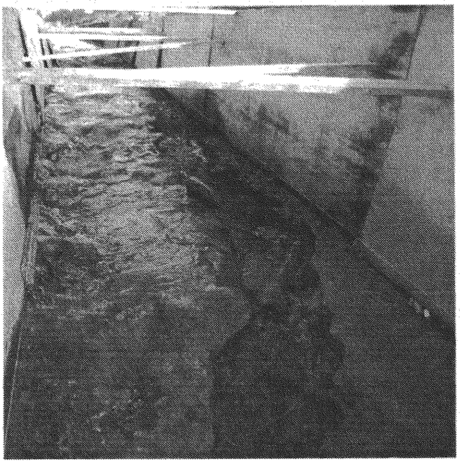

Photo 2 The bedrock channel observed in the field experiment conducted by Inoue et al. (1)

phenomena.

Moreover, there are a few numerical models which can deal with the bed evolution process in rivers with bedrocks. The various numerical models used to simulate river flows and bed deformations can be powerful tool for predicting changes in river beds. If we incorporate an erosion equation of bedrocks by using laboratory experiments and numerical models which can treat a bedrock evolution, it is useful to deal with the problems related to the degradation of river beds due to exposure of bedrocks to river flows.

As a first step for a general framework to examine bed degradation problems due to exposure of bedrocks to river flows, this study attempts to show the merits and the basic method for estimating the erosion velocity of bedrock by means of laboratory experiments, and also proposes a numerical model which can reproduce the bedrock channels which are typically observed on bedrocks in real bedrock rivers.

METHODOLOGY

Laboratory experiment

In this study, we propose a method for estimating the erosion velocity of bedrock by means of laboratory experiments. In order to understand the erosion mechanism and its rate of bedrock, Inoue et al. (1) conducted field experiments in the upper part of Ishikari River. These experiments revealed that the erosion velocity of bedrock is closely related to the product of the depth-averaged flow velocity and the bedload transport rate. Such field experiments can be useful to investigate the erosion velocity of bedrock in real scale rivers. However, it is difficult for us to investigate and show such relations in all real rivers which have problems related to the bedrock degradation because of the difficulty in setting up experiments and because of environmental damage caused by experiments. If we can predict such relations of the bedrock erosion by means of laboratory experiments, it is important to understand it from the viewpoint of costs and eco-friendliness. Moreover, laboratory experiments can change the details of experimental conditions, namely, flow velocity, sediment diameter and bedrock characteristics.

This study proposes an experimental method to determine the erosion velocity of bedrock by using a circular flume with artificial bedrocks made of gypsum, and discusses whether the proposed method can estimate the erosion velocity equation of bedrocks derived from real scale experiments conducted by Inoue et al. (1).

Numerical simulation

This paper also proposes a numerical model to simulate the bed deformation process of rivers with bedrocks. The bedrock erosion and bedrock channels observed in the field experiments shown in Photo 2 may have occurred because of the abrasion due to bedload transport. Thus, it is necessary to understand sediment transport on bedrocks for predicting the bed evolution process of rivers with bedrocks. However, this is usually unknown, since the sediment transport rate on bedrock is generally not equal to equilibrium sediment transport rate because of a lack of sediment on bedrocks and sediment supply from upstream of rivers. Furthermore, it is difficult to measure the movement of sediment on bedrocks in rivers, in particular during flooding time when active sediment transport can occur. This means that even if the erosion velocity of bedrock can be estimated by experiments, there are still cases where we cannot predict the bed degradation process and development of channels on bedrocks. Therefore, we need a model for calculating the bedload transport rate on bedrock to simulate bed degradation process and the expansion of the channels on bedrocks. Takebayashi et al. (2) proposed a two-dimensional morphodynamic model which can deal with the movement of sediment on non-erodible beds by using a bedload layer model. They applied this model to the bed deformation analysis of Tonle Sap River in Cambodia and discussed the bed evolution process and grain sorting in that river. This framework can be applied to the bed evolution process on bedrocks, which is the focus of this study, by coupling the erosion velocity equation of bedrocks estimated from the experiments.

This study incorporates the erosion velocity equation of bedrock obtained from the field experiments (1) and the bedload layer model (2) into the basic two-dimensional morphodynamic model (3) to simulate bed evolution process of rivers with bedrocks. The applicability of the proposed model to morphodynamic phenomena on bedrocks is explained by making a comparison with the experimental results by Inoue et al. (1).

LABORATORY EXPERIMENT

We conducted laboratory experiments to understand the erosion of bedrocks by using a circular flume with artificial bedrocks made of gypsum. The erosion velocity of bedrocks due to bedload transport was measured, and the relation between erosion velocity and flow velocity, sediment transport rate are discussed.

Experimental channel

In our experiments, we employed a circulation channel in width of 10cm and in outer bank radius of 1m shown in Photo 3. By using this channel, we need not supply the sand gravel continually and neglect the effects of boundary conditions of upstream and downstream end to the experimental results. Therefore, we can assume that all physical quantities, namely, flow velocity, sediment transport rate and erosion velocity of bedrocks, are equal in each cross section with respect to flow direction. The flow motion in this flume is generated by friction between the top board of the flume and the water surface induced by rotating of the top board of the flume. This means that the water surface slope in the transverse direction with respect to flow direction can be neglected. In addition, the flow speed can be adjusted by changing the rotation speed of the top board of the flume. We conducted eight experiments by changing the depth-averaged flow speed in range from 0 to 1.5 m/s.

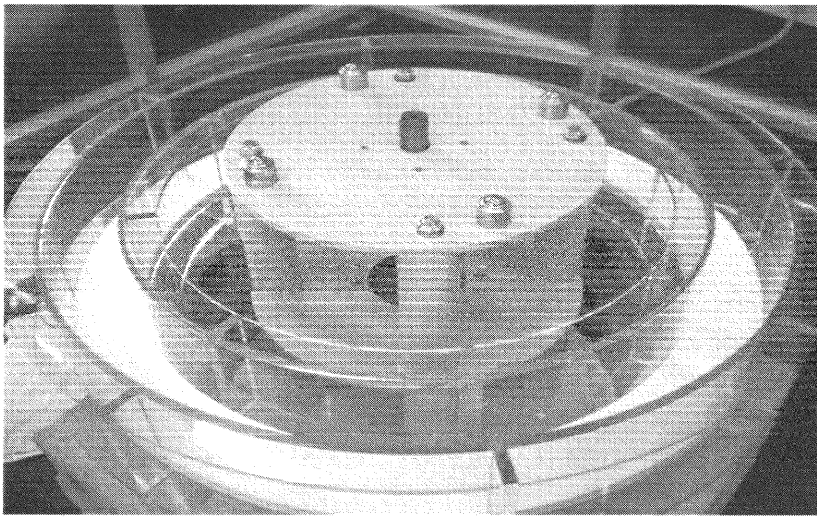

Photo 3 A circular channel without top board used in this experiment.

Bedrocks

In these experiments, the artificial bedrock made of gypsum was used as riverbed. Actually, the bedrocks which are exposed on the riverbed in the upper part of Ishikari River should be employed for the experiments, because this study discusses the possibility of the laboratory experiments to the estimation of the erosion velocity equation of its bedrock derived by field experiments by Inoue et al. (1). However, the type of bedrock in objective area is called "clay-slate" or "pelite", and these rocks have a weak feature to alternations between wet and dry conditions. If real bedrocks are used as bedrocks in the experiments, we should consider the erosion not only by salutation of bedload transport, but also by water flows. Moreover, bedload transport gradually increases due to sediment supply from the bedrocks due to bedrock erosion, and the sediment with different diameter is also added into the sediment over bedrock. This means that it is also difficult to keep the experimental conditions constant. Therefore, we used gypsum to make the artificial bedrock for experiments in order to overcome problems. Gypsum bed has advantages in that chemical erosion by water solubility, physical erosion by water flow, and slurry erosion by flaked gypsum flour itself can be negligible to the experimental results. Thus, we can focus on the erosion of bedrocks due to only sand gravel collision. This hypothesis is based on preliminary experiments in which we flow water and gypsum powder in experimental channel without feeding sand and cannot observe the erosion of bedrock in two hours experiment.

Inoue et al. (1) investigated the abrasion strength of bedrock in an actual river by conducting needle penetration tests. On the other hand, the present study employs the electric screwdriver penetration test to understand the abrasion strength of gypsum bed in experimental channel and to compare the strength of bedrock in Ishikari River and in experiments. In this test, a screwdriver was placed perpendicularly against the rock surface with five kilograms of weight to measure the penetration velocity of gypsum and bedrock, and to estimate the appropriate strength of gypsum to model the bedrock in the upper part of the Ishikari River. In these experiments, the intensity of gypsum can be adjusted by changing of water-gypsum ratio. Fig. 1 illustrates the penetration velocity of artificial bedrock made by gypsum with different water-gypsum ratio. From this relation, we can estimate that water-gypsum ratio for replicating the strength of bedrock in the upper Ishikari River is 0.789.

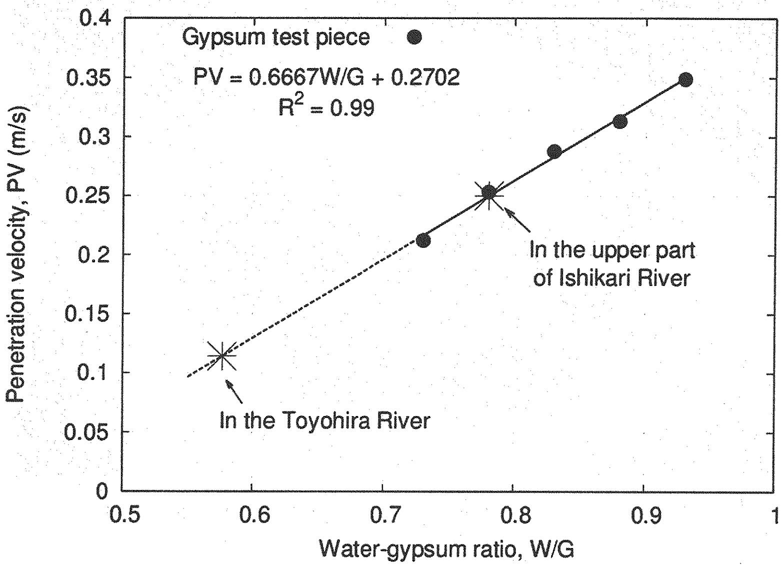

Fig. 1 The relation between penetration velocity and water-gypsum ratio.

Sand gravel

In the field experiments conducted by Inoue et al. (1), the ratio between the grain diameter and channel width is about one-tenth. Thus, these experiments employed gravel 5 and 10 mm in diameter by considering same grain diameter – channel width ratio in the field experiments. We chose gravel used in these experiments as near spherical shape. The density of sand gravel was 2.41 g/cm^3. In the experiments conducted by means of a circular channel, the sediment volume of feeding sand into the flume was not equal to bed load transport over the bedrock. From this reason, we randomly marked some sand particles in the feeding sands and counted the number of them that passed through the cross-section of channel to calculate the real bedload transport rate in experiments by using following equation:

$$q_{bs} = \pi\left(\frac{d^3}{6}\right)\left(\frac{100}{X}\right)\frac{N_x}{B\Delta t} \tag{1}$$

where, d denotes the grain diameter, Δt is the interval time to measure the sediment transport rate, N_x is the number of marking particle that passed through the cross-section surface of the channel during Δt, B is the width of channel, X is the percentage of the marking particles by the total sediment particles and q_{bs} is the bed load transport rate.

Depth-averaged flow velocity

In order to calculate the depth-averaged flow velocity in the circular flume, we put neutral particles together with sand gravel. By measuring the velocity of neutral particles obtained from the analysis of digital image in the experiments, we can determine the vertical velocity profile and the depth-averaged flow velocity.

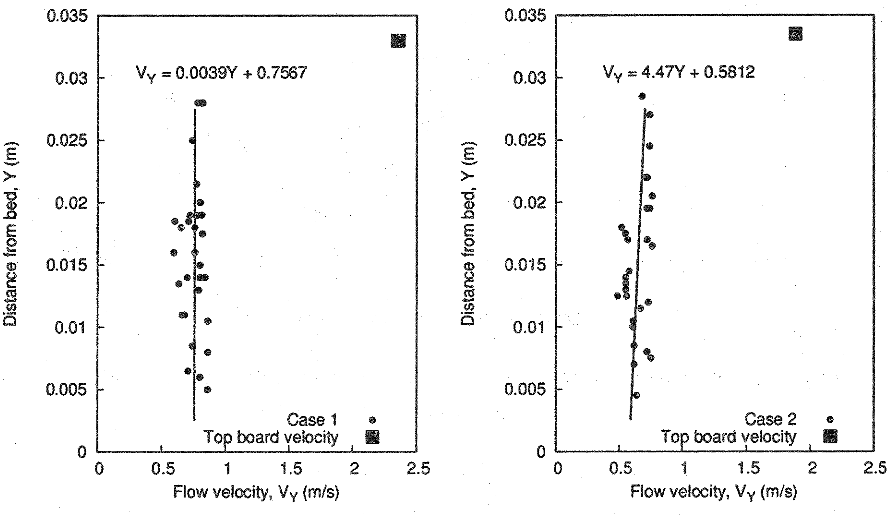

Fig. 2 The vertical velocity profile in the circular channel with gravels over the bedrocks. The velocity profile in vertical direction shows almost uniform.

Table 1 The experimental conditions and all results

	Case1	Case2	Case3	Case4	Case5	Case6	Case7	Case8
Sediment diameter (mm)	10	5	10	5	5	5	5	5
Number of sediment	330	2541	910	1313	3936	4936	6936	7936
Number of marking sediment	33	231	33	313	313	313	313	313
Bedload transport (m^2/h)	0.403	0.139	0.810	0.170	0.529	0.603	0.508	0.507
Depth-averaged flow velocity (m/s)	0.757	0.655	0.903	0.946	1.12	1.05	1.25	1.04
Test duration (h)	0.5	0.5	0.5	1.0	0.5	0.75	1.0	1.0
Mean erosion depth (cm)	-0.073	-0.018	-0.063	-0.019	-0.021	-0.032	-0.049	-0.042
Mean erosion velocity (mm/h)	1.46	0.35	1.26	0.19	0.43	0.42	0.49	0.42

In the experimental cases of feeding sand gravel, the velocity profile in a vertical direction becomes almost uniform as shown in Fig. 2. One of the main factors for this is agitation due to sand gravel. In the figure, the vertical velocity profiles over the bedrocks are averaged in a horizontal direction on the cross-section surface for each height from the bed.

Experimental results

Table 1 shows all the experimental conditions and the results of erosion velocity of artificial bedrock. In the experiments, we measured the cross-sectional bed profile after the removal of sediment at eight survey lines on the experimental channel with regular intervals. Each survey line has 18 points with a constant interval of 5mm, thus total measured point in an experiment is 144.

Fig. 3 demonstrates the one of cross sectional bed profiles measured in experiment Case7. This figure shows that the erosion of the bedrocks mainly occurs in the area from the channel centerline to the outer bank, while in the inner bank region, the bedrock is not eroded. This tendency was observed in all cross sections measuring the bed profile. This is caused by the abrasion due to the sediment transport over the bedrocks. During the experiments, in each case, we observed saltation of sand gravel over the bed. In addition, its plan distribution reveals a unique pattern, namely, the sediment which does not move by flow concentrates in the inner bank region, and active sediment transport occurred in the outer bank region. This suggests that the

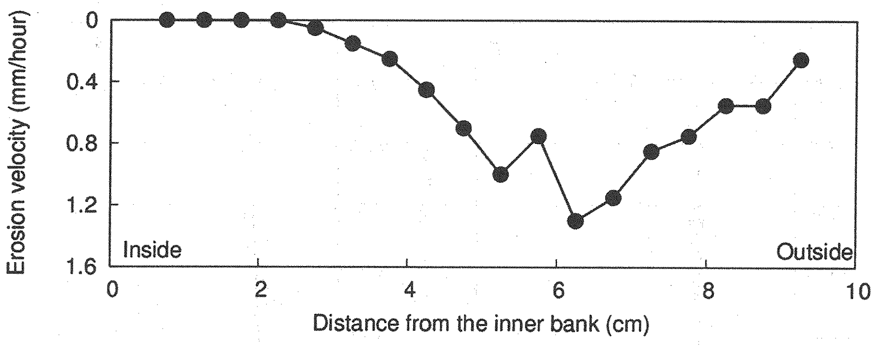

Fig. 3 The cross-sectional bedrock profile in Case7. The bed in outer bank region is mainly eroded, while no erosion can be seen in inner bank region.

sediment transport over the bedrock strongly affects the erosion of bedrock.

ESTIMATION OF EROSION VELOCITY EQUATION

This study discusses the possibility of estimation of erosion velocity equation given by field experiments (Inoue et al. (1)). We compared the relation between erosion velocity of bedrock, D, and the product of sediment transport rate, q_{bs}, and depth-averaged velocity, V, in present experiments and field experiments. Fig. 4 shows the experimental results and the erosion velocity equation derived by the field experiments. The erosion velocity by the sediment 10 mm in diameter is much larger than the results by using sediment 5mm in diameter, and these results indicate the grain size dependency for erosion velocity of bedrock. It is possible that the sediment 10mm in diameter creates a wider erosion channel on the bed compared cases where the sediment is 5 mm. However, as shown in Fig. 4, the linear relation between erosion velocity, D, and the product of sediment transport rate, q_{bs}, and depth-averaged velocity, V, could be found in the experimental results of 5 mm sediment case. Although the size dependency for erosion velocity could be observed in these experiments, we attempted to estimate the erosion velocity equation by using the results obtained from 5mm diameter cases.

Next, we corrected the bedload transport rate of these cases in order to compare both experimental results, because there were differences in measurement methods of sediment transport rate over the bedrock between the field and laboratory experiments. In the laboratory experiments, we measured the sediment transport rate over the bedrock by counting the amount of sediment through a cross-section of channel. On the other hand, in the field experiments, the bedload transport rate was counted by using a bucket. In this regard, the field experiments ignored the porosity of sand gravel in a bucket. By assuming the sediment used in the field experiments as completely spherical in shape, we can calculate the filling rate of the bucket as 74% at a maximum. Therefore, when we convert laboratory experimental results to one in the field experiments, we need to multiply the bedload transport by at least 100/74. As a result, the erosion velocity equation of the bedrock obtained by laboratory experiments can be written as follows:

$$D_{labor} = 9 \times 10^{-8} q_{bs} V + 0.0002 \tag{2}$$

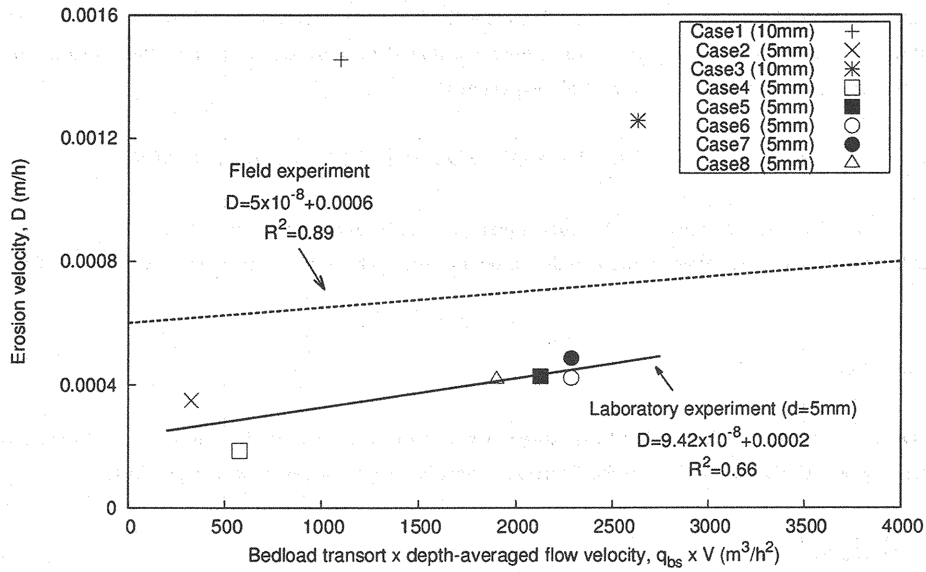

Fig. 4 Relation between erosion velocity and bed load transport rate times depth-averaged flow velocity.

$$\begin{aligned} D_{labor}' &= \frac{9\times10^{-8}}{100/74} q_{bs}V + 0.0002 \\ &= 6.66\times10^{-8} q_{bs}V + 0.0002 \end{aligned} \tag{3}$$

where, D_{labor}, D_{labor}' (unit : m/hour) denotes the erosion velocity of bedrock by means of laboratory experiments without and with the correlation of sediment transport rate, respectively. While, the erosion velocity of the bedrock by the field experiments, D_{field} (unit: m/hour) can be obtained as follows (Inoue et al. (1)).

$$D_{field} = 5\times10^{-8} q_{bs}V + 0.0006 \tag{4}$$

Because of the correction of sediment transport rate, both equations, D_{field} and D_{labor}', have the similar gradient. Although the experimental results also indicate the dependency of sediment size for erosion velocity equation of bedrocks, this experimental method could model the simple relation between bedrock erosion and sediment transport over bedrock derived by Inoue et al. (1). This result may suggest that the other dependency for bedrock erosion phenomena such as the strength of bedrock as well as more detailed physical mechanics between the sediment transport and bedrock erosion can be investigated by means of this experimental method.

Furthermore, we discuss the applicable scope of this erosion velocity equation. These experimental results assume that the all sediment can be moved and that the density of sediment over bed is small enough to attack the bed. This means that the derived erosion velocity equation cannot be applied under the hydrodynamic conditions where the bed shear stress is less than the critical bed shear stress or under conditions where sediment transport rate reaches near the equilibrium condition.

The intercept of this equation means the erosion velocity of bedrock by water flows. As previously mentioned, laboratory experiments can ignore the effect of erosion by water flows. Thus, the intercept in the erosion velocity equation obtained by laboratory experiments should be zero. However, bedrocks existing in actual rivers can become eroded not only by sediment transport, but also river flows over bedrocks. From this reason, the erosion velocity equation derived from the field experiments conducted by Inoue et al. (1) has an intercept as 0.33 mm/hour. Therefore, in the future we intend to work on estimating the erosion velocity of bedrock by water flow without conducting field experiments.

NUMERICAL SIMULATION OF BED EVOLUTION WITH BEDROCKS

In this section, we describe a two-dimensional morphodynamic model which considers the erosion of bedrocks by using a bedload layer model and discusses the applicability of model to bed evolution of river with bedrock.

Numerical model

A two-dimensional shallow water flow equation was utilized for simulating the flow field over river bed. The governing equations for the flow in the Cartesian coordinate system can be written as follows:

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (vh)}{\partial y}=0 \quad (5)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial H}{\partial x}-\frac{\tau_x}{\rho h}+\frac{\partial}{\partial x}\left(\nu_t\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_t\frac{\partial u}{\partial y}\right) \quad (6)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial H}{\partial y}-\frac{\tau_y}{\rho h}+\frac{\partial}{\partial x}\left(\nu_t\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_t\frac{\partial v}{\partial y}\right) \quad (7)$$

where, t denotes the time, x, y are the Cartesian coordinates, u, v are the depth-averaged flow velocity in x, y direction, respectively, h is the water depth, H is the water level, g is the gravitational acceleration (=9.8m/s^2) and τ_x, τ_y are the bed shear stress in x, y direction, respectively. The bed shear stress vectors were evaluated by using one of mean flow velocity formula, such as Manning's equation, as follows:

$$\tau_x=\frac{\rho g n^2 uV}{h^{1/3}},\quad \tau_y=\frac{\rho g n^2 vV}{h^{1/3}},\quad V=\sqrt{u^2+v^2} \quad (8)$$

where, ρ denotes the water density, V is the magnitude of depth-averaged velocity and n is the Manning's roughness coefficient. We used the zero-equation turbulence closure model for evaluating the eddy viscosity coefficient, ν_t, in flow model as follows:

$$\nu_t=\alpha u_* h \quad (9)$$

where, u_* denotes the shear velocity and α is the model constant for zero-equation turbulence model, which is given as 0.2 (Kimura et al. (4)). The advection terms in momentum equations were solved by using the CIP method which is known as high order scheme (Yabe and Ishikawa (5)).

In the natural bedrock rivers focused on this paper, we were able to observe bedrocks coexisting with sand covered bed. Thus, we have to consider the bed deformation on both sand bed and bedrocks. In addition, sediment transport rate over bedrocks is usually less than the equilibrium sediment transport rate. Therefore, the equilibrium sediment transport formula which is used to sand beds cannot be applied for estimating the sediment transport rate on bedrocks.

We adopted the bedload layer model which was used in the framework constructed by Takebayashi et al. (2) for simulating the bed deformation of rivers with a non-erodible bed. This model can capture the movement of sediment transport over the non-erodible bed by defining the bedload layer over river bed. The continuity equation of bed which considers only bedload transport can be written as follows:

$$\frac{\partial(c_b E_b)}{\partial t}+(1-\lambda)\frac{\partial z_b}{\partial t}+\frac{\partial q_{bx}}{\partial x}+\frac{\partial q_{by}}{\partial y}=0 \tag{10}$$

where, c_b denotes the sediment concentration in the bedload layer, E_b is the thickness of bedload layer, z_b represents the bed elevation, q_{bx}, q_{by} are the bedload transport rate in x, y direction, respectively and λ is the porosity of bed. The thickness of bedload layer was calculated by following equation:

$$E_b=\begin{cases} E_{be} & \cdots \quad E_{sd} > E_{be}\dfrac{c_b}{1-\lambda} \qquad (11) \\ E_{sd}\dfrac{1-\lambda}{c_b} & \cdots \quad E_{sd} < E_{be}\dfrac{c_b}{1-\lambda} \qquad (12) \end{cases}$$

where, E_{sd} denotes the thickness of sand over the bedrock and E_{be} is the thickness of bedload layer in equilibrium state, which can be obtained as follows (Egashira and Ashida (6)):

$$\frac{E_{be}}{d}=\frac{\tau_*}{c_b\cos\theta(\tan\varphi-\tan\theta)} \tag{13}$$

where, d is the sediment diameter, τ_* is the non-dimensional bed shear stress, θ is the angle of local bed elevation and φ is the angle of repose of sediment.

Equation (11) shows the existence of enough sand layer over bedrock for transporting the equilibrium bedload transport rate, while equation (12) expresses the state of exposed bedrocks. In case of exposed bedrocks, the continuity of sediment can be calculated from equation (10) by neglecting the second term of left hand side because of no sediment supply from the bedrocks.

When the bed is covered with enough sand, changes in the bed elevation can be calculated by time integration of second term on left hand side of equation (10). However, bed degradation of bedrocks cannot be calculated from the continuity of sediment expressed in equation (10). This model assumes that the erosion of bedrocks was caused by abrasion due to the bedload. The elevation of bedrock was updated as follows:

$$\frac{\partial z_b}{\partial t}+D=0 \tag{14}$$

where, D denotes the erosion velocity of bedrock. We adopted equation (4) as an erosion velocity equation derived from field experiments in the upper part of Ishikari River conducted by Inoue et al. (1).

The bedload transport rate was given by Ashida and Michiue's formula with the coefficient which expresses the existence of sediment in the bedload layer as follows:

$$\frac{q_{bs}}{\sqrt{sgd^3}} = 17\tau_*^{1.5}\left(1-\frac{\tau_{*c}}{\tau_*}\right)\left(1-\sqrt{\frac{\tau_{*c}}{\tau_*}}\right)r_b \quad (15)$$

where, s is the specific weight of sediment in a fluid, τ_{*c} is the non-dimensional critical bed shear stress and r_b is the coefficient which expresses the decreasing of bedload transport by exposing bedrocks. r_b was calculated by the ratio of the thickness of bedload layer between in equilibrium state and in calculation from equation (10) as follows:

$$r_b = \begin{cases} 1 & \cdots \quad E_{sd} > E_{be}\dfrac{c_b}{1-\lambda} \quad (16) \\ \dfrac{E_b}{E_{be}} & \cdots \quad E_{sd} < E_{be}\dfrac{c_b}{1-\lambda} \quad (17) \end{cases}$$

By calculating the total bedload transport rate and the depth-averaged flow velocity, we can update the elevation of bedrock by using equation (14).

Computational conditions

A proposed model was applied to the experiments conducted by Inoue et al. (1) for reproducing the bedrock channels which are typical river morphology on bedrock shown in Photo 2. However, it is difficult to prove the effectiveness of a numerical model by making a comparison with the qualitative experimental results, namely, channel number, channel width and depth because of lacking of experimental data focused on the bedrock channels. Thus, we focus only on the channel geometry on bedrock and the erosion process of it.

The Case7 in the field experiments conducted by Inoue et al. (1) was employed for computations. The channel is the straight and is 30m length and in width of 1m with the constant slope of 1/30. The Manning's roughness coefficient is 0.035. The input condition for discharge is $0.4\text{m}^3/\text{s}$ and for sediment supply is $0.000833\text{m}^3/\text{s}$. The sediment used in these experiments is almost uniform, and its diameter is 28.4mm. All the computational conditions were set to the same conditions of experiments with the exception of the channel length. We used a 90m channel for calculation. The grid number in longitudinal and lateral direction is 300 and 20, respectively. This model neglected the changing of bed roughness by the existing sediment on bedrocks, thus the bed roughness parameter which is expressed by Manning's roughness coefficient for calculation is constant in both space and time. Moreover, random perturbations in the range of $\pm$ 0.0125m were added to the initial flat bed as a trigger of channel incision.

Computational results

Fig. 5 shows the deviation of bedrock from the initial bed after one and three hours, respectively. In addition, Fig. 6 shows the bedload transport rate divided by input sediment transport rate in unit width after 300 seconds and three hours, respectively. The computational results indicate that the bedrock channels develop because of the concentration of the bedload transport. First, we discuss the plan distribution of sediment transport over bedrocks at early stage of computation. From Fig. 6, we can deduce that the feeding sediment from upstream end concentrates on some lines. This is the result of the effect of local slope of bedrocks associated with random perturbations added in initial bed for sediment movement. By concentration of

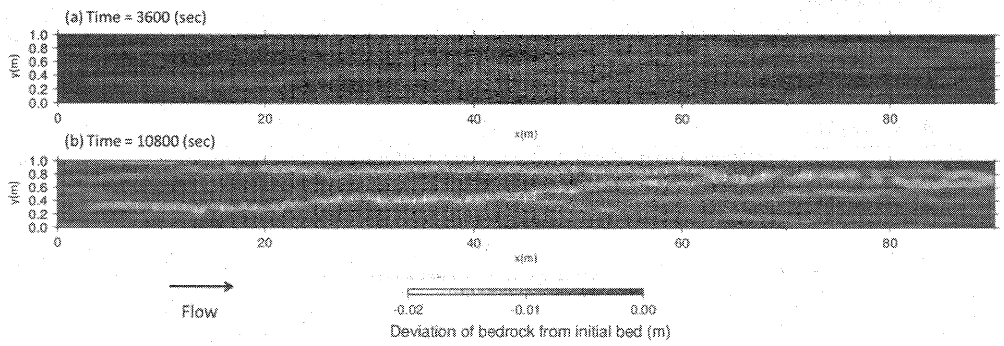

Fig. 5 The numerical results of deviation of bedrocks from initial bed after (a) an hour and (b) three hours, respectively. The flow direction is from left to right. After three hours, computational results show that clear bedrock channels develop on the bedrock.

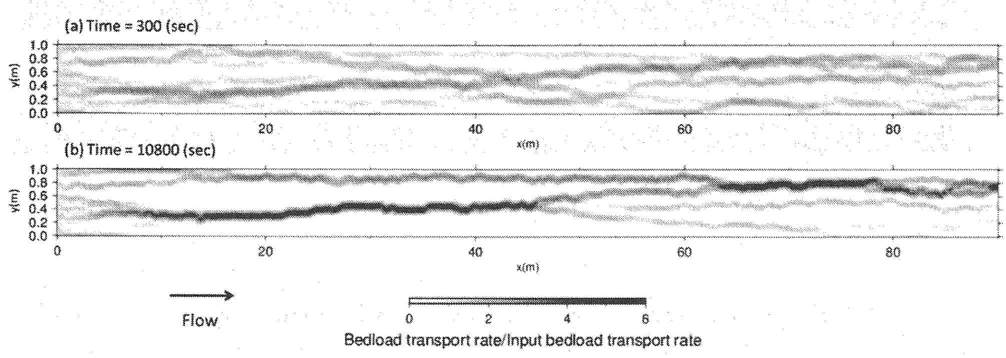

Fig. 6 The numerical results of the bedload transport rate divided by input sediment transport rate after (a) 300 seconds and (b) three hours, respectively. The flow direction is from left to right. The sediment mainly transport into the bedrock channels developed by sediment transport itself.

sediment on bedrock, it is locally eroded, thus bedrock channels develop as deep channels. Fig. 5 suggests that the depth of bedrock channels becomes 1 cm after an hour. By developing the bedrock channels, the sediment on the bedrock transports only into the bedrock channels developed by sediment transport itself. Fig. 7 displays the plan distribution of r_b, which is coefficient of decreasing of bedload transport by exposing bedrock. The distribution of r_b in early stage in Figure-7(a) shows a similar trend with distribution of bedload transport, and by concentration of bedload, r_b also becomes high into the bedrock channels.

After three hours of computation, similar bedrock channels shown in a real bedrock river are developed by that feedback process, and the input sediment almost transports into bedrock channels shown in Fig. 6. While, distribution of non-dimensional bed shear stress shown in Fig. 8 does not change by development of bedrock channels. Because under this condition the depth of the bedrock channel can be neglected with respect to the water depth. This suggests that the positive feedback between the development of bedrock channel and concentration of bedload is one of important mechanism describing the bed evolution process with bedrocks. However, Fig. 7(b) indicates that because of the concentration of bedload into the bedrock channels, r_b reaches

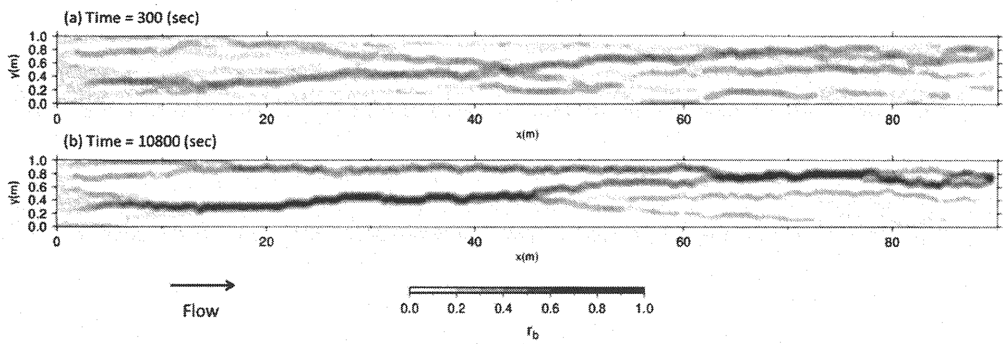

Fig. 7 The plan distribution of the coefficient, r_b, which express the decreasing of bedlaod transport by exposing bedrock (a) after 300 seconds and (b) three hours.

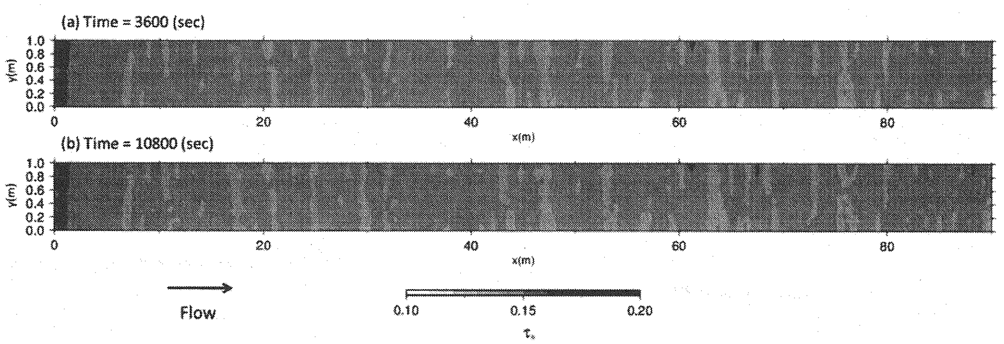

Fig. 8 The plan distribution of the non-dimensional bed shear stress, τ_*, (a) after an hour and (b) three hours.

near 1, which is the equilibrium situation for bedload transport, in the bedrock channels. This means that the bedrocks can be covered with sediment by a high concentration of bedload into bedrocks, and under this condition, the erosion of bedrocks did not occur. Therefore, such negative feedback is also a fundamental mechanism of the relationship between sediment transport and erosion in bedrock channels.

By incorporating the bedload layer model and the erosion velocity equation of bedrocks into a basic two-dimensional morphodynamic model, we can simulate bedrock channels which are typically observed in real bedrock rivers. However, the present numerical result underestimates the depth of bedrock channels observed in field experiment (1). The main reason may be that the bedload transport formula used in this model underestimates the bedload transport over bedrocks. As shown in equation (4), the erosion of bedrock is strongly influenced the bedload transport. Thus, the accuracy of bedload transport rate has dominant effect on the numerical result. The experimental result focusing on sediment transport processes over fixed bed or bedrocks revealed that sediment transport rate which is much larger than equilibrium sediment transport rate on sand bed can occur on the bedrocks (Watanabe et al. (7), Kyuka et al. (8)). In addition, a number of bedrock channels could not reproduced in numerical simulation. In the numerical model, two or three channels can be observed, while one main bedrock channel was developed in the field experiment. The amount of channels affects the concentration of sediment, and fewer channels can gather more sediment into the bedrock channels.

For these reasons, in particular the sediment transport rate over bedrock should be improved in this model to capture highly local erosion by bedload transport on bedrocks.

CONCLUSIONS

In this study, we investigated an experimental method to estimate a simple relation between bedrock erosion rate and sediment transport rate by using circular laboratory flume. In addition, a numerical model which can deal with the bed evolution process with bedrocks was proposed by incorporating the erosion velocity equation of bedrock and the bedload layer model into a classical plane two-dimensional morphodynamic model. The main conclusions of this paper are summarized as follows.

1. The proposed experimental method can model a simple relation between the bedrock erosion rate and the sediment transport rate derived by Inoue et al. (1). This suggests that the other dependency for bedrock erosion phenomena such as strength of bedrock as well as a more detailed physical mechanism between the sediment transport and bedrock erosion can be investigated by using this experimental method.
2. The initiation and evolution of bedrock channels on bedrocks can be reproduced by using our proposed numerical model. Positive feedback between the evolution of bedrock channels and the concentration of sediment transport is one of important mechanisms describing the bed evolution process with bedrocks.
3. The sediment transport rate over bedrocks should be improved in this model to express the erosion of bedrocks and the concentration of sediment transport into the bedrocks.

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APPENDIX-NOTATION

The following symbols are used in this paper.

B = the width of channel;
c_b = the sediment concentration in bedload layer;
d = the sediment diameter;
D = the erosion velocity of bedrock;
D_{labor}, D_{labor}', D_{field} = the erosion velocity of bedrock by laboratory experiments without and with the correlation of sediment transport rate and by the field experiments;
E_b = the thickness of bedload layer;
E_{be} = the thickness of bedload layer in equilibrium state;
E_{sd} = the thickness of sand over the bedrock;
g = the gravitational acceleration;
h = the water depth;
H = the water level;
n = Manning's roughness coefficient;
N_x = the number of marking particle that pass through the cross-section surface of the channel during Δt;
PV = the penetration velocity;
q_{bs} = the total bedload transport rate;
q_{bx} = the bedload transport rate in x direction;
q_{by} = the bedload transport rate in y direction;
r_b = the coefficient which expresses the decreasing of bedload transport by exposed bedrock;
s = the specific weight of sediment in fluid;
t = the time;
u, v = the depth-averaged flow velocity in x, y direction;
V = the depth-averaged flow velocity;
V_Y = the flow velocity measured in experiment at Y;
W/G = the water-gypsum ratio;
x, y = Cartesian coordinate;
X = the percentage of the marking particles by total sediment particles;
Y = the distance from bed of experimental channel;
z_b = the bed elevation;
α = the model constant for zero-equation turbulence model;
Δt = the interval time to measure the sediment transport rate;
θ = the angle of local bed elevation;
λ = the porosity of bed;
ν_t = the eddy viscosity coefficient;
ρ = the water density;
τ_x = the bed shear stress in x direction;
τ_y = the bed shear stress in y direction;
τ_* = the non-dimensional bed shear stress;
τ_{*c} = the non-dimensional critical bed shear stress; and
φ = the angle of repose.

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