

DEVELOPMENT OF RESERVOIR CONTROL OPTIMIZATION SIMULATOR BY INTEGRATING
A DISTRIBUTED-RAINFALL-RUNOFF-MODEL AND DYNAMIC PROGRAMMING

By

Takahiro SAYAMA

ICHARM, Public Works Research Institute, Japan

Yasuto TACHIKAWA

Graduate School of Engineering, Kyoto University, Japan

Hiroki KANNO

Nippon Steel Corporation, Japan

Kaoru TAKARA

Disaster Prevention Research Institute, Kyoto University, Japan

SYNOPSIS

A reservoir control optimization simulator is developed by integrating a distributed rainfall-runoff model and dynamic programming. The main objective of the system is to find effective reservoir operation rules that can reduce peak discharges even when an excessive flood event occurs. As this system uses a distributed rainfall-runoff model together with optimization algorithms for multiple reservoirs, it can provide optimized reservoir operation patterns under specific rainfall distributions with various objective functions assigned at multiple locations along river channels. The developed system is applied to the Kizugawa River basin with five major dam reservoirs. The optimization results suggest that changing the flood control only at Takayama Dam significantly reduces peak discharges of 200-year excessive flood events, whereas the current operation rules are almost ideal for other four dam reservoirs even for the excessive flood events.

INTRODUCTION

In recent years, much effort has been made to understand effective dam reservoir operations for extreme flood events. For example, Toya et al. (1) proposed a method to decide effective pre-release dam

operations based on catchment characteristics. In terms of optimization of dam operation and allocation, many studies have been conducted including integrated dam operation with multiple reservoirs. For example, Takeuchi (2) proposed a method to find an optimal reservoir control through a combination of dynamic and linear programming. Takasao et al. (3) addressed issues on optimal dam allocation and size. They used also dynamic programming to propose a solution for successfully balancing flood control and water use. On the other hand, real time optimal reservoir control was also challenged by many researchers, but it has been difficult to put into practice due to short flood arrival time of Japanese rivers, despite recent progress in hydrological observation. Therefore, the authors share the view that even in the future, it will still remain too difficult to optimize and execute complicated reservoir control on the real time basis under an extremely pressing situation during flooding. Our proposal here is to prepare emergency reservoir operation rules in advance and apply them for emergency situations as a feasible way of coping with excessive floods.

In order to investigate emergency dam operation rules, a close examination of several factors is necessary including progress in structural development in the upper reach of the river, spatial and temporal distribution characteristics of rainfall and runoff, and structural conditions of existing dams. Furthermore, it is important to identify reservoir control procedures to meet diversified flood-control purposes, including ones to protect an entire basin or to prevent devastating damage in specific places. In previously proposed optimization methods for reservoir control, a simulation was usually conducted with a simple function of rainfall-runoff and river routing. As a result, it was difficult to use discharge at an arbitrary point as a variable of an objective function or to reflect the influence of inflow from tributaries. That, in turn, led to difficulty in discussing dam operations in a complicated system. In this study, by combining dynamic programming and a distributed rainfall-runoff model developed for the Yodogawa River system (4) (5), we aimed to develop a reservoir control optimization simulator to help find an optimal operation based on different rainfall patterns and objective functions. The simulator was then applied to the Kizugawa River basin in the Yodogawa River system to discuss an optimal operation that can contribute to effective flood control during excessive flood events.

RESERVOIR CONTROL OPTIMIZATION SIMULATOR

Input and output information for the simulator

Input information for the Reservoir Control Optimization Simulator (ResCOS) includes rainfall for a distributed rainfall-runoff model, objective functions and constrained conditions used for dynamic programming. The rainfall is a design rainfall that is based on observed rainfall spatiotemporal distributions. The objective functions are defined using river discharges at a single point or multiple points. Since a distributed rainfall-runoff model is used to estimate river discharge, target points can be arbitrarily selected along the river channel. The simulator is designed to set conditions freely, including the lowest and highest water levels and discharges. As output information, ResCOS provides dam and river discharges as the result of optimal reservoir control. In addition, the simulator can output the dam storage volumes and water levels since they are used as simulation variables.

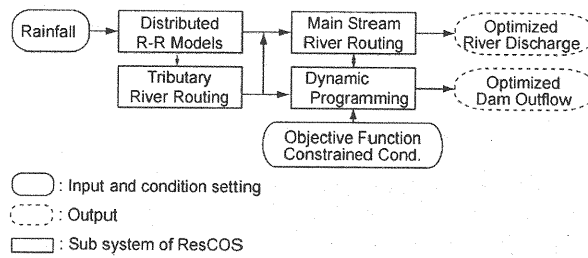


Fig. 1 Components and structure of the Reservoir Control Optimization Simulator (ResCOS)

System components

ResCOS consists of four sub-systems, which are illustrated as boxes in Fig. 1. The simulation is performed as follows: first, lateral inflows to tributaries are calculated by using distributed rainfall-runoff models applied for sub-catchments. Then discharges in the tributaries are calculated by river routing models. In this study, the main channel was defined as the rivers under the influence of the reservoir control and tributaries as those under no influence of the reservoir control. Since the lateral inflow to the main channel and tributary discharge are subject to no influence of the reservoir control, it is not necessary to repeat the calculations during the optimization process. On the other hand, the main channel requires repeating the calculations during the optimization process.

Distributed rainfall-runoff model

The sub-catchment rainfall-runoff models are developed based on a digital elevation model with 250 m resolution. A stage-discharge relationship simulating saturated, unsaturated subsurface and surface rainfall-runoff (Tachikawa et al. (6)) is applied to each grid-cell. Figure 2(a) shows a schematic diagram of the soil layer of the model. In this figure, the soil depth is D , the water depth corresponding to the water content is d_s , and the water depth corresponding to maximum water content in the capillary pore is d_c . Figure 2 (b) shows the stage-discharge relationship. Let k_c and k_a be saturated hydraulic conductivities in the capillary pore and in the non-capillary pore, respectively, and $v_c = k_c i$, $v_a = k_a i$ (i : slope), then the relationship between the discharge per unit width q_{slo} and the water depth h is described as follows:

$$q_{slo} = \begin{cases} v_c d_c \left(\frac{h}{d_c} \right)^\beta, & (0 \leq h \leq d_c) \\ v_c d_c + v_a (h - d_c), & (d_c < h \leq d_s) \\ v_c d_c + v_a (h - d_c) + \alpha (h - d_s)^m, & (d_s < h) \end{cases} \quad (1)$$

where, $\alpha = i^{1/2}/n$; n = Manning roughness coefficient. A non dimensional parameter β is introduced to

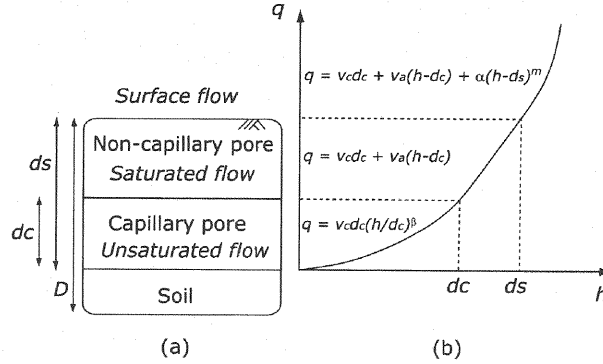


Fig. 2 Schematic diagram of (a) the soil layer and (b) the stage-discharge relationship of saturated-unsaturated subsurface and surface runoff model

describe the reduction of hydraulic conductivity in the capillary pore as the water content decreases. $\beta = k_a/k_c$ so as to keep the continuity of the depth-discharge relationship between the capillary pore and the non-capillary pore layers. Combining this depth-discharge relationship and the continuity equation, we can simulate rainfall-runoff from each grid-cell. The water flow is routed until it reaches a river segment. Model parameters are calibrated against typhoon flood events in 1982 and 1990. See Sayama et al. (4) (5) for the detailed model calibration and its performance.

Formulation of reservoir control optimization by dynamic programming

(a) Definition of variables

DP variables are defined as follows: discharge vector \mathbf{O}_t is the N -dimensional decision vector representing outflow from each dam ($n=1, \dots, N$). The subscript t represents the outflow during a time period between $t-1$ and t . N is the number of dams, and T is the discretized entire control period. \mathbf{S}_t is the N -dimensional storage volume vector. Vector \mathbf{q}_t is the combination of lateral inflow to main channels and tributary discharges. This vector is called environmental variable vector because it is independent from decision vector. \mathbf{Q}_t represents discharges in the main channel and it is called the synthetic variables vector because it is dependent on both decision and environmental variable vectors.

(b) Formulation of optimization

The optimization problem of ResCOS is stated as follows; "It optimizes the decision variable \mathbf{O}_t that minimizes the objective function value $J(\mathbf{Q}_t)$ with initial condition of $\mathbf{S}_0 = \mathbf{S}^0$ under constraint conditions of $g(\mathbf{S}_t, \mathbf{O}_t) \leq 0$ ". The dam storage volume also has to satisfy a continuity equation $\mathbf{S}_t = \mathbf{S}_{t-1} + \mathbf{I}_t \Delta t - \mathbf{O}_t \Delta t$. In addition, the river discharges have to satisfy $\mathbf{Q}_t = F(\mathbf{Q}_{t-1}, \mathbf{Q}_t, \mathbf{q}_t)$ based on flood routing models. Here, \mathbf{I}_t is the average inflow that is calculated from \mathbf{q}_t and \mathbf{Q}_t .

(c) Principle of optimality

As mentioned above, ResCOS optimizes reservoir operations using DP, which was originally developed by R. Bellman (7). The key feature of DP is to separate the entire optimization problem into sub-problems based on the principle of optimality defined as below and to provide an optimal solution for each sub-problem to reach the entire solution. Because of this feature, DP requires far less computational time and memory compared with other optimization techniques designed to obtain a solution at once.

The principle of optimality is defined as follows: "Optimal policy is characterized by the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy for the state resulting from the first decision."

The principle of optimization can be mathematically expressed as follows. First, $f_t(S_t)$ is defined as the value of objective function resulting from optimal policy taken at each decision stage from one to t . Based on the principle of optimization defined above, the relation between f_{t-1} and f_t can be described as follows:

$$\begin{aligned} f_t(S_t) &= \min_{\{Q_t\}} \{D(Q_t) + f_{t-1}(S_{t-1})\} \\ &= \min_{\{Q_t\}} \{D(Q_t) + f_{t-1}(S_t - I_t \Delta t + O_t \Delta t)\} \end{aligned} \quad (2)$$

where, $D(Q_t)$ = evaluation function at time t . The value of objective function J is expressed as the summation of the evaluation function as follows:

$$J = \sum_{t=1}^T \{D(Q_t)\} \quad (3)$$

Note that to solve the recursive formula (Eq. 2) requires the initial value of objective function $f_1(S_1)$ defined below as:

$$f_1(S_1) = D(Q_1) \quad (4)$$

(d) The solution of Dynamic Programming

In the DP solution, "Forward Process" determines the values of objective function f_t up to the last stage T based on Eqs. 2 and 4. Figure 3 shows a conceptual diagram of the Forward Process during the time period between $t-1$ and t . The circles in the figure correspond to the storage volume at a specific time during that period. For instance, if the maximum and minimum storage volumes of each dam are discretized into 50 sections, the number of the circles at each specific time will be 50 if only one dam is considered, and the number of the circles becomes $M = 50^N$ when the number of the dams are N .

To determine $f_t(S_t^j)$, the evaluation function $D(Q_t)$ must be calculated from all different stages S_{t-1}^j ($j=1, \dots, M$) to S_t^j , where the superscript of j represents the discretized state from 1 to M . Then it finds S_{t-1}^j that minimizes the function of $\{D(Q_t) + f_{t-1}(S_t - I_t \Delta t + O_t \Delta t)\}$. For each state S_t^j at time t , the value of objective function $f_t(S_t^j)$ and the corresponding state vector S_{t-1}^j are stored in computer memory. The calculation moves on to the next time step after calculating all the state conditions

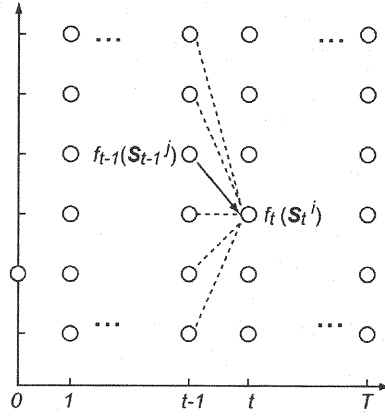


Fig. 3 Forward Process of Dynamic Programming (DP) optimization algorithm

($i=1, \dots, M$). The Forward Process comes to an end when the calculation is repeated up to time T .

When all $f_t(S_t^i)$ are determined, the storage volume vector at the final stage is now defined as S_T^{opt} , to which $f_t(S_t^i)$ is given. Then, the state vector at the time step right before S_T^{opt} is defined as $S_{T-1}^{opt} = S_{T-1}$. Repeating this procedure while going reverse in time, the series of storage volume vectors $S_t^{opt}(t=1, \dots, T)$ is obtained. This procedure is called the "Backward Process" of DP. All the optimization process is completed after the Forward Process and the Backward Process.

(e) Reduction of computational load with DDDP

One of the disadvantages of DP is a dramatic increase in computational load as N increases (so called "curse of dimensionality"). The required memory increases proportionally to the M th power of the discretizations. For instance, when optimization is attempted at a group of five dams by dividing each dam state into 100 storage volume sections, the calculation will require a storage capacity of an order of 10^{10} . This kind of number is still very difficult to cope with even for computers currently in use, and therefore some kind of approximation is necessary. Takasao et al. (3) found that the Discrete Differential Dynamic Programming (DDDP) (8) can be an effective approach to solve this computational problem of DP for reservoir control optimization problems. Thus the developed system in this study also employs DDDP.

Figure 4 shows the schematic diagram of DDDP. Each circle in the figure represents the state variable vector at a specific time. The main difference between the original DP and DDDP is that DDDP introduces the concept of a trial trajectory, an optimal trajectory and corridors explained below.

The DDDP algorithm works as follows. First, an initial trajectory that satisfies constrained conditions is defined as a trial trajectory. Then DP is executed to obtain an optimal trajectory around the neighboring corridors. The obtained optimal trajectory is treated as another trial trajectory, which in turn is used to obtain the next optimal trajectory. Repeating this procedure eventually leads to the final optimal trajectory. In this procedure, the corridor refers to the state variable vectors obtained from one unit larger and one unit smaller at each dam reservoir. Note that the number of the state variable vector (M') explored in DDDP is $M' = 3^N$ when the number of dams is three, although Fig. 4 simply shows three

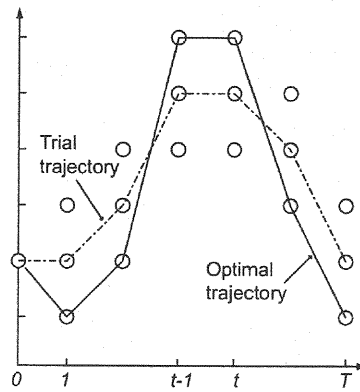


Fig. 4 Schematic diagram of Discrete Differential Dynamic Programming (DDDP)

circles at each stage as trial trajectory. DDDP requires a convergence test since it repeats calculation. In this study, however, the calculation is continued until a trial trajectory matched the optimal trajectory.

Algorithm of ResCOS

The optimization algorithm used in ResCOS is summarized as follows:

- 1) Execute rainfall-runoff models applied for sub-catchments and river routing models applied for tributaries to calculate $q_i (i = 1, \dots, T)$, which is a discharge vector that is not affected by dam control.
- 2) Initialize the main stream discharge vector as Q_0 .
- 3) Initialize a trial trajectory of the state variable vector. In this study, it was defined as the lowest water levels for all the dam reservoirs.

Steps 1)-3) are the initialization procedures. The subsequent steps 4)-6) are the Forward Process to proceed from $t-1$ to t .

- 4) Discharge vector O_t is calculated for the state variable shift from S_{t-1}^j to S_t^i . If there are reservoirs arranged in series, O_t must be calculated after river routing calculations for the main streams. As for the river routing calculations, the Muskingum-Cunge method is used in this study.
- 5) The evaluation function value $D(Q_t)$ is obtained based on river routing calculations from the state variable S_{t-1}^j to S_t^i . Note that, the initial condition at $t-1$ is necessary for running the river routing model. In this study, the initial condition is determined as $Q_{t-1}^{temp}(S_{t-1}^j)$ that is obtained from 6) at the previous time step.
- 6) By executing steps 4) and 5), $f_t(S_t^i)$ is determined for each $j (j = 1, \dots, M')$. The system stores the information of S_{t-1}^j corresponding to $f_t(S_t^i)$ as $S_{t-1}^{from}(S_t^i)$. In addition, it stores also the corresponding discharge as $Q_t^{temp}(S_t^i)$.

The Forward Process ends when the calculations 4) to 6) are repeated from $t = 1$ to T . The process then moves on to the Backward Process.

- 7) S_T^{opt} is determined as the state variable vector S_T^i that reaches to $f_T(S_T^i)$ at time T .
- 8) The Backward Process finds an optimal trajectory based on $S_{t-1}^{opt} = S_{t-1}^{from}(S_t^{opt})$ in backward.

9) When $S_i^{opt}(t=1,...,T)$ differs from the trial trajectory, it means that it is not yet the optimal solution. In such a case, the calculation is repeated from step 4) by using $S_i^{opt}(t=1,...,T)$ as the next trial trajectory. When the obtained optimal trajectory matches with the trial trajectory over the entire time step, the process moves on to the final step 10).

10) Based on the optimized state variable vector $S_i^{opt}(t=1,...,T)$, corresponding outflow from dam reservoirs O_i^{opt} and river discharges Q_i^{opt} are simulated with river routing models.

SYSTEM APPLICATION

Simulation conditions

(a) The Kizugawa River basin and its dam reservoirs

The developed system is applied to the Kizugawa River basin (the upper reach of the Kamo point: 1469 km²) in the upper reach of the Yodogawa River system. There are seven major multi-purpose dams in total in the Yodogawa River system (more specifically, the upper reach of the Hirakata point), five of which are located in the Kizugawa River basin (Fig. 5).

Under the current reservoir control practice, dams are operated for flood control based on the operation rules, in which the discharge from each dam is determined based on the water level and the inflow discharge. Preliminary release operation is exceptionally performed at the Shorenji and the Murou dams, in which the dams reduce their water levels prior to flooding based on rainfall forecasting.

The designing target points in the Kizugawa River basin are the Kamo and Ieno points (located in the upper reach of the Takayama dam) with the design high water discharges of 6,100 m³ and 3,800 m³, respectively. The capacity of the river at the Kamo point is estimated at about 3,400 m³ as of 1998.

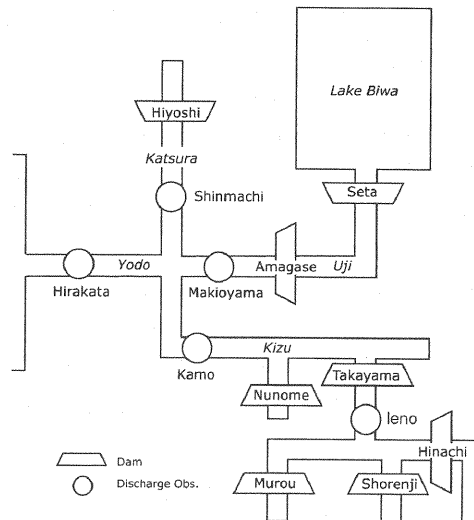


Fig. 5 The Yodogawa basin and its dam reservoirs. This study focuses on the upper reach of Kamo point with five multi-purpose dams.

(b) Objective function

This study used an objective function shown as Eq. 3. The below Eq. 3 was used to give an evaluation function $D(Q_i)$ in the objective function expressed by Eq. 3. In Eq. 5, D was the square sum of the quotients of the discharges at the Kamo and Ieno points both divided by its respective design high water discharge.

$$D = \left(\frac{Q_i^{kamo}}{6100} \right)^2 + \left(\frac{Q_i^{ieno}}{3800} \right)^2 \quad (5)$$

The function expressed above is obtained on the assumption that damage such as levee breaches will increase as discharge increases. Different optimization solution can be obtained by setting different evaluation points and function suitable for a specific flood control objective.

(c) Constrained condition

The constrained condition for dam water-level was specified so that the water level was neither below the normal water level in flood season nor above the surcharge water level. However, for the dams which perform preliminary release operation according to their reservoir control guidelines, it was acceptable to reduce the water level down to the preliminary water level stated in the guidelines. The discharge condition was also set to avoid an unrealistic reservoir control whereby a dam releases an extremely large amount of water in normal times or an extremely small amount of water during flooding. In this study, the dams were required to keep the discharge less than the flood control starting discharge when the inflow is less than that level, and also keep the discharge larger than the flood control starting discharge if the inflow is larger than that.

(d) State discretization

DP performs optimization by discretizing state variables. In this study, the difference between the maximum and minimum storage volumes was finally divided into 50 equal sections for each dam after applying different numbers for discretization through trial and error. When this discretization number was applied to the five dams in the Kizugawa River basin, a personal computer (Xeon, 3.0 GHz) repeated the calculation about 60-70 times and completed it after in 30 minutes.

(e) Initial conditions

The dam water level to start the calculation was set at the normal water level in flood seasons. The initial trajectory was set for each dam to be consistent when the water level was maintained at the normal water level. In other words, the initial trajectory was defined as a balanced state in which the inflow and discharge were controlled to be equal.

(f) Rainfall event

This study selected four flooding events over the Kizugawa River basin after 1982 and used the spatially distributed rainfall that is interpolated from hourly ground rainfalls at about 30 locations. The rainfalls were extended to correspond with the 2-day rainfall at the upper reach of the Kamo point of the 50-, 100-, 150- and 200-year return periods.

Simulation results

A comparison was made between the discharges at the Kamo point under the current reservoir control and the optimal control simulated with ResCOS. The simulation under the current control was performed by using a distributed runoff model combined with dam operation models. It was based on detailed information about the current reservoir operation rules including emergency operation rule, preliminary release and other procedures during flooding. A simulation was also performed under the no-reservoir condition.

Figure 6 shows the simulation results of four flooding events in 1990, 1994, 1995 and 1997 with the rainfalls extended to the 200-year return period level. The results indicated that under the no-dam condition, the peak discharge reached 12,000 m^3/s during the 1994 flooding. In the other three events also, it reached 10,000 m^3/s , far exceeding the design high water discharge of 6,100 m^3/s at the Kamo point.

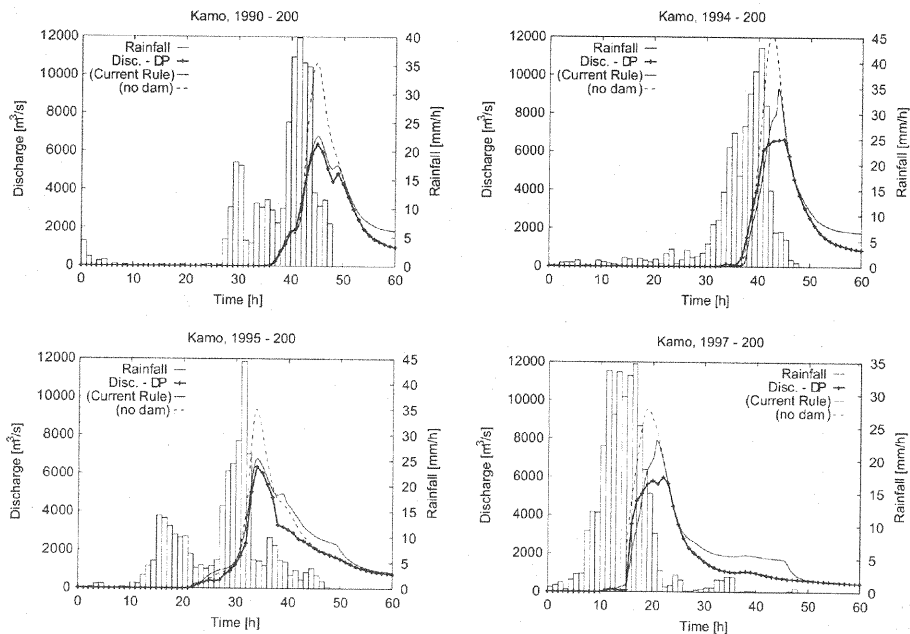


Fig. 6 River discharges at Kamo point optimized by ResCOS (Disc. - DP) and simulated by a rainfall-runoff model with current operation rule (Current Rule) and no dam condition (no dam). Four different rainfall distributions (1990, 1994, 1995, 1997) are used for the model input by extending the rainfall amount corresponding to 200-year return period.

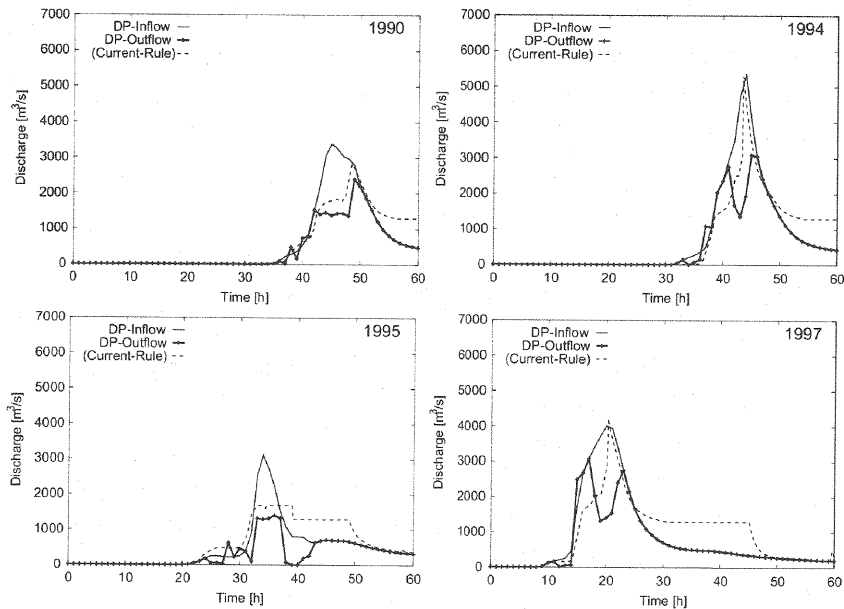


Fig. 7 Outflow from Takayama Dam under DP optimized condition (DP-Outflow) and with current operation rule (Current-Rule). DP-Inflow represents the inflow to Takayama Dam under the optimized condition.

Under the current control condition (sim), the peak discharges reduced down to almost the same level as the design high water discharge in 1990 and 1995. However, in 1994 and 1997, the peak discharges were 9,300 m^3/s and 7,900 m^3/s , respectively, indicating that the dams contributed little to the reduction of those discharges. As the Current Rule lines in Fig. 7 show, the main reason for the results in 1994 and 1997 is that the emergency operations were carried out at the Takayama Dam before the flooding reached its peak. At the Takayama Dam, when the inflow volume exceeded 1,300 m^3/s , the flood control operation would start with a certain release rate. This operation was performed in the 1994 and 1997 flooding events and successfully reduced the discharge before the flooding. The simulation results clearly show that the dams located upstream could delay the peak arrival time at the Kamo point.

Figure 6 also shows the results from optimization based on ResCOS (DP). The peak discharges at the Kamo point were reduced successfully down to the design high water discharges in 1994 and 1997 and almost to the same level as the results under the Current Rule conditions in 1990 and 1995. Figure 7 shows an optimal discharge pattern at the Takayama Dam (DP-Outflow). The distinct two-peak pattern appeared both in 1994 and 1997, in which the discharge increased up to 3,000 m^3/s in the initial stage of the flooding, decreased down to 1,300 m^3/s at the flood peak, and increased again after the peak. However, the 1990 discharge pattern looked similar to that under the current reservoir control, in which the discharge was released at a consistent rate in the first half of the flooding and under the emergency operation in the second half. That was also the case with the 1995 event.

Overall, the simulation results revealed that in case of an excessive flood, which far exceeded the

design high water discharge, the discharge at the Kamo point could be reduced approximately to the design high water discharge by applying the two-peak discharge pattern to the Takayama Dam operation. In addition, the current operation was found to be almost the same as the optimal operation in the case of the other dams. In this respect, the current reservoir control can be considered as a nearly ideal one even in case of an excessive flood.

CONCLUSION

In this study, we developed a reservoir control optimization simulator by integrating a distributed rainfall runoff model and devised dynamic programming to find an effective control of multiple dams in case of excessive flood events. The simulator was designed to reproduce a detailed flooding pattern by using a distributed rainfall runoff model and optimize the objective function for reservoir control based on the simulation result. It was applied to the Kizugawa River basin in the Yodogawa River system. The application revealed that the optimal discharge pattern appeared different from that under the current control, showing a two-peak discharge pattern in the case of the Takayama Dam located downstream of the other study dams. It was also found that the current control is nearly optimal for the dams located upstream of the Takayama Dam. In an actual reservoir control, it is impossible to apply an optimal control to all the dams. Therefore, it is advisable to prepare a control procedure for excessive flooding in advance based on an optimal trajectory found through this type of simulation, and switch from a regular flood operation to such a prepared emergency procedure when excessive flooding is expected.

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