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A NEW SEMI-ANALYTICAL SOLUTION FOR A SLUG TEST IN A CONFINED AQUIFER UNDER THE EFFECTS OF WELL PARTIAL PENETRATION

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SYNOPSIS

This paper proposes a new semi-analytical solution for a slug test performed in a partially penetrating well within a confined aquifer system. A constant-flux solution, including well partial penetration and wellbore storage, is first derived and then converted into the slug test solution by means of the formula of Peres et al. (12). The new solution in the cases of well full penetration can be reduced to Cooper et al.'s solution (3) and coincides with the KGS model (Hyder et al. (7)); yet, in regard to well partial penetration, it differs from the KGS model (Hyder et al. (7)) due to the different assumption on the well boundary condition. An example demonstrates the applicability of the new solution to a field problem. The simulated results in a partially penetrating well reveal that a smaller screen length, anisotropic ratio, or specific storage can result in a slower recovery rate of the normalized head.

INTRODUCTION

A slug test is a practical method of identifying aquifer parameters, e.g., hydraulic conductivity and storativity, in the vicinity of test well. The method is performed by suddenly removing/ (adding) a small amount of water from/ (into) the well and measuring the change of well water level simultaneously. Subsequently, the aquifer parameters are identified by either numerical methods or graphical methods by matching the slug test data with an appropriate type curve. The slug test is widely used to assess aquifer site characterization because of the advantages of low costs, being easy and rapid to perform the test, no demand in post-treatment of large contaminated water, and minor disturbances on the existing ground-water system.

A slug test well usually partially penetrates the thickness of an aquifer. The flow toward a fully penetrating well is expected to be horizontal, while the flux toward a partially penetrating well includes both the horizontal and vertical components. The characteristics of a partially penetrating well, i.e., the length and position of well screen, can affect the well water level. Thus, the presence of well partial penetration may lead to significant errors in the estimation results if the mathematical model used for the test-data analysis is not able to describe the behavior of well partial penetration.

A number studies of mathematical models developed for analyzing slug test data have been published over the past four decades. In the case of well full penetration, Cooper et al. (3) proposed an analytical solution and gave a corresponding type-curve method considering a finite-diameter well in a confined aquifer system. However, the lack of sensitivity in type-curve matching makes the determination of storage coefficient questionable. In the case of well partial penetration in a confined aquifer, Dougherty and Babu (4) and Hyder et al. (7) both proposed a semi-analytical solution for the slug test performed in a partially penetrating well and constructed in a double-porosity reservoir.

Unlike traditional type-curve approaches, Peres et al. (12) proposed a different concept of data analysis on the slug test. They established a mathematical relationship between the solutions of slug test and constant-flux test. The constant-flux test is referred to as a pumping test. Such a test is performed in an aquifer with a constant discharge and the aquifer drawdown is recorded in the meantime. Furthermore, the constant-flux solution is referred to as the solution describing the temporal and spatial drawdown distribution in response to the constant-flux test. In Peres et al. (12), these two related solutions involve the effects of wellbore storage and well full penetration in a confined aquifer system. The mathematical relationship shows clearly that the time derivative of the constant-flux solution is proportional to the slug test solution. Based on Duhamel's theorem, they showed that the derived relationship can be valid in any well and aquifer constructions. Accordingly, the slug test data may be converted to the pumping test data and analyzed by means of a corresponding pumping test model.

Based on the formula given by Peres et al. (12), the pumping test solution is used to find the slug test solution for a confined aquifer with a partially penetrating well. Thus, the literature concerning the constant-flux pumping test is also briefly reviewed hereinafter. As for the well radius, the literature of the constant-flux pumping test may be classified into two types. One type does not consider the well radius, that is, the well is treated as a line source (e.g., Streltsova and McKinley (14) and Theis (15)). The other type considers the finite-radius well in a pumping test model (e.g., Ingersoll et al. (9 and 10) and Papadopulos and Cooper (11)). As for the well partial penetration, Hantush (6) made a detailed investigation of the problem in a confined aquifer system.

The purpose of this paper is to develop a new mathematical model to describe the response of well water level for a slug test performed in a partially penetrating well of confined aquifers. This new model is indeed a Laplace-domain solution derived from the pumping test solution based on the formula given by Peres et al. (12). The results of head response versus time computed from our model are compared to those of Cooper et al. (3) and the KGS model (Hyder et al. (7)). An example is presented to illustrate the applicability of the new solution for a field problem. In addition, the influences of well partial penetration, aquifer anisotropy, and aquifer specific storage on the slug test results are examined by means of this new model.

METHODOLOGY

This section includes two parts. In the first part, we develop a mathematical model for a constant-flux pumping test performed in the confined aquifer and partially penetrating well when considering the effect of wellbore storage. The constant-flux solution is derived by a series of integral transformations. In the second part, the constant-flux solution is converted into the slug test solution based on the formula given by Peres et al. (12).

Model of Constant-Flux Pumping Test

Figure 1 represents the well and aquifer configurations. The assumptions of the constant-flux pumping test model are as follows: (1) the aquifer is confined, homogeneous, anisotropic, infinite lateral extent, and with a constant thickness; (2) the finite-radius well is partially penetrated with the effect of wellbore storage; and (3) the initial hydraulic head is uniform throughout the entire aquifer. It should be noted that the gray vertical strips shown in Fig. 1 represent the filter pack, not the skin zone. The governing equation of the ground-water flow can be described as

$$k_r\left(\frac{\partial^2 h_c}{\partial r^2}+\frac{1}{r}\frac{\partial h_c}{\partial r}\right)+k_z\frac{\partial^2 h_c}{\partial z^2}=S_S\frac{\partial h_c}{\partial t} \qquad (1)$$

where h_c (or $h_c(r, z, t)$) = hydraulic head during a constant-flux pumping test, r = radial distance from the centerline of well, z = vertical direction, t = time from the start of test, S_S = specific storage, k_r = radial component of hydraulic conductivity, and k_z = vertical component of hydraulic conductivity.

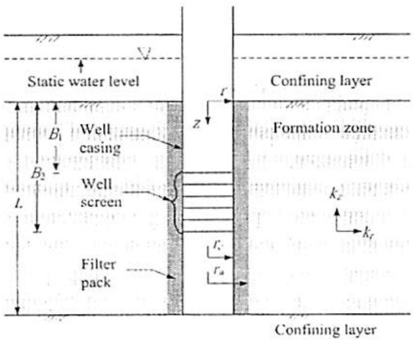

Fig. 1 Schematic representation of a partially penetrating well and aquifer configurations.

The initial hydraulic head for the whole aquifer equals zero, that is,

$$h_c(r,z,0)=0 \qquad (2)$$

Equation (1) is subject to two boundary conditions in r-direction. One of the boundary conditions presents at infinity and the hydraulic head is equal to zero, that is

$$h_c(\infty,z,t)=0 \qquad (3)$$

The other condition describes the conservation of the mass flux along the well screen:

$$\left(Q - \pi r_c^2 \frac{\partial h_c(r_w, z, t)}{\partial t}\right)\left[U(z - B_1) - U(z - B_2)\right] = -2\pi r_w k_r (B_2 - B_1)\left(\frac{\partial h_c(r, z, t)}{\partial r}\right)_{r=r_w} \tag{4}$$

where Q = constant discharge rate pumped from the test well, r_c = radius of the casing, r_w = effective radius of the well, B_1 and B_2 are the length from the top of aquifer to the top and bottom of screen, respectively, and $U(z - B_i)$ is the unit step function which equals one when $z \geq B_i$, and equals zero when $0 \leq z \leq B_i$ for i = 1 or 2. It should be noted that the second term on the left-hand side (LHS) of equation (4) reflects the effect of wellbore storage and the term on the right-hand side (RHS) represents the total flux flows across the wellbore screen. In short, equation (4) states that the rate of constant-flux pumping is equal to the sum of the rate of flow into the well and the rate of decrease in the volume of water within the well.

For a confined aquifer system, the upper and lower boundaries are impermeable, i.e.,

$$\frac{\partial h_c(r, 0, t)}{\partial z} = \frac{\partial h_c(r, L, t)}{\partial z} = 0 \tag{5}$$

where L denotes the aquifer thickness.

The solution of the constant-flux pumping test described by equations (1) to (5) can be derived via the Laplace transform and finite Fourier cosine transformation. Thus, a Fourier-domain solution can be easily obtained. Furthermore, by using the inverse finite Fourier cosine transformation, the Laplace-domain solution of the constant-flux pumping test model can be expressed as

$$\begin{aligned}\tilde{h}_c(r, z, p) = \frac{1}{L}&\left\{\frac{\left(\frac{Q}{p}\right)(B_2 - B_1)K_0(\alpha_1 r)}{\pi r_c^2 p K_0(\alpha_1 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_1 K_1(\alpha_1 r_w)}\right\} \\ &+ \frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{\left(\frac{Q}{p}\right)W_1 K_0(\alpha_2 r)\cos(w_n z)}{\pi r_c^2 p K_0(\alpha_2 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_2 K_1(\alpha_2 r_w)}\right\}\end{aligned} \tag{6}$$

where $\tilde{h}_c(r, z, p)$ = Laplace-domain solution of hydraulic head; p = Laplace-transform parameter; K_0 and K_1 respectively represent the modified Bessel functions of the second kind of order zero and one; and

$$w_n = \frac{n\pi}{L}, \quad n = 1, 2, 3, \ldots \tag{7}$$

$$W_1 = \frac{1}{w_n}\left(\sin w_n B_2 - \sin w_n B_1\right) \tag{8}$$

$$\alpha_1 = \sqrt{\left(\frac{S_s}{k_r}\right)p}\,, \quad for\ n = 0 \tag{9}$$

with

$$\alpha_2 = \sqrt{\left(\frac{S_s}{k_r}\right)p + \left(\frac{k_z}{k_r}\right)w_n^{\,2}}\,, \quad n = 1, 2, 3, \ldots \tag{10}$$

The detailed derivation of equation (6) is shown in Appendix 1.

Solution of Slug Test

Peres et al. (12) express the following relationship between the slug test and constant-flux solutions in the Laplace domain as

$$\tilde{h}(r,z,p) = \frac{\pi r_c^{\,2} H_0 p}{Q}\tilde{h}_c(r,z,p) \tag{11}$$

where H_0 = well water level at the beginning of slug test, and $\tilde{h}(r,z,p)$ = hydraulic head of slug test in the Laplace domain. Combining equation (11) with the constant-flux solution, i.e., equation (6), yields the slug test solution. Thus, the Laplace-domain solution of slug test in an arbitrary point is described as

$$\begin{aligned}\tilde{h}(r,z,p) = &\frac{1}{L}\left\{\frac{\pi r_c^{\,2} H_0 (B_2 - B_1) K_0(\alpha_1 r)}{\pi r_c^{\,2} p K_0(\alpha_1 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_1 K_1(\alpha_1 r_w)}\right\} \\ &+ \frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{\pi r_c^{\,2} H_0 W_1 K_0(\alpha_2 r)\cos(w_n z)}{\pi r_c^{\,2} p K_0(\alpha_2 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_2 K_1(\alpha_2 r_w)}\right\}\end{aligned} \tag{12}$$

In addition, the water level in the test well can be obtained by averaging the hydraulic heads of equation (12) along the screened interval, that is,

$$\begin{aligned}\tilde{h}_w(p) = &\frac{1}{L}\left\{\frac{\pi r_c^{\,2} H_0 (B_2 - B_1) K_0(\alpha_1 r_w)}{\pi r_c^{\,2} p K_0(\alpha_1 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_1 K_1(\alpha_1 r_w)}\right\} \\ &+ \frac{2}{(B_2 - B_1)L}\sum_{n=1}^{\infty}\left\{\frac{\pi r_c^{\,2} H_0 W_1^{\,2} K_0(\alpha_2 r_w)}{\pi r_c^{\,2} p K_0(\alpha_2 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_2 K_1(\alpha_2 r_w)}\right\}\end{aligned} \tag{13}$$

Thus, the average of well water level for an observation well can be described precisely as

$$\tilde{h}(r,p)=\frac{1}{L}\left\{\frac{\pi r_c^2 H_0 (B_2-B_1) K_0(\alpha_1 r)}{\pi r_c^2 p K_0(\alpha_1 r_w)+2\pi r_w k_r (B_2-B_1)\alpha_1 K_1(\alpha_1 r_w)}\right\} + \frac{2}{(B_2^{'}-B_1^{'})L}\sum_{n=1}^{\infty}\left\{\frac{\pi r_c^2 H_0 W_1 W_1^{'} K_0(\alpha_2 r)}{\pi r_c^2 p K_0(\alpha_2 r_w)+2\pi r_w k_r (B_2-B_1)\alpha_2 K_1(\alpha_2 r_w)}\right\} \tag{14}$$

with

$$W_1^{'}=\frac{1}{w_n}\left(\sin w_n B_2^{'}-\sin w_n B_1^{'}\right) \tag{15}$$

where $B_1^{'}$ and $B_2^{'}$ denote the length from the top of aquifer to the top and bottom of the screen, respectively.

Since equations (12) to (14) are complicated; the inversions of these equations to the time domain analytically may not be tractable. The routine of DINLAP of IMSL (8) is thus utilized to perform the numerical Laplace inversion, and the results are accurate to the fifth decimal place. The routine DINLAP is a FORTRAN code of modified Crump algorithm and IMSL is an acronym for the International Mathematical and Statistical Library provided by Visual Numerics Inc.

RESULTS AND DISCUSSION

In this section, we first compare our model with two existing slug test models and a field example. Then the influences of well partial penetration, aquifer anisotropy, and aquifer specific storage on the slug test data are investigated by using our model.

Model Comparisons

The new model which we derived is compared to two existing slug test models. One is Cooper et al.'s model (3), which was performed on condition that a finite-diameter and full-penetration well is constructed in a confined aquifer system. The other is Hyder et al.'s model (7), also known as the KGS model, which was derived for a partially penetrating well constructed in a confined or unconfined aquifer system with or without the effect of wellbore skin.

To compare our slug test model with that of Cooper et al. (3), the effect of well partial penetration in our solution should be disregarded. That is, the solution is independent of z. Hence, when removing the second term on the RHS of equation (12), the equation can be reduced to

$$\tilde{h}(r,p)=\frac{r_w S H_0 K_0(\alpha_1 r)}{T\alpha_1\left[r_w \alpha_1 K_0(\alpha_1 r_w)+\left(\frac{2 r_w^2 S}{r_c^2}\right)K_1(\alpha_1 r_w)\right]} \tag{16}$$

where T = transmissivity and S = storativity. Notice that equation (16) is the same as the solution of Cooper et al. (3) in the Laplace domain.

The KGS model (Hyder et al. (7)) was developed for slug tests performed in a partially penetrating well. Their boundary conditions for describing the flux at the interface between the screen and aquifer were represented as (Hyder et al. (7))

$$2\pi r_w k_{r1} \frac{\partial h(r_w, z, t)}{\partial r} = \frac{\pi r_c^2}{(B_2 - B_1)} \frac{dH(t)}{dt}\left[U(z - B_1) - U(z - B_2)\right], \quad t > 0 \tag{17}$$

and

$$\frac{1}{B_2 - B_1} \int_{B_1}^{B_2} h(r_w, z, t)\, dz = H(t), \quad t > 0 \tag{18}$$

The notations defined in Hyder et al.'s equations are as follows: h = hydraulic head of slug test in the formation; H = depth-averaging head in the well. Equation (17) represents that the rate of flow out/in of the well equals the rate of water volume decrease/increase within the well. The RHS of equation (17) is independent of depth; however, the LHS of equation (17) is depth-dependent. Obviously, the derivation of equation (1) with the boundary conditions of equations (17) and (18) resulted in an approximate solution.

Our model is compared to Cooper et al.'s model (3) and the KGS model (Hyder et al. (7)) by means of an aquifer-test analysis software known as AQTESOLV (Duffield (5)). Assume that slug test is performed in a 10 m thick aquifer. The aquifer is confined and isotropic with $k_r = 10^{-4}$ m/sec and $S_S = 10^{-4}$ m^{-1}. The test well has r_w and r_c of 0.0915 m and 0.0508 m, respectively. The initial water level H_0 in the test well is 1 m. Figure 2 illustrates that our model exactly matches that the model of Cooper et al. (3) for the case of well full penetration, i.e., B_1 and B_2 are respectively set to 0 m and 10 m. Figure 3 illustrates the comparison between our model and the KGS model (Hyder et al. (7)). Note that the normalized head used in the figure is a dimensionless variable defined as the well water level in a test or observation well divided by the initial water level in the test well. Figure 3(a) shows the normalized heads predicted by these two models at radial distances 0.0915, 0.305 and 0.914 m when $B_1 = B_1' = 4.5$ m and $B_2 = B_2' = 5.5$ m. Figure 3(b) illustrates the normalized head responses observed in a fully penetrating observation well for a slug test performed in a partially penetrating well. The screen of the test well is situated at the center of the confined aquifer with a penetration ratio such as 1.0, 0.75, 0.5 or 0.1. The observation well is located at a radial distance 0.305 m from the center of the test well. Figure 3(c) shows the normalized head differences for various penetration ratios shown in Fig. 3(b).

As shown in Fig. 3, the differences in the head response of a slug test become insignificant as the radial distance increases. On the other hand, the head responses computed by our model are obviously lower than those computed by the KGS model (Hyder et al. (7)) at intermediate time but are slightly higher than those at late time. The differences between the results computed from our model and the KGS model (Hyder et al. (7)) diminish when the penetration ratio increases. The results of our model also coincide with those of the KGS model (Hyder et al. (7)) when the test well is fully penetrated. Such differences are due to the fact that the KGS model (Hyder et al. (7)) used the derivative of depth-averaging hydraulic head with respect to time to produce the flow for the flux boundary condition whereas our model did not take into account the depth-averaging work for constructing the related boundary condition. Physically, the boundary condition used in our model is more realistic to represent the flux across the well screen. Therefore, the hydraulic conductivity may be underestimated if the KGS model is chosen to identify the aquifer parameters when the well is partially penetrating.

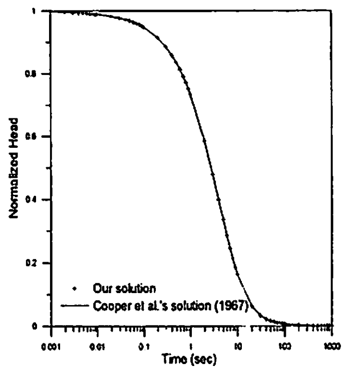

Fig. 2 A plot of normalized head responses versus logarithm time predicted by our solution and Cooper et al.'s (3) solution for a slug test performed in a fully penetrating well.

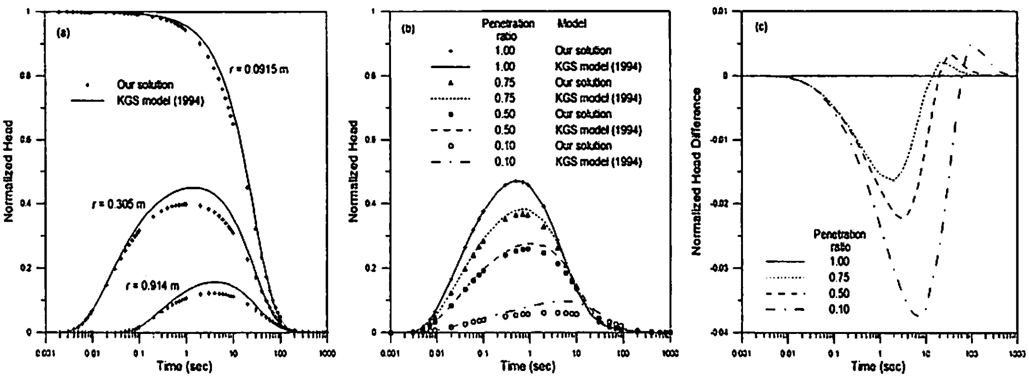

Fig. 3 The normalized head responses depicted by our solution and the KGS model (Hyder et al. (7)) for the cases of (a) an observation well located at radial distance 0.0915, 0.305 or 0.914 m and the penetration ratio of the test and observation wells are both 0.1; (b) a fully penetrating observation well at radial distance 0.305 m and the penetration ratio of test well is 1.0, 0.75, 0.5 or 0.1; and (c) the normalized head differences of case (b).

Field Example

An example is used to illustrate the applicability of our solution in characterizing the hydraulic behavior during a field slug test. The test data were collected in well 3 at the Pratt County site, Kansas in October 1993 (Butler (1), Table 5.4). At this site, the aquifer was 47.87 m thick with interbedded clays and consisted mostly of sands and gravels. The well partially penetrated the aquifer with well screen opened from $B_1 = 35.37$ to $B_2 = 36.89$ m. The values of r_w and r_c of the well were 0.125 m and 0.064 m, respectively. Figure 4 shows that there is a close match between the normalized field data plot and the normalized head curve produced by our model with $k_r = 3.1\times10^{-4}$ m/sec, $k_z = 9.3\times10^{-6}$ m/sec, and $S_s = 10^{-6}$ m^{-1}. At the early stage of testing, the normalized data had considerable fluctuations due to the test initiation. Therefore, the fluctuated data should not be used for evaluating the matching quality of a theoretical model (Butler (1)). Furthermore, the data were also analyzed by using Cooper et al.'s model (3) and the KGS model (Hyder et al. (7)) with the results both presented in Butler (1). The estimated $k_r = 4.9\times10^{-4}$ m/sec and $S_s = 3.3\times10^{-6}$ m^{-1} while using Cooper et al.'s model (3) (Butler (1), Fig. 5.14). In addition, by assuming that the aquifer is isotropic, the KGS model (Hyder et al. (7))

gives the following results: $k_r = 2.0\times10^{-4}$ m/sec, $k_z = 2.0\times10^{-4}$ m/sec, and $S_s = 3.4\times10^{-6}$ m^{-1} (Butler (1), Fig. 5.18). The normalized head curve predicted by our model is very similar to that of the KGS model.

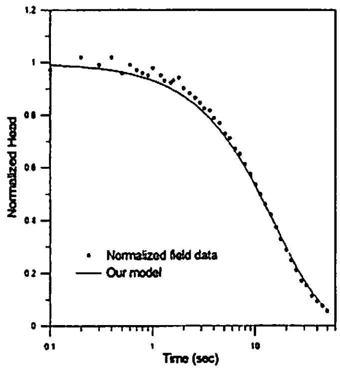

Fig. 4 Plots of the normalized data for the Pratt County test in Kansas (Butler (1)) and the normalized head curve drawn based on our model.

Effect of Well Partial Penetration

Two issues concerning well partial penetration, i.e., the penetration ratio and position of well screen, are discussed in this section. Assume that a slug test operates in a 10 m thick confined and isotropic aquifer with $k_r = 10^{-4}$ m/sec and $S_S = 10^{-4}$ m^{-1}. The values of r_w and r_c are 0.0915 m and 0.0508 m, respectively. The initial water level H_0 in the test well is 1 m. Six cases, with the penetration ratios of 0.9, 0.5 or 0.1 and the screen opens at the top or the center of isotropic aquifer, were designed to investigate the effects of well partial penetration on the normalized head responses. Figure 5 plots the six curves of the normalized head responses and the head differences versus dimensionless time. The dimensionless time is defined as the elapse time multiplied by the radial hydraulic conductivity and is divided by the radius of well casing.

Figure 5(a) shows the recovery rate of the normalized head decreases with the screen length. Thus, in a small penetration ratio case, disregarding the well partial penetration in the slug test data analysis will cause a notable underestimation of the hydraulic conductivity. In regard to the location of well screen, the recovery rate for a well screen situated at the top of confined aquifer is slower than that situated at the center, especially in the case of small penetration ratio.

The normalized head difference in Fig. 5(b) represents the difference in normalized heads between the discussed case and a fully penetrating well case. Accordingly, Fig. 5(b) shows the influences when one simply employs the model of full-penetration well for analyzing a set of test data with a partially penetrating well. As illustrated in Fig. 5(b), it is apparent that the normalized head difference increases as the penetration ratio decreases. For instance, the largest difference in normalized head between a fully penetrating well and a well with a penetration ratio of 0.9 is about 0.02 occurring at the dimensionless time of 0.006. However, the largest normalized head differences between a fully penetrating well and the wells with a penetration ratio of 0.5 and 0.1 are respectively around 0.20 happened at the dimensionless time 0.01 and 0.55 appeared at the dimensionless time 0.02.

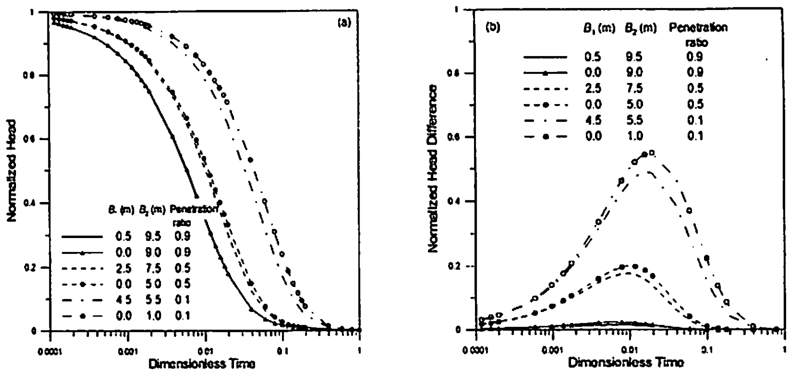

Fig. 5 Plots for the screen situated at the top or center of isotropic confined aquifer when the penetration ratios equal to 0.9, 0.5 or 0.1. (a) Curves of normalized head responses versus dimensionless time. (b) Curves of normalized head differences versus dimensionless time.

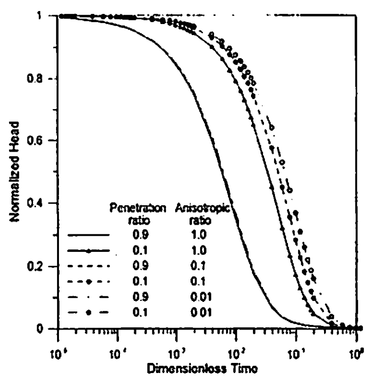

Fig. 6 Curves of normalized head responses versus dimensionless time for the penetration ratio of 1.0 or 0.2 when the anisotropic ratio (k_z/k_r) equals to 1.00, 0.10 or 0.01.

Effect of Anisotropic Ratio

The flow toward a partially penetrating well contains not only the horizontal flow but also the vertical flow. Hence, the aquifer anisotropic ratio, k_z/k_r, is also an important factor on the slug test response under the anisotropic condition. We assume that two well screens centered in an aquifer have the penetration ratios of 0.9 and 0.1. Three different values for the anisotropic ratios of 1, 0.1 and 0.01 are considered, and the effect of aquifer anisotropy is presented in Fig. 6. A smaller anisotropic ratio results in a definitely slower head response in small penetration-ratio cases as indicated in the Fig. 6. However, in large penetration-ratio cases, the normalized-head curves which represent different anisotropic ratios are difficult to differentiate.

Effect of Aquifer Specific Storage

Generally speaking, the value of storativity in a confined aquifer ranges from 10^{-3} to 10^{-5} (Schwartz and Zhang (13)). Thus, three different values of S_S, 10^{-4} m^{-1}, 10^{-5} m^{-1} and 10^{-6} m^{-1}, are considered for a hypothetical aquifer of 10 m thick.

In addition, two conditions of well construction are chosen; one is a fully penetrating well and the other is a partially penetrating well with B_1 and B_2 of 4 m and 6 m, respectively. Figure 7 illustrates the influence of aquifer specific storage on the slug test data. Under the same penetration ratio, smaller specific storage results in a slower head response. The normalized head distributions between different values of specific storage, though significantly different in a fully penetrating well, are similar to the penetration ratio of 0.2 as indicated in Fig. 7. This evidence suggests that the normalized head distribution is not sensitive to the change of the specific storage for a small penetration ratio case.

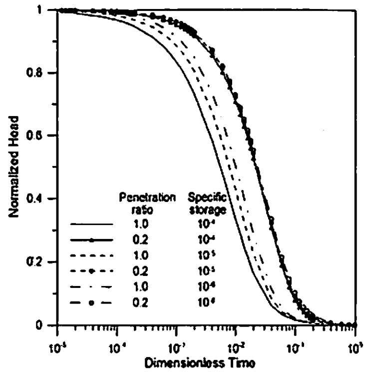

Fig. 7 Curves of normalized head responses versus dimensionless time for the penetration ratio of 1.0 or 0.2 when the aquifer specific storage equals to 10^{-4}, 10^{-5} or $10^{-6}\,m^{-1}$.

CONCLUSIONS

A new slug test solution for the head response was developed based on the formula given by Peres et al. (12) when considering a partially penetrating well constructed in a confined aquifer system. A significant difference of head response was observed between our model and the KGS model (Hyder et al. (7)) by virtue of these models with different flux boundary conditions along the wellbore. Nevertheless, the new solution can coincide with that of Cooper et al. (3) and the KGS model (Hyder et al. (7)) if the test well is fully penetrated. Moreover, an example at the Pratt County site provides evidence of the applicability of our solution to analyze a set of field slug test data collected from a partially penetrating well.

Our solution was adopted to explore the influences of well partial penetration, aquifer anisotropy, and aquifer specific storage on the slug test data analysis. When a slug test is performed in a partially penetrating well, a smaller screen length, anisotropic ratio, or specific storage can result in a slower recovery rate of the normalized head. In addition, in regard to the screen location, the recovery rate for a well screen situated at the top of aquifer is significantly slower than that situated at the center for a well of small penetration ratio. Thus, in a small penetration ratio case, disregarding the well partial penetration or aquifer anisotropy on the test-data analysis, can cause an apparent underestimation on the hydraulic conductivity. Yet, the normalized head distribution is not sensitive to the change of specific storage in a well of small penetration ratio.

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APPENDIX 1 – DERIVATIONS OF EQUATION (6)

To solve equation (1), we apply the Laplace and finite Fourier cosine transforms to the governing equation and related boundary conditions. The Laplace transform of function $f(t)$ is defined as (Carslaw and Jaeger (2))

$$\tilde{f}(p) = \int_0^\infty e^{-pt} f(t)dt \qquad \text{(A1)}$$

Thus, applying the Laplace transform to equation (1) results in

$$\frac{\partial^2 \tilde{h}_c(r,z,p)}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{h}_c(r,z,p)}{\partial r} + \frac{k_z}{k_r}\frac{\partial^2 \tilde{h}_c(r,z,p)}{\partial z^2} = \frac{S_s}{k_r} p\,\tilde{h}_c(r,z,p) \tag{A2}$$

where $\tilde{h}_c(r,z,p)$ = hydraulic head in the Laplace domain. In addition, the boundary conditions are rewritten as follows

$$\tilde{h}_c(\infty,z,p) = 0 \tag{A3}$$

$$\frac{\partial \tilde{h}_c(r,0,p)}{\partial z} = \frac{\partial \tilde{h}_c(r,L,p)}{\partial z} = 0 \tag{A4}$$

and

$$\left(\frac{Q}{p} - \pi r_c^2 p \tilde{h}_c(r_w,z,p)\right)\left[U(z-B_1) - U(z-B_2)\right] = -2\pi r_w k_r (B_2 - B_1)\left(\frac{\partial h_c(r,z,p)}{\partial r}\right)_{r=r_w} \tag{A5}$$

The finite Fourier cosine transform is defined as (Carslaw and Jaeger (2))

$$\bar{f}(w_n) = \int_0^L \tilde{f}(z)\cos(w_n z)\,dz \tag{A6}$$

Applying the finite Fourier cosine transform to equation (A2) gives

$$\frac{\partial^2 \bar{h}_c(r,w_n,p)}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{h}_c(r,w_n,p)}{\partial r} = \alpha\,\bar{h}_c(r,w_n,p) \tag{A7}$$

with

$$\alpha = \sqrt{\left(\frac{S_s}{k_r}\right)p + \left(\frac{k_z}{k_r}\right){w_n}^2}\ ,\ n = 0, 1, 2\ldots \tag{A8}$$

where $\bar{h}_c(r,w_n,p)$ = hydraulic head in the Fourier-Laplace domain. The transformed boundary equations are

$$\bar{h}_c(\infty,w_n,p) = 0 \tag{A9}$$

and

$$\left(\frac{Q}{p}w_1 - \pi r_c^2 p\bar{h}_c(r_w, w_n, p)\right) = -2\pi r_w k_r (B_2 - B_1)\left(\frac{\partial h_c(r, w_n, p)}{\partial r}\right)_{r=r_w} \quad \text{(A10)}$$

The general solution of equation (A7) can be obtained as

$$\bar{h}_c(r, w_n, p) = C_1 I_0(\alpha r) + C_2 K_0(\alpha r) \quad \text{(A11)}$$

where I_0 = modified Bessel function of the first kind of order zero. In addition, C_1 and C_2 can be obtained by substituting equations (A9) and (A10) into equation (A11). One finds

$$C_1 = 0 \quad \text{(A12)}$$

and

$$C_2 = \frac{\left(\frac{Q}{p}\right)W_1}{\pi r_c^2 pK_0(\alpha r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha K_1(\alpha r_w)} \quad \text{(A13)}$$

where

$$W_1 = \frac{1}{w_n}\left[\sin(w_n B_2) - \sin(w_n B_1)\right] \quad \text{(A14)}$$

Hence,

$$\bar{h}_c(r, w_n, p) = \frac{\left(\frac{Q}{p}\right)W_1 K_0(\alpha r)}{\pi r_c^2 pK_0(\alpha r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha K_1(\alpha r_w)} \quad \text{(A15)}$$

Further, employing the inverse finite Fourier cosine transform on equation (A15), the hydraulic head of constant-flux pumping test can be expressed as

$$\tilde{h}_c(r, z, p) = \frac{1}{L}\left\{\frac{\left(\frac{Q}{p}\right)(B_2 - B_1)K_0(\alpha_1 r)}{\pi r_c^2 pK_0(\alpha_1 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_1 K_1(\alpha_1 r_w)}\right\} + \frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{\left(\frac{Q}{p}\right)W_1 K_0(\alpha_2 r)\cos(w_n z)}{\pi r_c^2 pK_0(\alpha_2 r_w) + 2\pi r_w k_r (B_2 - B_1)\alpha_2 K_1(\alpha_2 r_w)}\right\} \quad \text{(A16)}$$

where

$$\alpha_1 = \sqrt{\left(\frac{S_s}{k_r}\right)p} \tag{A17}$$

and

$$\alpha_2 = \sqrt{\left(\frac{S_s}{k_r}\right)p + \left(\frac{k_z}{k_r}\right){w_n}^2}, \quad n = 1, 2, 3\ldots \tag{A18}$$

APPENDIX 2—NOTATION

The following symbols are used in this paper:

B_1 = distance from the top of aquifer to the top of test-well screen;
B_2 = distance from the top of aquifer to the bottom of test-well screen;
B_1' = distance from the top of aquifer to the top of observation-well screen;
B_2' = distance from the top of aquifer to the bottom of observation-well screen;
h_c = hydraulic head of constant-flux pumping test;
$\tilde{h}$ = hydraulic head of slug test in the Laplace domain;
$\tilde{h}_c, \bar{h}_c$ = hydraulic head of constant-flux pumping test in the Laplace and Fourier-Laplace domains, respectively;
$\tilde{h}_w$ = well water level of slug test in the Laplace domain;
H_0 = initial water level in a test well;
I_0 = modified Bessel functions of the first kind of order zero;
k_r, k_z = radial and vertical component of hydraulic conductivity, respectively;
K_0, K_1 = modified Bessel functions of the second kind of order zero and one, respectively;
L = thickness of aquifer;
p = Laplace variable;
Q = constant-flux rate through the wellbore;
r = radial distance from the centerline of well;
r_c = radius of casing;
r_w = effective radius of test well;
S = storativity of aquifer;
S_s = specific storage;
t = time since the start of test;
T = transmissivity of aquifer;
U = unit step function

$w_n \quad = \frac{n\pi}{L}$, $n = 1, 2, \ldots$;

$W_1 \quad = \frac{1}{w_n}\left(\sin w_n B_2 - \sin w_n B_1\right)$;

$W_1' \quad = \frac{1}{w_n}\left(\sin w_n B_2' - \sin w_n B_1'\right)$;

z = vertical direction;

$\alpha \quad = \sqrt{\left(\frac{S_s}{k_r}\right)p + \left(\frac{k_z}{k_r}\right){w_n}^2}$, $n = 0, 1, 2, \ldots$;

$\alpha_1 \quad = \sqrt{\left(\frac{S_s}{k_r}\right)p}$; and

$\alpha_2 \quad = \sqrt{\left(\frac{S_s}{k_r}\right)p + \left(\frac{k_z}{k_r}\right){w_n}^2}$, $n = 1, 2, 3, \ldots$

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