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PARTICLE-BASED VS. GRID-BASED SIMULATION OF PLUNGING BREAKING WAVES; A BASIC STUDY

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SYNOPSIS

A basic study is carried out to investigate the performance and accuracy of two particle methods; namely the MPS and the Incompressible SPH (ISPH) methods, in comparison to a VOF-type grid-based model. Such methods are applied to simulate a plunging wave breaking and resultant splash-up. Qualitative and quantitative comparisons of simulation results with experimental data highlight the superiority of the MPS method to the ISPH and VOF methods for the simulation of plunging wave breaking and post-breaking.

INTRODUCTION

Wave breaking is a complex yet important process as it plays a significant role in transferring mass and momentum in coastal zones. Because of the importance of wave breaking, extensive research is being carried out on this process or the related processes in the surf zone.

Although a variety of numerical approaches have been applied to the study of breaking waves, the most direct and reliable one is to solve the Navier-Stokes equation. Based on two different numerical points of view, namely, the Lagrangian and Eulerian view points, different numerical algorithms have been developed to solve the Navier-Stokes equation and to simulate the breaking waves. Most Eulerian solvers of the Navier-Stokes equation use grids; hence, they should be coupled with a mathematical treatment of the water surface. Reconstructing volume tracking methods, among which is the VOF method (Hirt and Nichols (10)), are the most popular and robust water surface tracking methods incorporated in Eulerian grid-based solvers of Navier-Stokes equation. The VOF method has been extensively used and modified to study the breaking waves (e.g. Lemos (17), Lin and Liu (21)). Despite their popularity and large range of applicability, the VOF-based models have a drawback because of numerical diffusion arising from the successive interpolation of the advection terms in both the VOF function transport equation and the Navier-Stokes momentum equation. The numerical diffusion becomes more significant when the free surface undergoes large deformations (as in the case of plunging breaking waves) and especially when fragmentation and coalescence of water exist (as in the process of splash-up at post-breaking stage).

A recent point of interest has been on the development of the next generation computational methods, namely, the mesh-free methods. The Lagrangian mesh-free methods or the particle methods are classified into those based on field approximations, as the Element-Free Galerkin (EFG) method, and those based on kernel approximations, as the Smoothed Particle Hydrodynamics (SPH; Lucy (22) and Gingold and Monaghan (4)) or Moving Particle Semi-implicit (MPS; Koshizuka and Oka (15)) methods. In contrast to the conventional grid-based methods, particle methods are free of numerical diffusion as the advection terms are directly calculated by moving particles. Moreover, particle methods do not require an additional water-surface tracker as the free surface particles can be efficiently detected by a simple condition.

In this paper, a basic study is performed to evaluate the capability and the accuracy of two particle methods; namely, the MPS and the Incompressible SPH (ISPH) (Shao and Lo (27)) methods, in comparison to an Eulerian VOF-type model (Khayyer *et al.* (12)). Followed by a brief explanation of basic concepts, the standard codes are applied to plunging wave breaking and post-breaking on a uniform slope. The results of simulations are compared to the experimental data by Li and Raichlen (18, 19). The deficiencies and capabilities of each method and the essential future works are discussed.

NUMERICAL MODELS

MPS Method

Since the MPS method was developed by Koshizuka *et al.* (16), it has been applied in a wide variety of problems such as elastic structures (Koshizuka *et al.* (13)), nuclear reactor safety (Koshizuka *et al.* (14)) or blood flow simulation

(Tsubota *et al.* (28)). The MPS method has been improved and also adopted in coastal engineering to study wave breaking (Gotoh and Sakai (7); Gotoh *et al.* (5)) and two-phase sediment-water interactions (Gotoh *et al.* (8)). In the MPS method, fluid is modeled as an assembly of interacting particles, the motion of which is calculated through the interactions with neighboring particles. The governing equations are the continuity equation and the Navier-Stokes equation describing the motion of a viscous incompressible flow:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{g} \tag{2}$$

where $\boldsymbol{u}$=particle velocity vector, t=time, ρ=density of fluid, p=pressure, ν=kinematic viscosity, and $\boldsymbol{g}$= gravitational acceleration vector. The above equations are discretized by the particle interaction models, namely the gradient and Laplacian operators defined as (Koshizuka and Oka (15)):

$$\langle \nabla \phi \rangle_i = \frac{D_0}{n_0} \sum_{j \neq i} \frac{(\phi_j - \phi_i)}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i)\, w\left(|\boldsymbol{r}_j - \boldsymbol{r}_i|\right) \tag{3}$$

$$\langle \nabla^2 \phi \rangle_i = \frac{2 D_0}{n_0 \lambda} \sum_{j \neq i} (\phi_j - \phi_i) w\left(|\boldsymbol{r}_j - \boldsymbol{r}_i|\right) \tag{4}$$

where ϕ = an arbitrary physical quantity at particle i ($=\phi_i$) or its neighboring particle j ($=\phi_j$), D_0=number of space dimensions, $\boldsymbol{r}$=coordinate vector of fluid particle, $w(r)$=the kernel function, n_0=the constant particle number density, and λ=a coefficient defined as:

$$\lambda = \frac{\sum_{j \neq i} w\left(|\boldsymbol{r}_j - \boldsymbol{r}_i|\right) |\boldsymbol{r}_j - \boldsymbol{r}_i|^2}{\sum_{j \neq i} w\left(|\boldsymbol{r}_j - \boldsymbol{r}_i|\right)} \tag{5}$$

The particle number density is calculated as follows:

$$\langle n \rangle_i = \sum_{j \neq i} w\left(|\boldsymbol{r}_j - \boldsymbol{r}_i|\right) \tag{6}$$

The iterative prediction-correction process of the MPS method involves two steps. The first prediction step is an explicit integration in time without enforcing incompressibility, while, the second correction step is an implicit computation of a divergence free velocity field. In the first process, intermediate temporal particle velocities and positions are obtained without considering the pressure term. In this process the mass conservation or the incompressibility of fluid is not satisfied; in other words, the number densities n^* that are calculated at the end of first process deviate from the constant n_0; therefore, a second corrective process is required to adjust the number densities to initial constant values prior to the time step. In the second process, the intermediate particle velocities are updated by solving the Poisson equation of pressure (Koshizuka *et al.* (16)):

$$\left(\nabla^2 p_{k+1}\right)_i = \frac{\rho}{(\Delta t)^2} \frac{(n_k^*)_i - n_0}{n_0} \tag{7}$$

where Δt = the calculation time step; and the subscript k shows the step of calculation. When the MPS method as well as the ISPH method are applied, the particle number density at the free surface drops sharply; thus, the free surface particles can be effectively detected. In the particle-based simulations of the present study, a particle is regarded as a surface particle if the temporal particle number density for that particle is over 3% below the initial one.

Incompressible SPH Method

As one of the earliest particle methods, the SPH method was originally developed by Lucy (22) and Gingold and Monaghan (4) for astrophysical applications but has also been applied to simulate a wide range of engineering applications including the incompressible free-surface flows. The SPH-based simulation of incompressible flows consist of those which treat the fluid as weakly compressible or those which strictly enforce the incompressibility by applying a two step projection method by solving a Poisson pressure equation similar to the Eq. 7 in MPS method. From this view point, the strictly incompressible SPH methods are analogues to the MPS method.

Another categorization of SPH-based methods can be made on the basis of viscosity treatment. While in many SPH calculations the effect of viscosity is represented by a so-called artificial viscosity term (e.g. Monaghan (23) and Dalrymple and Rogers (3)) several attempts have been made to apply a tensor-type strain-based viscosity which can model the

viscosity in a realistic manner (e.g. Watkins *et al.*(29), Shao(26)). In the current paper, a strictly incompressible SPH calculation is carried out, while, tensor-type strain-based viscosity is treated.

The governing equation and the equation solution process are the same for both the MPS and the ISPH methods; yet, the discretization of the terms is quite different. The basis of all kernel-based particle methods is a reproducing kernel approximation of an arbitrary function $f(x)$ in terms of a kernel function $W(x,h)$:

$$\langle f(x)\rangle = \int_{\Omega} f(\xi) W(x-\xi,h)\, d\xi \tag{8}$$

in which h is the smoothing length taken slightly larger than particle diameter (h=1.2d in this study). Computationally, the above integration is evaluated in a discrete manner as follows:

$$\langle f(x_i)\rangle \approx \sum_{j\neq i}^{M} f(x_j) W(x_i - x_j, h) V_j \quad ; \quad V_j = \frac{m_j}{\rho_j} \tag{9}$$

where M is the number of neighboring particles of particle i and m_j, ρ_j and V_j are the mass, the density and the tributary (or statistical) volume associated with particle j, respectively. V_j is defined as:

$$V_j^{-1} = \sum_{i\neq j}^{M} W(x_j - x_i, h) \tag{10}$$

The integration is calculated for each particle in a circular domain or the influence circle of particle i, the radius of which is usually $2h$ as in cubic B-Spline kernel (e.g. Monaghan(24)) or $3h$ as in Quintic-Spline kernel (e.g. Morris *et al.*(25)). The main difference between the MPS and SPH-based methods arise in the derivative of the function $f(x)$. In SPH-based methods the derivative of a function is expressed as:

$$\langle \nabla f(x_i)\rangle \approx \sum_{j\neq i}^{M} f(x_j) \nabla W(x_i - x_j, h) V_j \tag{11}$$

Nevertheless, in the MPS method, the derivative of a field variable is obtained by local weighted averaging of the variable derivatives estimated between a pair of particle i and its neighboring particle j (as in Eq. 3). For this reason, a kernel gradient is not calculated in the MPS method.

From equation (11), the SPH-based velocity divergence of particle i can be formulated as:

$$\nabla \cdot \boldsymbol{u}_i = \sum_j V_j \boldsymbol{u}_j \cdot \nabla W(x_i - x_j, h) \tag{12}$$

With the density placed inside operators (Monaghan(24)):

$$\nabla \cdot \boldsymbol{u} = [\nabla \cdot (\rho \boldsymbol{u}) - \boldsymbol{u} \cdot \nabla \rho] / \rho \tag{13}$$

Eq. 12 is rewritten as:

$$\nabla \cdot \boldsymbol{u}_i = \sum_j V_j (\boldsymbol{u}_j - \boldsymbol{u}_i) \cdot \nabla W(x_i - x_j, h) \tag{14}$$

The pressure gradient and the viscous stress terms are derived in a similar manner and are written in symmetrical forms as (Shao(26)):

$$\left(\frac{1}{\rho}\nabla p\right)_i = \sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right) \nabla W(x_i - x_j, h) \tag{15}$$

$$\left(\nu \nabla^2 \boldsymbol{u}\right)_i = \left(\frac{1}{\rho}\nabla \cdot \boldsymbol{T}\right)_i = \sum_j m_j \left(\frac{\boldsymbol{T}_i}{\rho_i^2} + \frac{\boldsymbol{T}_j}{\rho_j^2}\right) \nabla W(x_i - x_j, h) \tag{16}$$

where $\boldsymbol{T}$ is the viscous stress tensor which can be written in terms of the rate of strain tensor. The Laplacian for pressure is formulated as a hybrid of a standard SPH first derivative with a finite difference approximation for the first derivative and represented also in a symmetrical form as (Shao(26)):

$$\nabla \cdot \left(\frac{1}{\rho}\nabla P\right)_i = \sum_j m_j \frac{8}{(\rho_i + \rho_j)^2} \frac{P_{ij} \boldsymbol{r}_{ij} \cdot \nabla W(x_i - x_j, h)}{\left|\boldsymbol{r}_{ij}\right|^2} \tag{17}$$

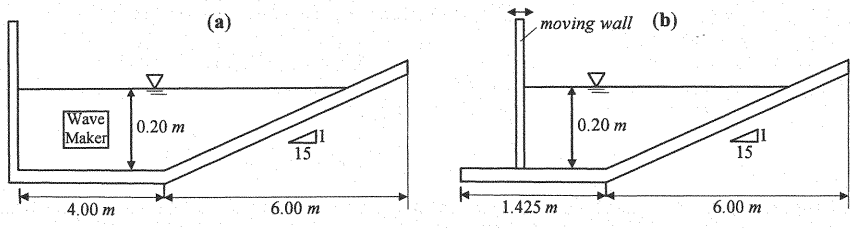

Fig. 1 Sketch of the computational domain for (a) VOF (b) MPS and ISPH calculations

VOF Method

The VOF method (Hirt and Nichols (10)) has been frequently applied for the grid-based simulation of interfacial flows. In the VOF methodology, the characteristic function F denotes the fraction of a cell filled with a fluid. Away from the interface F=0 or 1; in cells cut by the interface F has a value between zero and unity. Detailed interface information cannot be extracted from the discrete volume data F until an interface is reconstructed. Hence, the first major step in all volume tracking algorithms involves reconstructing the interface geometry according to some approximations. The approximation used for the interface reconstruction in the original VOF method (10) is the piecewise constant/stair stepped scheme. Once an interface is reconstructed at one time step, the fluid volumes are advected in cells by solving an advection equation for F written in a conservative form as:

$$\frac{\partial F}{\partial t}+\frac{\partial (Fu)}{\partial x}+\frac{\partial (Fv)}{\partial y}=0 \qquad (18)$$

where u and v = components of the fluid velocity in x and y directions, respectively. Since the F function is discontinuous, it is necessary to devise appropriate methods for the time integration of F advection equation that preserve a sharp interface definition. One popular and reliable method is the Donor-Acceptor algorithm (Chorin (2)). In the current study, the original VOF method of Hirt and Nichols (10) is applied together with the Donor-Acceptor algorithm of Chorin (2) (Khayyer *et al.* (12)).

BREAKING AND POST-BREAKING OF SOLITARY WAVES ON A PLANE SLOPE

Qualitative Comparison

In this section, the numerical models described earlier are applied to simulate the breaking and post-breaking of a solitary wave over a slope (=s) of 1:15. The incident relative wave height; which is defined as the ratio of offshore wave height (=H_0) to offshore water depth (=h_0), is H_0/h_0=0.40. The computational domains corresponding to the grid-based calculation (VOF) and particle-based calculations (MPS and ISPH) are shown in Fig. 1. The difference in computational domains arises from the different wave generators applied in grid-based and particle-based computations. In the VOF-type model, the solitary wave is generated by means of a mass source function (Lin and Liu (20)) located inside the computational domain, while, in particle-based models the solitary waves are generated by use of a moving wall. The initial offshore water depth is 0.200 *m* for all the models. In the VOF simulation, the entire domain is discretized by means of a finely square mesh composed of 2000×90 cells the size of each is 0.005 *m* being exactly equal to the particle size in particle-based simulations. About 20000 particles are located inside the particle-based computational domains. In both MPS and ISPH calculations, the waves are generated by a moving wall (which initially moves backward) and a constant number of particles are employed. The incident wave height and the offshore water depth for particle-based simulations are 0.075 *m* and 0.187 *m*, respectively. For all the grid-based and particle-based calculations, the increment of the calculation time step is set according to the Courant stability condition and a time resolution of 0.0005 *s*. The territory area (or the influence circle) of one particle is the same (r_e=2h=2.4d) in both MPS and ISPH calculations.

The conditions of H_0/h_0=0.40 and s=1:15 cause a strong plunging breaking in which the plunging jet hits the still water ahead of the wave (and not the dry slope); consequently a secondary shoreward-directed jet is generated followed by the plunging jet impact. The splash of water in the form of a secondary jet, which is known as the splash-up, is due to the penetration of plunging jet and the momentum exchange between the plunging jet and the wedge shape still water at the toe of the wave. Although the splash-up is very complex, it is an important process as it is responsible for the generation of large-scale vortices and plays a significant role in the dissipation of wave energy and momentum transfer. The simulation of a highly non-linear process such as the splash-up requires accurate numerical solution of the governing equation.

In grid-based methods, the convective flux of momentum is combined with the momentum already present in each cell; thus, an average value of momentum is obtained through a sort of interpolation for each cell at each time step. When advanced in time, this interpolated value is passed on to the next cell in the direction of flow. Repetition of the simplified interpolations contributes to a "diffusion" of momentum in the direction of flow. The conventional grid-based methods usually apply a linear interpolation for the time and spatial discretization; for this reason, the computation of a highly

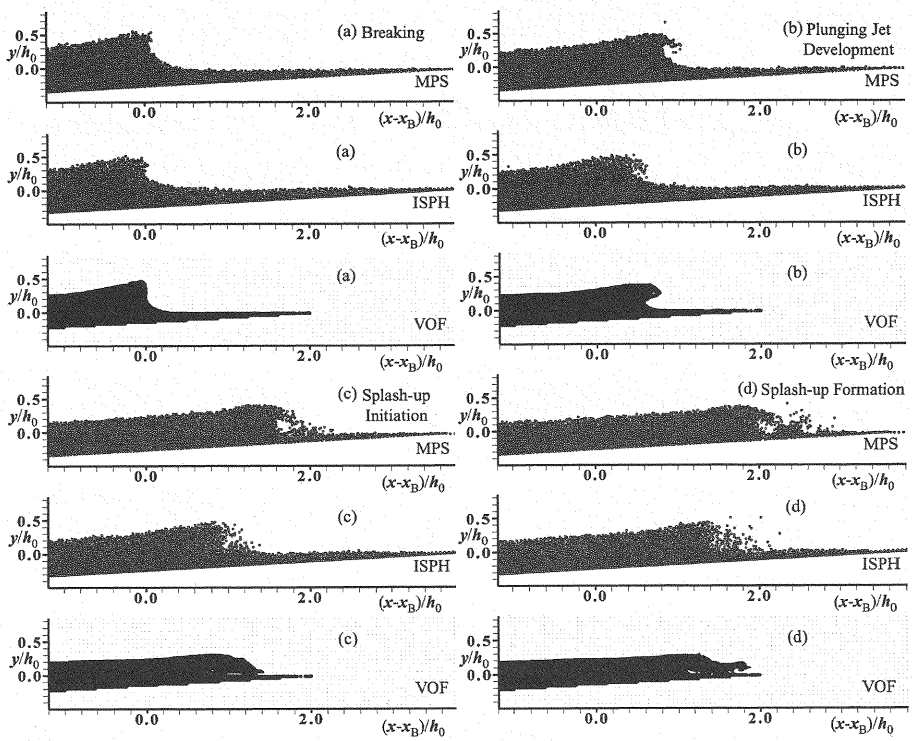

Fig. 2 MPS, ISPH and VOF snapshots illustrating (a) breaking (b) plunging jet development (c) splash-up initiation and (d) splash-up formation for a strong plunging breaker (H_0/h_0=0.40)

non-linear process with such numerical models result in considerable numerical errors. The Constrained Interpolation Profile (CIP) method (Yabe *et al.*(30)) is one of the approaches that has been introduced to minimize the numerical diffusion in grid-based calculations. In the CIP method, the spatial profile within each grid is interpolated with a cubic polynomial rather than a linear one, accordingly the results show higher accuracy, yet the computational effort seems to be considerably higher too. In contrast to the conventional grid-based methods, particle methods are free of numerical diffusion as the discretization of advection terms is accomplished by moving particles rather than fixed computational grids. In addition, due to the discrete nature of moving particles, particle methods prove to have a substantial potential for the simulation of problems characterized by large deformations and fragmentations such as the splash-up.

Fig. 2 shows MPS, ISPH and VOF snapshots of water profile representing the initiation of breaking, development of the plunging jet, initiation of the resultant splash-up and the splash-up formation. A general image of the breaking process can be illustrated by all the three numerical models, although ISPH computation of wave breaking is accompanied by considerable particle scattering. Compared to the MPS and ISPH methods, the VOF method overestimates the breaking point (=the place where the front face of the wave becomes vertical=x_B), as for each method, the x-axis zero point is set at the breaking point corresponding to the results of that method. The overestimation of the breaking point by the VOF method is probably due to the excessive energy dissipation as a result of numerical diffusion. Such excessive energy dissipation causes the wave energy, height and propagation speed to be lower than reality during the breaking process. Fig. 3 is constructed from laboratory photographs (Li and Raichlen(18)) and the typical MPS, ISPH and VOF snapshots. From Fig. 3(b-c), compared to the ISPH and VOF methods, the MPS method has finely reproduced the plunging jet and its impingement with less particle scattering as seen in corresponding ISPH snapshots. The geometrical shape of the plunging jet and the air chamber beneath it are in line with the laboratory photographs. On the other hand, in VOF snapshots the thickness of the jet is far greater than what is observed in the laboratory photographs. Accordingly, the air chamber beneath the plunging jet is not simulated well by the VOF method. The ISPH method can not either portray a clear image of the plunging jet as the particles forming the jet tend to become considerably scattered during the development of the plunging jet.

The initiation and the formation of the splash-up are better reproduced by the MPS method (Fig. 3(d-e)) than by the ISPH or VOF methods. The VOF method can also reproduce the initiation of splash-up; however, the formation of splash-up does not well match the laboratory photograph as the reflected jet angel is much less than the experiment. A clear reproduction of the splash-up initiation and formation cannot be seen in case of the ISPH snapshots. From Fig. 3(f), the development of the splash-up is moderately well simulated by the MPS method. Although a rough image of the splash-up is

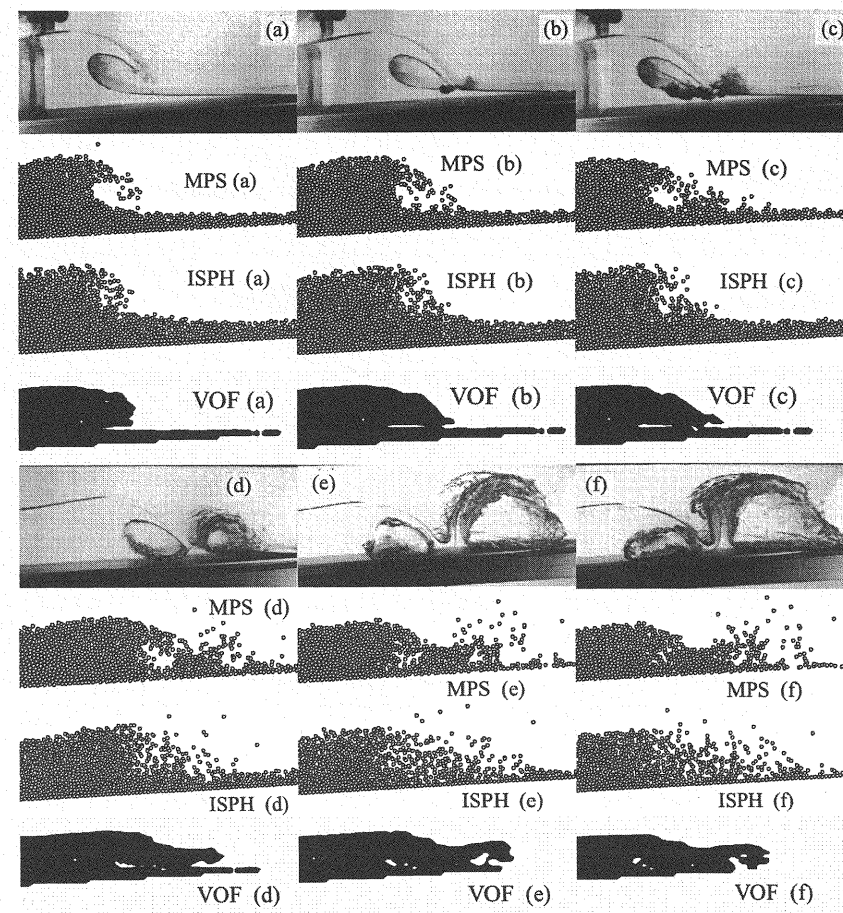

Fig. 3 A strong plunging breaker and the resultant splash-up (H_0/h_0=0.40) - qualitative comparison of laboratory photographs (18) with MPS, ISPH and VOF snapshots

depicted by the MPS snapshot, details and the exact conditions of the splash-up are not well simulated. The ISPH snapshot cannot even illustrate a rough image of the splash-up process as most of the particles in the vicinity of the wave front become fully dispersed. The splash-up is reproduced by the VOF method, yet, its height is noticeably underestimated. Moreover, the fragmented splash of water is simply reproduced by a single continuous jet. As the splash-up proceeds, the reflected jet curls back toward the incident jet and eventually becomes nearly vertical. This phenomenon is moderately simulated by the VOF method in Fig. 3(g). From Figs. 2 and 3, the MPS results are superior to those of ISPH and VOF in the reproduction of a plunging wave breaking and formation of splash-up.

Quantitative Comparison

The accuracy of the numerical models in the tracking of water surface during a plunging breaking should be further investigated through a quantitative comparison of simulation results with the experimental data. Hence, in this section another case of solitary plunging breaking with H_0/h_0=0.45 is simulated over a slope of 1:15. Fig. 4 shows a comparison of the variation in wave height H/H_0 among the MPS, ISPH and VOF methods and the experimental data (Li and Raichlen (19)). In the figure x_s is the starting location of the slope. The wave height in MPS and ISPH methods is determined as the distance between the wave crest particles and the still water. A fully-isolated particle (an isolated particle for which there is no neighboring particle) is not taken into account for the determination of wave height. From Fig. 4, one can conclude that both VOF and ISPH methods do not provide an accurate estimation of the wave height during breaking and post-breaking. The ISPH prediction of wave height works quite well prior to the impact point of the plunging jet. Nevertheless, near the impact point an abrupt change is present in the trend of the wave height variation leading to inaccurate results. The VOF method significantly overestimates the breaking point and results in a wave height variation curve which does not well

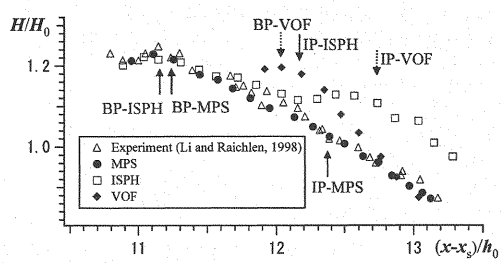

Fig. 4. Comparison of variation in wave height during breaking and post-breaking (H_0/h_0=0.45)
- BP and IP correspond to Breaking Point and Impact Point, respectively

agree with the experiment. On the other hand, the MPS method accurately predicts the wave height during the entire processes of breaking and post-breaking.

CONCLUDING REMARKS AND FUTURE WORKS

A basic study is carried out to investigate the performance and the accuracy of two particle methods, namely the MPS and ISPH methods, in comparison to a VOF-type grid-based model. The three methods are applied to the simulation of plunging breaking and post-breaking of solitary waves on a plane slope and are compared to the experimental data. The processes of plunging breaking and splash-up formation are well reproduced by the MPS method. On the other hand, both the ISPH and the VOF methods could not portray a precise image of the plunging breaking and resultant splash-up. The MPS method accurately predicted the variation of wave height during breaking and post-breaking. In contrast, neither the ISPH nor the VOF methods could provide an accurate estimation of wave height variation through the breaking process. In addition to providing accurate results, the MPS method is conceptually simpler than both the ISPH method and VOF-type models. As a consequence, MPS-based code writing, modification and extension are easier. In conclusion, the MPS method is a reliable and an easy-to-implement technique for calculating hydrodynamic free surface flows such as the breaking waves.

Because the ISPH applies a tensor-type viscosity, which produces anisotropic interacting viscous stresses (or forces), the conservation of angular momentum can not be guaranteed in discretization of ISPH. This fact may lead to an excessive dispersiveness of secondary jet in ISPH. The exact conservation of momentum can be guaranteed by the introduction of corrective terms that modify the kernel or its gradient (Bonet and Lok(1)). The development of Corrected Incompressible SPH has been carried out by the authors (Khayyer *et al.*(11)). In addition to the momentum conservation several other issues such as modeling of Sub-Particle-Scale turbulence (Gotoh *et al.*(9)), modeling of gas-liquid behaviour and the 3D model should be considered in a particle-based simulation of breaking waves (Gotoh and Sakai(6)).

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APPENDIX - NOTATION

The following symbols are used in this paper:

d = particle diameter;
D_0 = number of space dimensions;
F = VOF function;
$\boldsymbol{g}$ = gravitational acceleration vector;
h = smoothing length in ISPH method;
h_0 = offshore water depth;
H = wave height;
H_0 = offshore wave height;
m = mass of a particle;
n, n_0 = particle number density and its initial constant value;
n^* = temporal particle number density;
p = pressure;

$\boldsymbol{r}$ = coordinate vector of fluid particle;
s = beach slope;
$\boldsymbol{T}$ = viscous stress tensor;
$\boldsymbol{u}$ = particle velocity vector;
V = statistical volume of fluid particle;
w = kernel function in MPS method;
W = kernel function in ISPH method;
x, y = Cartesian coordinates of fluid particle;
x_B, x_s = breaking point and slope starting location;
Δt = time step of calculation;
ϕ = an arbitrary physical quantity;
λ = coefficient in MPS Laplacian model;
ν = kinematic viscosity; and
ρ = density of fluid.

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