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AN IMPROVED METHOD FOR PREDICTING SOLIDS CONCENTRATION PROFILES IN SLURRY FLOW THROUGH PIPELINE, RECTANGULAR DUCT AND OPEN CHANNEL

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SYNOPSIS

The experimental data reported in literature were compared with the predictions by Karabelas (9) model for solids concentration profiles in the slurry flow of a variety of materials through a pipe, a rectangular duct and an open channel with wide range of flow velocities and efflux concentrations of solids. The Karabelas model was found to be able to predict reasonably accurately at lower efflux concentrations, however it failed to predict the solids concentration profiles accurately at higher levels of efflux concentrations. Modifications were proposed in the Karabelas model to remove the deficiencies by alleviating some of its restrictive assumptions. In particular, the effects of local concentration on the particle settling was considered, and particle diffusivity was taken as variable with local concentration and particle size. The modified model predicted the solids concentration profiles with reasonable accuracy in all the three geometries for the entire range of efflux concentration and flow velocity considered in the present study.

INTRODUCTION

A liquid-solid two phase flow through a pipe, rectangular duct and open channel are widely employed in the chemical and mining industries and are encountered in natural phenomena such as river mechanics. In any practical situation, the transported solids are multisized and their size may span three orders of magnitude. The flow of slurry is very complex. Researchers around the world have attempted to develop accurate models for concentration distribution in slurry transportation through pipes, rectangular ducts and open channels. These models may be used to determine the parameters of direct importance (mixture and solid flow rates and pressure drop in pipe and rectangular duct and critical bed slope in open channel) and the secondary effects such as wall abrasion and particle degradation. The advection/diffusion method has been widely used to predict the concentration distribution. By means of this method, the rate of upward transfer of suspended particles due to turbulence $\left(-\varepsilon_s \, \partial C/\partial y\right)$ is in equilibrium with the downward exchange due to gravitational forces wC, where C = concentration; y = a coordinate perpendicular to the bed, w is the particle settling velocity; and ε_s is the particle diffusivity defined as $\beta\varepsilon_l$, where β = dimensionless particle diffusivity; and ε_l = liquid diffusivity or momentum diffusivity of liquid. This method is effective in that only the parameter, which is not very well quantified, is the dimensionless particle diffusivity β. The model of particle suspension is based on the work of O' Brien (20), through Rouse (24), Ismail (7), Hunt (5, 6), Karabelas (9), Walton (29), Kaushal *et al.* (11, 12) and Kaushal and

Tomita (13, 14) to the lucid accounts given in many text books (Wasp *et al.*, 30; Raudkivi, 22; Govier and Aziz, 3).

Hunt (5) modified the basic diffusion equation by taking into account the liquid volume displaced upwards by an equal volume of particles moving in the lower strata of a horizontal flow field. The modified equations proposed by Hunt are more suitable for studies of relatively concentrated suspensions with wide particle size distribution. Karabelas (9) worked out the general solution of the system of equations proposed by Hunt (5) for the flow of multisized particulate slurries through pipes, rectangular ducts and open channels.

On the basis of extensive analysis of their own experimental data and Mukhtar's (19) data for flow of multisized particulate slurries through pipeline, Kaushal *et al.* (11) observed discrepancies in the prediction of composite concentration profiles by Karabelas (9) solution at higher efflux concentrations, and modified the Karabelas (9) model by considering the effects of efflux concentration on particle diffusivity ε_s and settling velocity. By comparing with experimental data, it was observed that the modified Karabelas model also showed good agreements for composite concentration profiles at higher efflux concentrations. Kaushal and Tomita (13) compared the solids concentration profiles based on the modified Karabelas model with experimental data and observed that the model predicts more asymmetric profiles for larger particles and less asymmetric profiles for finer particles. They also compared their pressure drop data with the modified Wasp model by examining the effects of efflux concentration on dimensionless particle diffusivity β suggested by Kaushal *et al.* (11) and modified Wood's equation proposed by Mukhtar (19) and found good agreements at higher flow velocities; however, deviations were still large at lower flow velocities near deposition velocity.

Some of the earlier studies (Wasp *et al.*, 30 and Walton, 29) revealed an increase in the value of particle diffusivity ε_s with particle size. Kaushal and Tomita (14) modified their previous model (Kaushal and Tomita , 13) for composite and solids concentration profiles in the pipe flow of multisized particulate slurries by examining the dependence of particle diffusivity on particle size and efflux concentration. The modified model is also used for promoting an improved method for predicting pressure drop along the slurry pipeline, and compared predicted pressure drops, composite and solids concentration profiles with experimental data and observed excellent agreements.

Prasad *et al.* (21) compared experimentally the head loss for solid-liquid flow in a circular pipe with that of rectangular ducts and concluded that rectangular duct of a particular aspect ratio proved to be better from an energy requirement point of view. Kaushal and Tomita (15) compared measured pressure drops and solids concentration profiles for multisized particulate slurry through $200\text{mm}\times 50\text{mm}$ rectangular duct and 105mm diameter pipe. It was observed that not only the secondary flow inside a rectangular duct but also the larger bed area was a factor for head loss reduction.

On the basis of extensive analysis of their own experimental data and Ismail (7) data for multisized particulate slurries flow through rectangular duct, Kaushal *et al.* (12) observed discrepancies in the prediction of composite concentration profiles by Karabelas (9) solution at higher levels of efflux concentrations. Kaushal *et al.* (12) modified the Karabelas (9) model by considering the effects of efflux concentration on particle diffusivity ε_s and settling velocity. On the basis of comparison with experimental data, Kaushal *et al.* (12) observed that modified Karabelas model showed good agreements for slurry transportation at higher levels of efflux concentrations also through a rectangular duct.

In comparison to rectangular duct, literature on slurry flow through open channels is abundant. Starting from Rouse (24), several researchers have proposed models in order to predict the vertical concentration distribution based on the concept of uniform sediments. Antsyferov and Kos'yan (1) proposed a two layer model which was extended by McTigue (17) on the basis of mass and momentum balance equations for both phases. Some of the other models were those proposed by Samaga *et al.* (25) and Itakura and Kishi (8).

Except for Karabelas (9) and Samaga *et al.* (25), all of the equations discussed above for open channel flows were developed essentially for uniform sediments. Also, Samaga *et al.* (25) described only the functional relationships without giving well defined expressions for different parameters used in the model. In any rational calculation of the concentration distribution, it is necessary to calculate it for various size fractions as the distribution of one size will be affected by the presence of other sizes.

In experimental studies on particle distributions in open channels available in literature (Samaga *et al.*, 25; Van Rijn *et al.*, 27; Vanoni, 28; Winterwerp *et al.*, 31 and Morales, 18), only Samaga *et al.* (25) determined the concentration distribution of the individual fractions in a mixture. In the other experimental studies only overall sediment concentration distributions were measured.

All these studies reveal that a universal prediction tool for the prediction of solids concentration profiles is of great importance, so that optimum particle size distribution can be determined for efficient operation of pipe, rectangular duct and open channel transporting slurries. For this purpose, the existing Karabelas (9) model was modified in the present study by examining the effects of local concentration on the particle settling and particle diffusivity as variable with local concentration and particle size. Predicted concentration profiles by means of the proposed modified model and the models available in literature were compared with the experimental data collected in pipelines, rectangular ducts and open channels.

A BRIEF DESCRIPTION OF KARABELAS MODEL FOR PREDICTION OF SOLIDS CONCENTRATION PROFILE IN PIPELINE, RECTANGULAR DUCT AND OPEN CHANNEL

Hunt (6) simplified his equations [Hunt (5)] describing a vertical diffusion of particles by assuming solids diffusivity for all size fractions n uniform and equal to the liquid diffusivity as:

$$\varepsilon_s \frac{\mathrm{d}C_j}{\mathrm{d}y} + C_j\left[w_j - w_y\right] = 0 \tag{1}$$

where C_j and w_j = concentration and settling velocity of j th size particles; and w_y is the liquid velocity in vertical direction:

$$w_y = \sum_{i=1}^{n} w_i C_i \tag{2}$$

For horizontal conduits, the set of Eqs. (1) admits the following general solution (Karabelas, 9).

$$C_j(y) = \frac{G_j \exp\{-w_j f(y)\}}{1 + \sum_{i=1}^{n} G_i \exp\{-w_i f(y)\}} \;;\; j = 1,\ 2,\ \ldots,\ n \tag{3}$$

where $f(y) = \int \frac{dy}{\varepsilon_s(y)}$; and G_j = a set of coefficients characteristic of each size fraction but independent of space co-ordinates. In order to proceed in the development of the distribution function $C_j(y)$ the following two assumptions are made:

(i) The dimensionless momentum diffusivity of liquid ξ is a constant and independent of solid concentration and space coordinates, that is,

$\varepsilon_s = \xi R u_*$ for pipe, where R = radius of pipe

$= \xi_1 H u_*$ for rectangular duct, where H = height of rectangular duct

$= \xi_2 H u_*$ for open channel (4)

(ii) The solids concentration is a function of vertical co-ordinate y only.
Using Eq. (4) for $\varepsilon_s(y)$, $f(y)$ can be expressed as

$$f(y) = \int_0^y \frac{\mathrm{d}y}{\xi u_* R} = \frac{y'}{\xi u_*} \quad \text{for pipe}$$

$$= \int_0^y \frac{\mathrm{d}y}{\xi_1 u_* H} = \frac{y'}{\xi_1 u_*} \quad \text{for rectangular duct}$$

$$= \int_0^y \frac{\mathrm{d}y}{\xi_2 u_* H} = \frac{y'}{\xi_2 u_*} \quad \text{for open channel} \tag{5}$$

where $y' = y/R$ varies from -1 to 1 for pipe and

$y' = y/H$ varies from 0 to 1 for rectangular duct and open channel.

Substitution of $f(y)$ into Eq. (3) leads to

$$C_j(y') = \frac{G_j \exp(-k_j y')}{1 + \sum_{i=1}^{n} G_i \exp(-k_i y')} \quad ; \; j = 1,\, 2,\, \ldots,\, n \tag{6}$$

where $k_j = w_j / \xi u_*$ for pipe,

$k_j = w_j / \xi_1 u_*$ for rectangular duct,

$k_j = w_j / \xi_2 u_*$ for open channel and $j = 1,\, 2,\, \ldots,\, n$ (7)

The values of ξ, ξ_1 and ξ_2 in the above equation depend on the geometry of flow passage, namely, pipe, rectangular duct and open channel, respectively.

A transformation is required to determine the parameter G_j. For this purpose, first the following local and mean relative concentrations are defined as

$$v_j(y) = \frac{C_j(y)}{1 - \sum_{i=1}^{n} C_i(y)} ; \quad \bar{v}_j = \frac{C_{vjf}}{1 - \sum_{i=1}^{n} C_i} = \frac{C_{vjf}}{1 - C_{vf}} \tag{8}$$

where bar denotes the quantities averaged over the duct cross-section. In terms of the relative concentration $v_j(y)$, Eq. (6) can be expressed as follows

$$v_j(y') = G_j \exp(-k_j y') \; ; \quad j = 1,2,\ldots,n \tag{9}$$

Thus it is assumed that the flow is steady, and that there is no particle deposition at the bottom, hence the mean concentration of each particle size C_{vjf} in the duct cross-section is constant and already known. Thus the mean relative concentration $\bar{v}_j$ is also constant. Therefore the integration of Eq. (9) over the duct of cross-sectional area A leads to:

$$\bar{v}_j = \frac{1}{A} \int_A v_j(y') \mathrm{d}A = G_j \frac{1}{A} \int_A \exp(-k_j y') \mathrm{d}A$$

$$G_j = \frac{\bar{v}_j}{E(k_j)} \quad ; \; j = 1,2,\ldots,n. \tag{10}$$

with the coefficients $E(k_j)$ defined as follows :

$$E(k_j) = \frac{1}{A} \int_A \exp(-k_j y') \mathrm{d}A \quad ; \; j = 1,2,\ldots,n \tag{11}$$

The approximate form of $E(k_j)$ is obtained for pipe

$$E(k_j) = 1 + \frac{k_j^2}{8}\left(1 + \frac{k_j^2}{24}\right) + O(k_j^6)$$

by expanding exp (-kjy′) and for rectangular duct and open channel

$$E(k_j) = \frac{1-\exp(-k_j)}{k_j} \; ; \; j = 1,2,...,n$$

The final solution for the concentration distribution for each size fraction as a function of y' for all the three geometries; namely, the pipe, the rectangular duct and the open channel is

$$C_j(y') = \left\{\frac{\overline{v}_j \exp(-k_j y')}{E(k_j)}\right\}\left\{1+\sum_{i=1}^{n}\frac{\overline{v}_i \exp(-k_i y')}{E(k_i)}\right\}^{-1} \; , \; j = 1,2,...,n. \qquad (12)$$

A BRIEF DESCRIPTION OF EXISTING MODELS FOR PREDICTION OF SOLIDS CONCENTRATION PROFILE IN OPEN CHANNEL

Rouse (24) Model

Rouse (24) proposed the sediment distribution equation as

$$\frac{C}{C_a} = \left\{\left(\frac{H-y}{y}\right)\left(\frac{a}{H-a}\right)\right\}^{Z_0} \qquad (13)$$

where, C_a = concentration at $y = a$; C = concentration at any level y; H = depth of solid-liquid mixture in the open channel; and Z_o = sediment distribution exponent. The theoretical expression of Z_o is given as $w_o/u_*\kappa$, where w_o = unhindered settling velocity of particle size d; u_* = shear velocity and κ = von Karman constant, which was taken to be 0.4.

Itakura and Kishi (8) Model

Itakura and Kishi (8) argued that the value of κ for sediment-laden flows is not different from its value for clear water flow. They proposed a concentration distribution equation as

$$\frac{C}{C_a} = \left\{\left\{\frac{H-y}{H-a}\right\}^{1+D_*}\frac{a}{y}\right\}^{Z_o} \qquad (14)$$

where

$$D_* = \frac{(2.8g\Delta\rho_s H w_o \overline{C})}{\rho_l u_*^3}$$

$$\Delta\rho_s = \rho_s - \rho_l$$

where ρ_s = mass density of sediment; ρ_l = mass density of liquid; and $\overline{C}$ = average volumetric concentration of sediment.

Antsyferov and Kos'yan (1) Model

Antsyferov and Kos'yan (1) proposed a two layer model for the distribution of sediment. They argued that the particle diffusivity ε_s differs considerably from the liquid diffusivity ε_l close to the boundary. Introducing an additive term for ε_s below $y = 0.2H$, they expressed the sediment distribution equation as

$$\frac{C}{C_a} = \exp\left\{ -w_o \int_a^y \frac{\mathrm{d}y}{\frac{u_*^2 y^2 (1-y/H)}{2.8u_* y + 15.7\nu} \tanh \frac{u_* y}{w_o H} + \frac{g\rho_f}{0.14\Delta\rho_s} \left(\frac{\nu^2}{g}\right)^{1/3} (u_o - w_o) \exp(-30y/d)} \right\} \quad (15)$$

where u_o = mixture velocity close to the bottom given by

$$\frac{V_m}{u_o} = \frac{1.25}{\log(8.8H/d)}$$

V_m is the mean velocity of flow and ν is the kinematic viscosity of the mixture.

McTigue (17) Model

McTigue (17) proposed a mixture theory using a two layer model. He considered a liquid and a solid phase and noted the mass and momentum balance equations for both phases. By using turbulent diffusion equation, he derived the following equation:

$$\frac{C}{C_a} = \exp\left\{ \frac{-w_o(y-a)}{K_1 u_* H} \right\} \quad \text{for} \quad y \geq 0.2H \qquad (16a)$$

where $a = 0.2H$ and $K_1 = 0.11$. He also wrote

$$\frac{C}{C_a} = \left(\frac{y}{a}\right)^{\frac{-w_o}{K_2 u_*}} \quad \text{for} \quad y \leq 0.2H \qquad (16b)$$

where $K_2 = 0.35$.

EXPERIMENTAL DATA USED IN THE PRESENT STUDY FOR COMPARISON

The experimental data used for comparison with predictions of Karabelas model are as follows:

Rectangular Duct Data

The rectangular duct data for composite and solids concentration profiles used in the present study are those of Ismail (7) and Kaushal (10). In the present study, five composite concentration profiles reported by Ismail (7) with different efflux concentrations and mean flow velocities using fine sand slurry were considered. Along with Ismail's data, zinc tailing slurry data reported by Kaushal (10) with 15 composite and 90 solids concentration profiles for different combinations of efflux concentration and velocity were also considered in the present study. These experimental data are shown in Table 1 and together cover a wide range of concentration and velocity.

Pipe Data

The pipe data used in the present study were generated primarily in the Fluid Mechanics Laboratory at I.I.T. Delhi by Mukhtar (19), Seshadri *et al.* (26) and Kaushal (10) in 105mm and 54mm diameter pipe for zinc tailings slurry. A total of 56 composite and 310 solids concentration profiles covering a concentration range of 3.8 % to 26 % by volume and a velocity range of 1.25 m/s to 4.0 m/s were generated. The particle size range of zinc tailing was wide enough to cover the range expected in commercial slurries. The ranges covered for different parameters are given in Table 1.

Table 1 Experimental data used in study

S. No.	Author	Geometry	Material	Particle Size Distribution			V_m (m/s)	C_{vf} (%)
				Size fractions range (μm)	d_j (μm)	Fraction (%)		
1.	Ismail (7)	Rectangular Duct (width = 270 mm, height = 75 mm)	Sand mixed with water (d_{50} = 100 μm)	118-134 112-118 98-112 82-98 78-82	126 115 105 90 80	20 20 20 20 20	0.5 to 1.5	0.153 to 0.72
2.	Vanoni (28)	Open Channel (width = 845 mm, depth of flow = 72 mm to 140 mm)	Sand mixed with water (d_{50} = 100 μm)	20-60 60-100 100-140 140-180	40 80 120 160	25 25 25 25	1.0 to 1.5	0.0075 to 0.052
			(d_{50} = 133 μm)	43-83 83-163 163-203 203-283	63 123 183 243	25 25 25 25		
3.	Kaushal (10)	Rectangular Duct (width = 200 mm height = 50 mm) Pipe (diameter = 105 mm)	Zinc tailings mixed with water (d_{50} = 50 μm)	297-1180 210-297 150-210 106-150 75-106 0-75	740 255 180 128 91 38	3.52 10.00 5.73 19.33 13.86 47.56	2.0 to 3.5	3.8 to 26
4.	Mukhtar (19)	Pipe (diameter = 105 mm)	Zinc tailings mixed with water (d_{50} = 91 μm)	297-1180 150-297 106-150 75-106 53-75 0-53	740 224 128 91 64 26.5	3 19.8 26.9 12 8.6 29.3	1.4 to 3.1	4.0 to 25.8
5.	Seshadri et al. (26)	Pipe (diameter = 54 mm and 105 mm)	Zinc tailings mixed with water (d_{50} = 30 μm)	0-75 75-106 106-150 150-300 300-600	38 91 127 219 438	53.67 9.01 23.16 11.73 2.43	1.25 to 4.0	7.6 to 23.75
6.	Morales (18)	Open Channel (width = 200 mm, depth of flow = 53 mm to 123 mm)	Sand mixed with water (d_{50} = 120 μm)	0-30 30-70 70-100 100-500	15 50 85 300	35 25 20 20	1.25 to 2.11	6.0 to 31.2
7.	Winterwerp et al. (31)	Open Channel (width = 300 mm, depth of flow = 53 mm to 99 mm)	Sand mixed with water (d_{50} = 120 μm)	0-80 80-120 120-200 200-240	40 100 160 220	12.5 12.5 37.5 37.5	1.01 to 2.05	3.0 to 30.0
8	Samaga et al. (25)	Open Channel (width = 400 mm, depth of flow = 35 mm to 65 mm)	Sand mixed with water (d_{50} = 155 μm)	105-120 120-155 155-170 170-205	112.5 137.5 162.5 187.5	25 25 25 25	1.0 to 2.5	0.05 to 0.8

Open Channel Data

The open channel data reported in literature by Samaga *et al.* (25), Vanoni (28), Morales (18) and Winterwerp *et al.* (31) were adopted for comparison and are listed in Table 1. Only Samaga *et al.* (25) reported solids concentration profiles. The total sets of experimental data for open channel considered in the present study were 48 composite and 15 solids concentration profiles and the ranges covered for different parameters are also given in Table 1.

The complete data collected in the present study (121 composite and 415 solids concentration profiles) were for the velocities where the particles were fully suspended. There was no deposition at the bottom of the geometries.

COMPARISON BETWEEN MEASURED AND PREDICTED CONCENTRATION PROFILES BASED ON KARABELAS MODEL

A computer program for the Karabelas final solution given by Eq. (12) was developed for a pipe, a rectangular duct and an open channel flow. The solid particles were divided into four to six size fractions and the unhindered settling velocity w_j for the mean diameter of each size fraction was calculated using the drag relationships given in Table 2.

Table 2 Drag relationships

Fall regime and range of particle Reynolds number (R_{ed})	Relation for drag coefficient (C_D)
Stoke's law $R_{ed} \leq 1.0$	$C_D = 24\ R_{ed}^{-1}$
Intermediate $1 < R_{ed} \leq 1000$	$C_D = 24\ R_{ed}^{-1}\ (1+0.15\ R_{ed}^{0.687})$
Newton's law $1000 < R_{ed} \leq 2 \times 10^5$	$C_D = 0.44$

The shear velocity u_* is evaluated as

$$u_* = \sqrt{g r i_m} \tag{17}$$

where $r =$ hydraulic radius $= D/4$ for pipe $= \{HW/(2H+2W)\}$ for the rectangular duct $= \{HW/(2H+W)\}$ for the open channel and i_m is the measured pressure drop in terms of water column per unit length of pipe or rectangular duct at the given concentration and flow velocity and this is replaced by i (slope) in case of open channel.

In the Karabelas model ε_s is assumed to be constant throughout the cross-section and equal to ε_l and thus considering the dimensionless particle diffusivity β equal to 1.0, the following values of ε_s are assumed in the calculations

$\varepsilon_s = 0.044 H u_*$ or $\xi_1 = 0.044$ for rectangular duct, as given by Ismail (7)

$= 0.100 H u_*$ or $\xi_2 = 0.10$ for open channel, as given by Van Rijn (27)

$= 0.07 R u_*$ or $\xi = 0.07$ for pipe as given by Reynolds analogy

Predictions based on Karabelas model for the rectangular duct, pipe and open channel data were compared with experimental data quantitatively in Figures 1 to 3. The comparison for the rectangular duct and open channel were made at $y/H = 0.1$ and 0.9 whereas for pipe at $y/D = 0.1$ ($y' = -0.8$) and 0.9 ($y' = 0.8$). Figure 1 shows a comparison of the rectangular duct with Karabelas model. It was observed that for almost all the data, the concentration profiles obtained from the Karabelas model were more

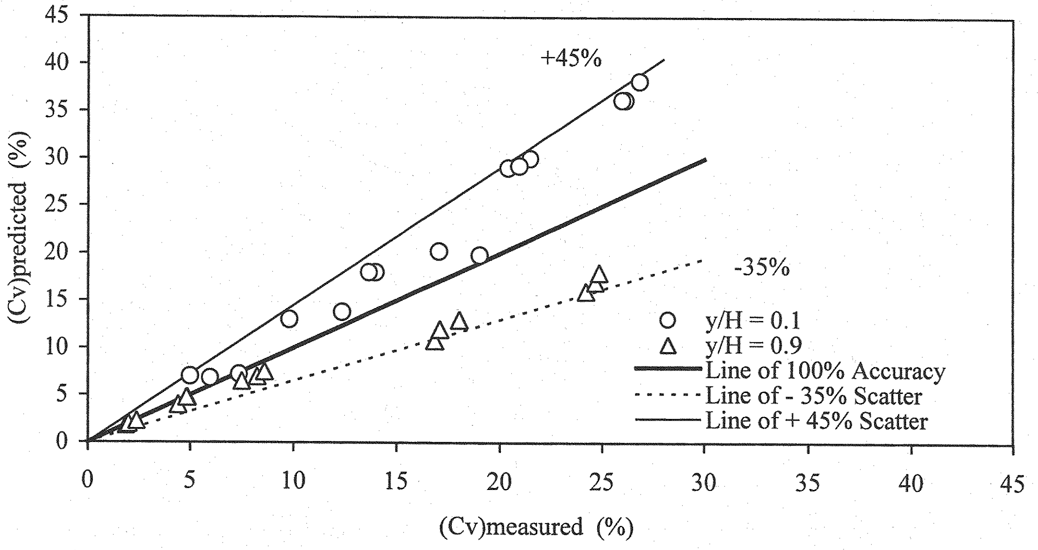


Fig. 1 Comparison between measured and predicted (by Karabelas model) overall concentration profiles for Ismail (7) and Kaushal (10) data in rectangular duct

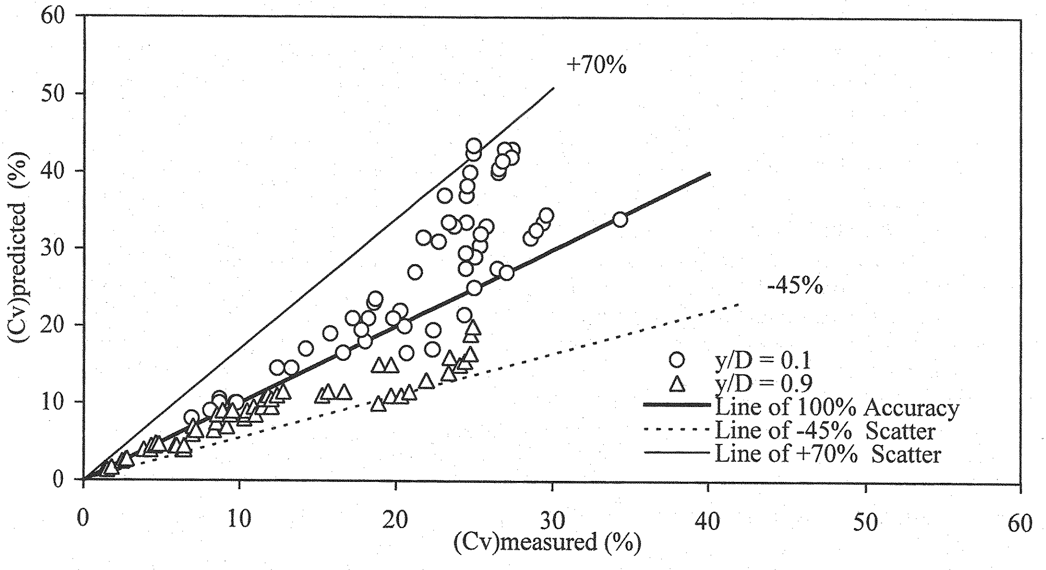


Fig. 2 Comparison between measured and predicted (by Karabelas model) overall concentration profiles for Kaushal (10), Mukhtar (19) and Seshadri et al. (26) data in pipe

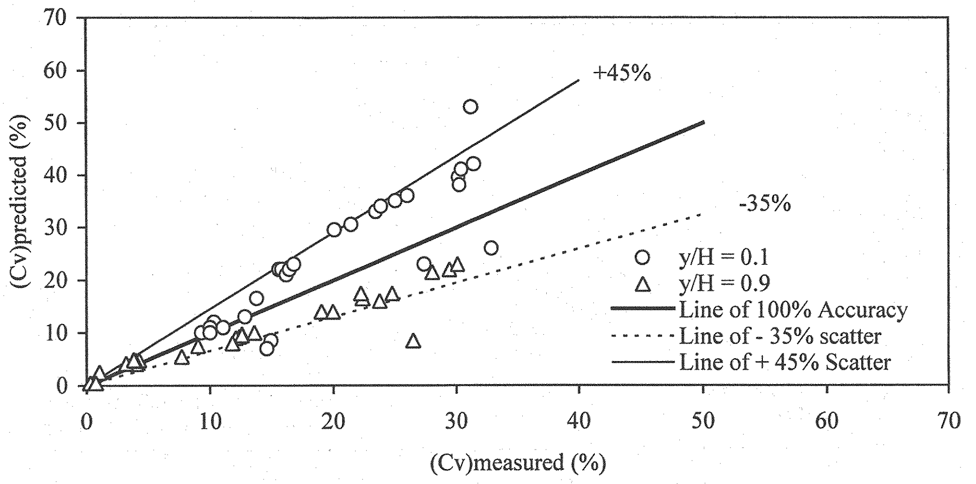


Fig. 3 Comparison between measured and predicted (by Karabelas model) overall concentration profiles for Vanoni (28), Samaga et al. (25), Morales (18) and Winterwerp et al. (31) data in open channel

asymmetric than those obtained experimentally. There was nearly perfect agreement at all the velocities for low efflux concentration ($C_{vf} = 4\%$). For efflux concentrations C_{vf} greater than 4 %, the deviations were systematic and sometimes over-prediction was as high as 45% at the bottom ($y/H = 0.1$) and under prediction is as low as 35% at the top ($y/H = 0.9$) of the rectangular duct. Figure 2 depicts a similar type of graph for Kaushal (10), Mukhtar (19) and Seshadri *et al.* (26) data in pipe. It was observed that for almost all the data the concentration profiles by the Karabelas model were more asymmetric than those measured. For almost all the 56 data points, the deviations were systematic. Only some of the data corresponding to lower efflux concentrations ($C_{vf} = 4\%$) showed good agreement and for C_{vf} greater than 4 %, there is over prediction by as much as 70% at the bottom ($y/D = 0.1, y' = -0.8$) of the pipe and under prediction by as much as 45% at the top ($y/D = 0.9,\ \ y' = 0.8$) of the pipe. Figure 3 presents a similar type of graph for Vanoni (1942), Samaga *et al.* (26), Morales (18) and Winterwerp *et al.* (31) data for open channel. It was observed that for almost all the data, the concentration profiles obtained from the Karabelas model were more asymmetric than those obtained experimentally. For almost all the 48 data points, the deviations were systematic and there was a maximum over prediction of approximately 45% at the bottom ($y/H = 0.1$) of the open channel and under prediction of approximately 35% at the top ($y/H = 0.9$) of the open channel.

From the above comparison, it is evident that Karabelas model predicts the concentration distribution well at lower concentrations but yields large errors at higher concentrations. It was also observed that all of these errors at higher concentrations were systematic. The Karabelas model tends to over predict the concentration at the bottom and under predicts at the top of the pipeline, rectangular duct and open channel.

Furthermore, quantitatively, the concentration profiles predicted by Karabelas model show progressively greater deviations from the measured ones as the solid concentration in the slurry increases. These deviations are expected because the key parameter in the model is $k_j = w_j/(\xi u_*)$. The large value of k_j is usually associated with non uniform solid distribution and small value with uniform distribution. It is also known that value of ξ can only be determined with some confidence for single phase. From these observations, it is clear, that Karabelas model does not account for the changes in fluid and flow properties, which occur with increasing efflux concentrations. The causes for failure of Karabelas model at higher efflux concentrations have been identified as:

(i) The Karabelas model uses unhindered settling velocity in his calculations by not considering the effect of concentration, particle size distribution and duct walls.

(ii) Particle diffusivity ε_s is assumed as constant and is equal to liquid diffusivity ε_l. Further ε_s was assumed to be independent of particle size and solid concentration, which may not be true.

COMPARISON BETWEEN MEASURED AND PREDICTED CONCENTRATION PROFILES IN OPEN CHANNEL BASED ON EXISTING MODELS

Figures 4 and 5 show a comparison of measured and predicted overall concentration profiles in the open channel by Karabelas (9) model using measured efflux concentration as a known parameter for all the data. These figures also show the comparison of measured and predicted overall concentration profiles by Rouse (24), Itakura and Kishi (8), Antsyferov and Kos'yan (1), McTigue (17) model using the measured concentration at reference height of $y' = 0.2$ as a known parameter for all the data. Hirano *et al.* (4) reported that the selection of reference height greatly affects the prediction of solids concentration profile. However, Antsyferov and Kos'yan (1), McTigue (17) and Samaga *et al.* (25) proposed considering the reference height as $y' = 0.2$. The choice of reference height as $y' = 0.2$ is justified by the fact that the predicted concentration profiles by different methods available in literature are crossing the measured profiles with both larger and smaller gradients. From these figures it is observed that most of the models available in literature for open channel flows can predict with reasonable accuracy at lower concentrations but fails to do so at higher concentrations.

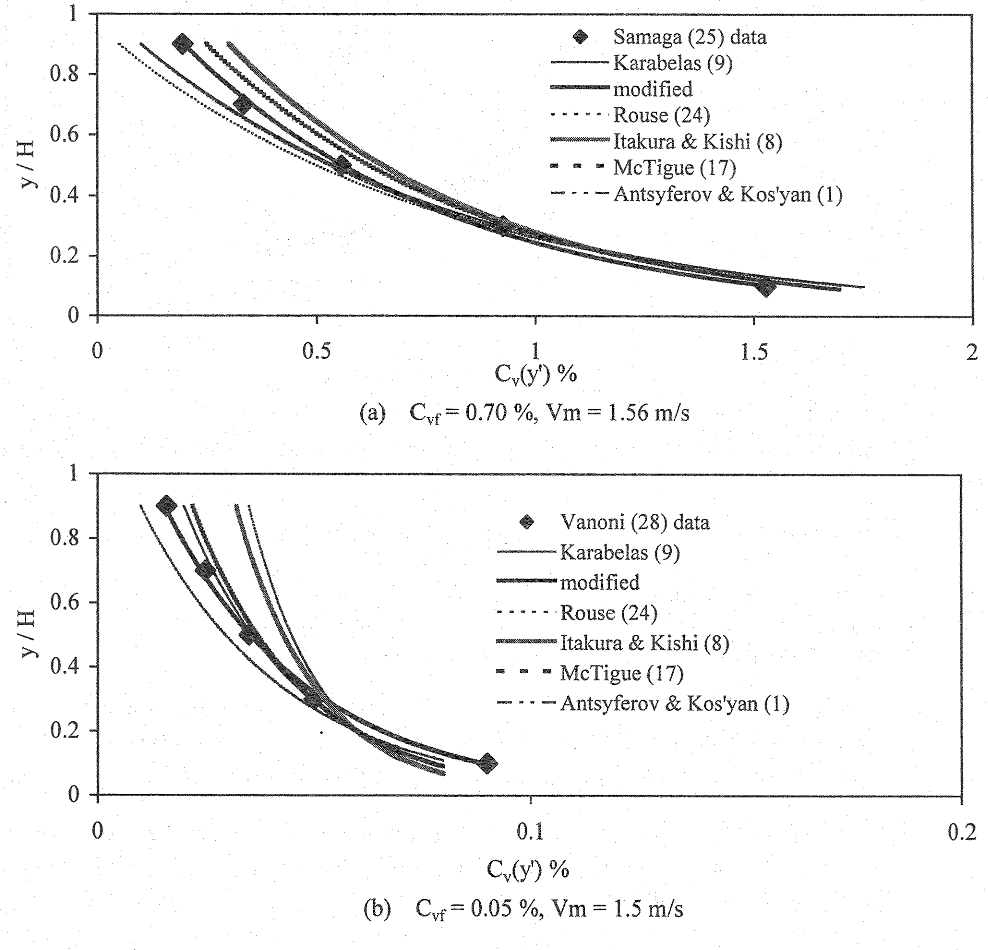
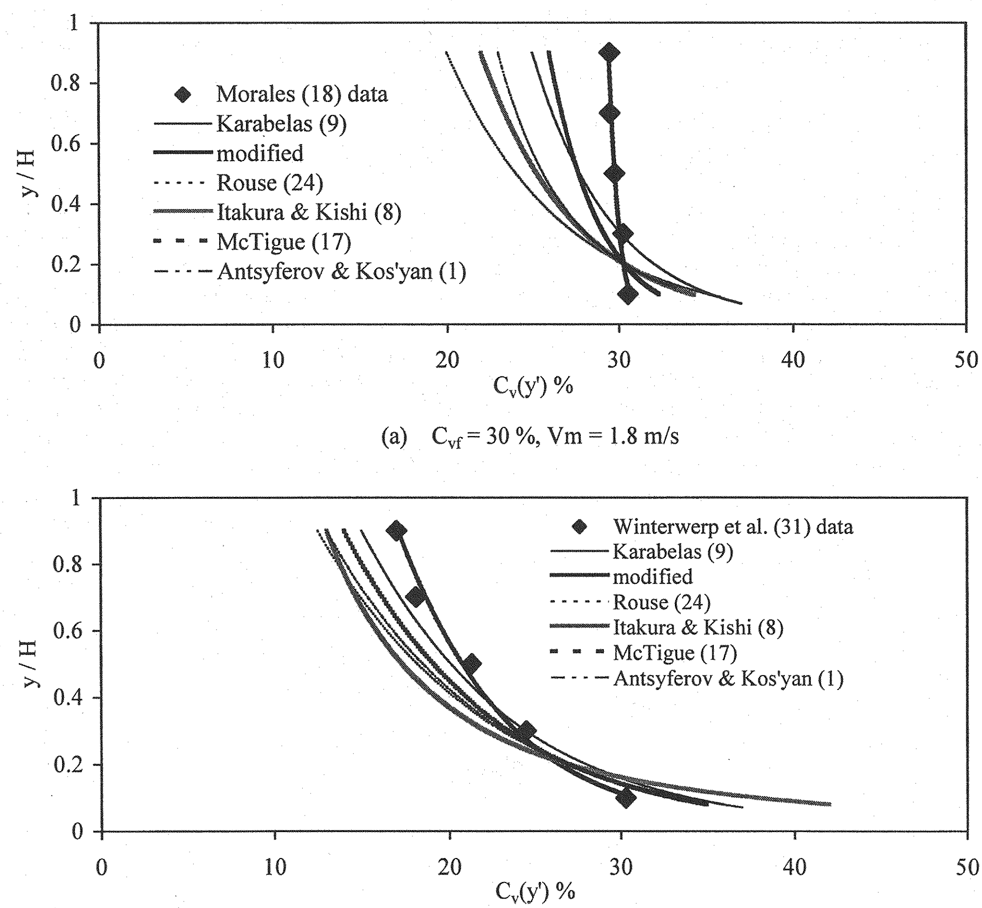


Fig. 4 Measured and predicted composite concentration profiles in open channel for data with lower efflux concentrations



(a) $C_{vf} = 30$ %, Vm = 1.8 m/s

(b) $C_{vf} = 20$ %, Vm = 2 m/s

Fig. 5 Measured and predicted composite concentration profiles in open channel for data with higher efflux concentrations

DESCRIPTION OF MODIFICATIONS INCORPORATED IN KARABELAS MODEL

To overcome the shortcomings in the Karabelas model, a general solution given by Eq. (3) is used instead of a closed form solution given by Eq. (12). The steps taken in the calculations for the modified model as well as the modifications incorporated are given below:

Step 1 Mean relative concentration $\bar{v}_j$ for each size fraction for different experimental data is calculated using Eq. (8). Unhindered settling velocity w_{jo} for the mean diameter of each size fraction is calculated using standard drag relationships (Table 2).

Step 2 Hindered settling velocity w_j for the mean diameter of each size fraction is calculated using the formula given by Richardson and Zaki (23) which accounts for the effect of other particles, concentration and duct walls:

$$w_j = w_{jo}(1-C)^Z \tag{18}$$

for $0.002 < R_{ed} \le 0.2$; $z = 4.65 + 19.5\left(\frac{d_i}{D}\right)$

$0.2 < R_{ed} \le 1.0$; $z = \left\{4.35 + 17.5\left(\frac{d_i}{D}\right)\right\} R_{ed}^{-0.03}$

$1.0 < R_{ed}$; $z = \left\{4.45 + 18.0\left(\frac{d_i}{D}\right)\right\} R_{ed}^{-0.1}$

where R_{ed} = particle Reynolds number; D = diameter of pipe $= H$ = height of rectangular duct or depth of flow in open channel.
In the current modifications, a provision for allowing any variation of w_j across the duct cross-section was incorporated.

Step 3 The value of shear velocity u_* is calculated in the same way as in the case of Karabelas model. It is evaluated from the measured pressure drop for any given concentration and mean velocity for pipe and rectangular duct, and bed slope for open channel.

Step 4 The liquid momentum diffusivity due to turbulent motion is not constant across the duct cross-section and its variation depends on several parameters like duct geometry, flow conditions, *etc*. In order to improve the accuracy of prediction, it is necessary to incorporate a realistic variation of ε_l across the cross section of the duct. By means of experiments, several variations for ε_l across the cross-section of the pipe, rectangular duct and open channel was proposed. In the present modified model, following variations of ε_l were incorporated:
Longwell (16) model for ε_l in pipe:

$$\varepsilon_l = 0.369 y u_* \left(1 - \frac{2y}{D}\right) \text{ for } 0 \le y/D \le 0.33 \text{ and } 0.66 \le y/D \le 1.0$$

$$\varepsilon_l = 0.0775 R u_* \text{ for } 0.33 \le y/D \le 0.66 \qquad (19)$$

Brooks and Berggren (2) model for ε_l in a rectangular duct:

$$\varepsilon_l = \kappa u_* y \left(1 - \frac{2y}{H}\right) \text{ for } 0 \le y/H \le 0.337 \text{ and } 0.663 \le y/H \le 1.0$$

$$\varepsilon_l = 0.11 \kappa H u_* \text{ for } 0.337 \le y/H \le 0.663 \qquad (20)$$

Van Rijn (27) model for ε_l in open channel:

$$\varepsilon_l = \kappa u_* y \left(1 - \frac{y}{H}\right) \text{ for } 0 \le y/H \le 0.5$$

$$\varepsilon_l = 0.25 \kappa H u_* \text{ for } 0.5 \le y/H \le 1.0 \qquad (21)$$

Step 5 The assumption that ε_s is equal to ε_l (or $\beta = 1$), which is not valid, was modified by assuming dimensionless particle diffusivity β to be a function of concentration and particle size and variable across the duct cross-section as given by Kaushal and Tomita (14):

$$\beta_j(y') = 1.0 + 0.125 \{d_j / d_{wmd}(y')\} \exp\{4.22\, C_v(y')/C_{vss}\} \quad (22)$$

where $d_{wmd}(y')$ = local weighted mean diameter defined as

$$d_{wmd}(y') = \frac{\sum\{C_j(y')\, d_j\}}{\sum C_j(y')} \quad ; j = 1, 2, \ldots n \quad (23)$$

$C_v(y')$ is the local concentration and C_{vss} is the static settled concentration.

Step 6 The variation of $f(y)$ across the cross-section for a particular value of dimensionless particle diffusivity $\beta_j(y)$ is calculated as $f(y) = \int_0^y \frac{dy}{\varepsilon_s(y)}$. As the value of w_j has already been calculated, G_j is calculated using the Eq. (10) for the pipe, rectangular duct and open channel by integrating numerically by Simpson's rule. Then, by using G_j, $f(y)$ and w_j, concentration distributions of individual size fractions $C_j(y)$ are calculated for different data with the help of Karabelas general solution given by Eq. (3). The overall concentration profile is predicted by summing up all the concentration distributions of individual size fractions.

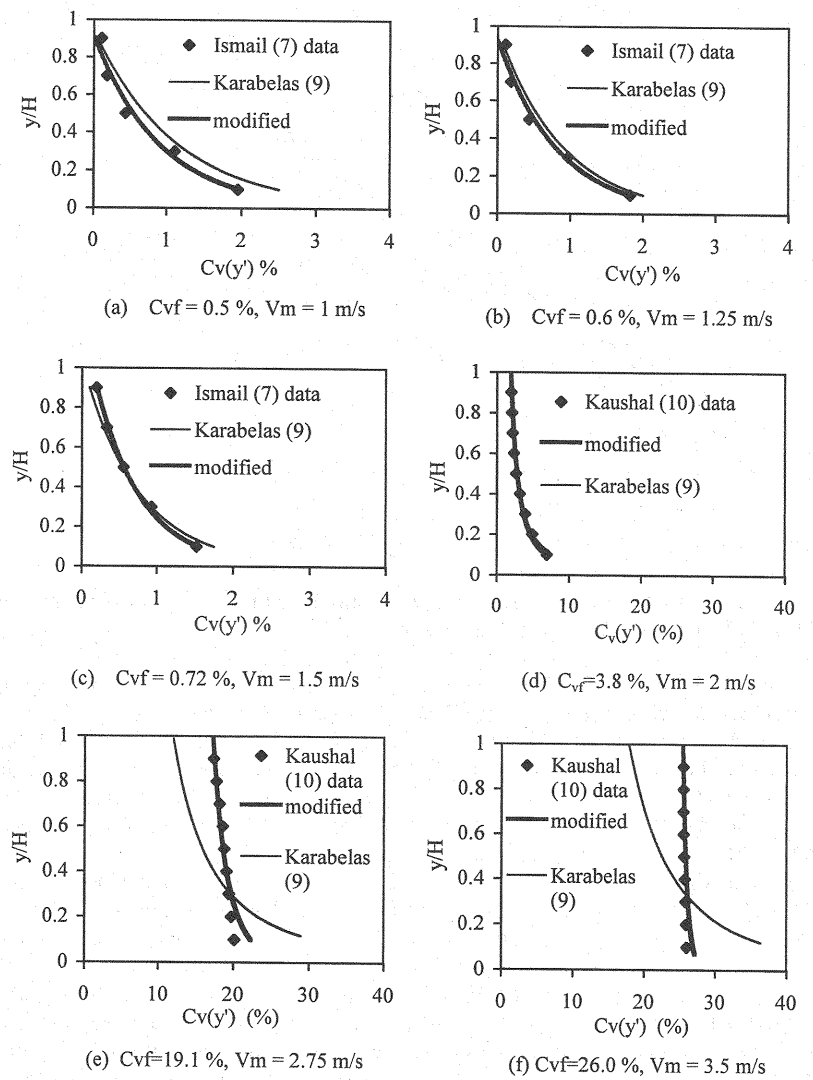
Step 7 During the first iteration, the distribution of individual size fractions and overall concentration profiles are obtained by calculating hindered settling velocity w_j and dimensionless particle diffusivity β_j by Eqs. (18) and (22), respectively, with $C_v = C_{vf}$ for all the data in pipe, rectangular duct and open channel. In the successive iteration, the values of w_j and β_j are calculated using overall local concentration. It should be noted that both w_j and β_j vary across the cross section. Thus by assuming dimensionless particle diffusivity β_j and settling velocity w_j to depend on functions of overall local concentration, overall concentration profiles are calculated by summing up the distributions of individual size fractions. This process is continued until the overall concentration profile converges in successive iterations.

RESULTS AND DISCUSSION

The predictions by the modified Karabelas model for the overall concentration profiles are shown in Figures 4 to 10. Figures 4 and 5 present the comparison of measured and predicted overall concentration profiles in an open channel by the proposed modified model and various models available in literature. It was observed that the proposed modified model yielded better predictions than the models available in literature.

Figures 6 and 7 show the comparison for Ismail and Kaushal data in rectangular duct. It is observed that for almost all the data, the modified Karabelas model yields almost exact fit between measured and predicted overall concentration profiles. One should recall that in the earlier predictions by Karabelas (9) model (Fig.1) the deviations were systematic and sometimes as high as 45%. This is evidence that the predictions by modified Karabelas model for overall concentration profile (Fig. 7) in the rectangular duct are more accurate than the predictions by Karabelas (9) model for the data collected by Ismail (7) and Kaushal (10).

Figures 8 and 9 present the comparison of the measured and predicted concentration profiles by modified Karabelas model for Kaushal, Mukhtar and Seshadri data in pipe. It is observed from Figure 9 that 95% of the data points lie within an error band of ±15%. Further, the data points are distributed randomly around the ideal line. Thus it can be concluded that the modified model predicts the concentration profile with higher accuracy as compared to the Karabelas model. It should noted that in the earlier prediction by Karabelas model (Fig. 2) the deviations were systematic and were sometimes as high as 70%.



(a) Cvf = 0.5 %, Vm = 1 m/s

(b) Cvf = 0.6 %, Vm = 1.25 m/s

(c) Cvf = 0.72 %, Vm = 1.5 m/s

(d) C_{vf}=3.8 %, Vm = 2 m/s

(e) Cvf=19.1 %, Vm = 2.75 m/s

(f) Cvf=26.0 %, Vm = 3.5 m/s

Fig. 6 Some measured and predicted (by modified and Karabelas (9) model) composite concentration profiles in rectangular duct

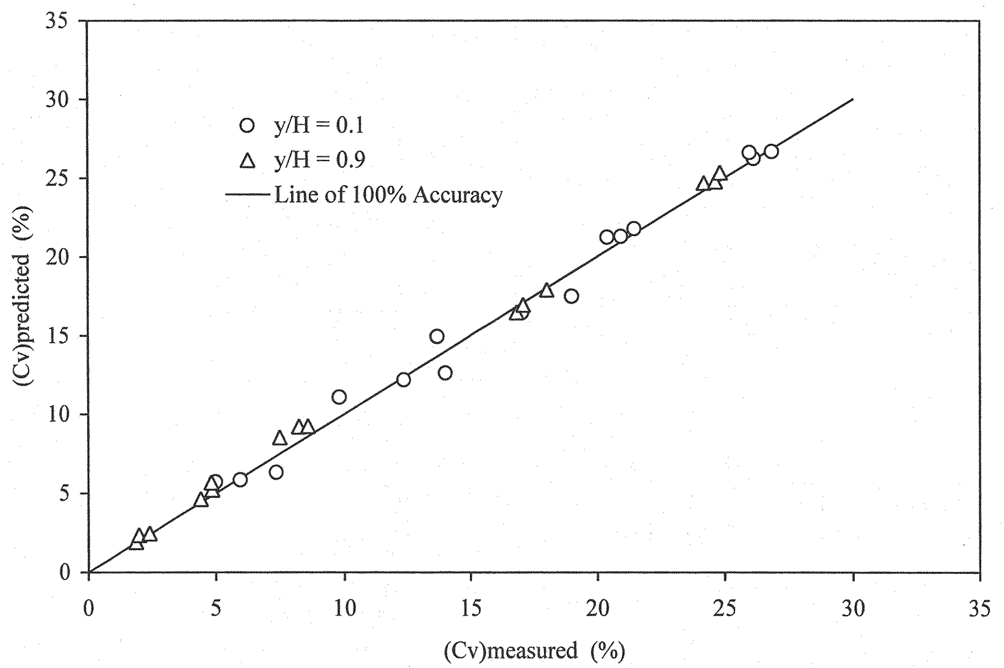
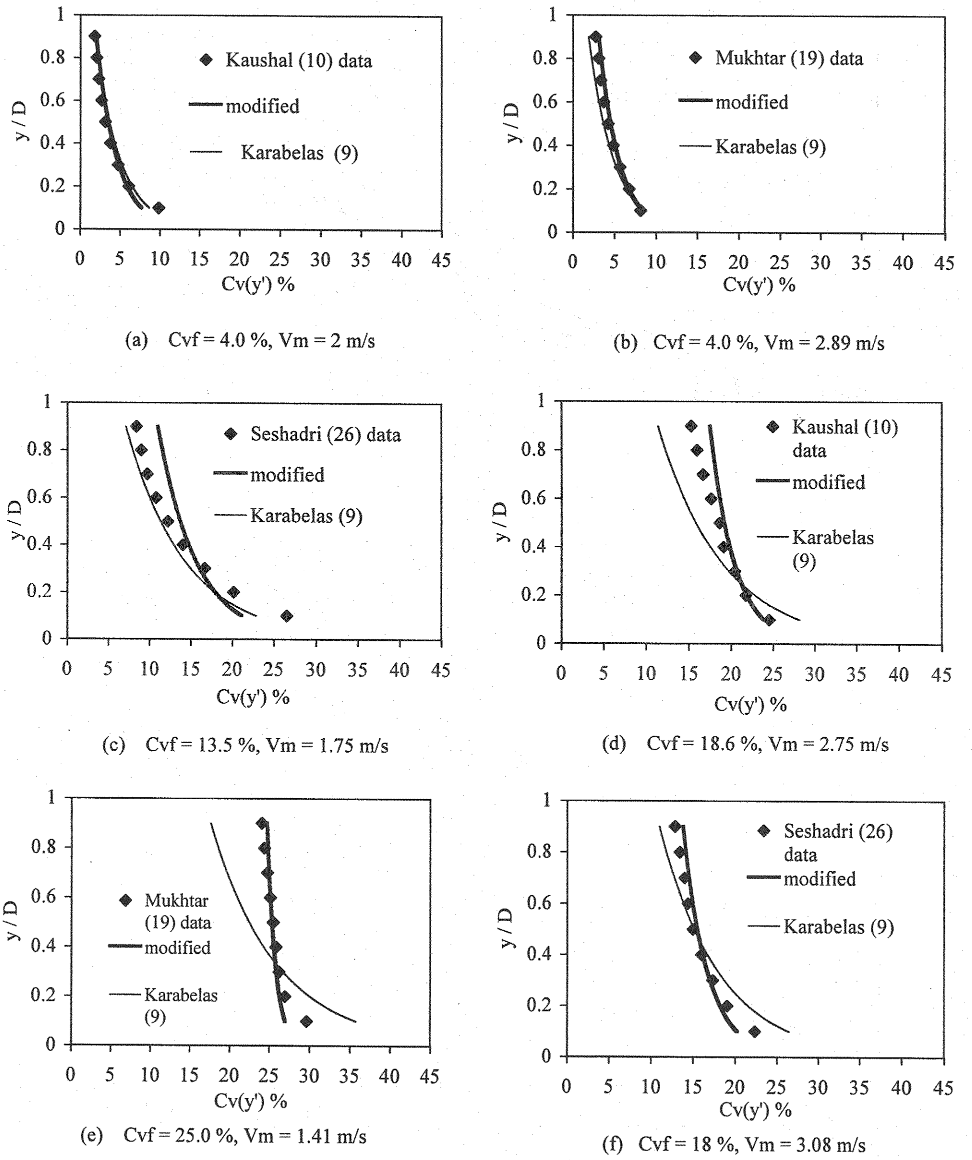


Fig. 7 Comparison between measured and predicted (by modified model) overall concentration profiles for Ismail (7) and Kaushal (10) data in rectangular duct

Figure 10 shows the comparison of measured and predicted overall concentration profiles by the modified Karabelas model for open channel flow data reported by Samaga *et al.* (25), Vanoni (28), Morales (18) and Winterwerp *et al.* (31). It is observed that for almost all the data, the modified Karabelas model gives exact fit between measured and predicted overall concentration profiles. While, for the same set of experimental data in open channel, the comparison for concentration profile as predicted by Karabelas (9) model showed an over prediction of approximately 45% at the bottom and under prediction of approximately 35% at the top of the open channel (Fig.3).

The discrepancies of the order of ± 15 % between the measured and predicted concentration profiles by modified Karabelas model in the pipe may be due to the fact that the Karabelas model considers the concentration to vary only in vertical plane. However, it has already been established experimentally that in pipe, concentration varies in horizontal plane also. The modified Karabelas model gives almost exact predictions in the cases of the rectangular duct and open channel as in most of the data, width to depth of flow ratio is more than 4.0 and hence it is fair to assume the concentration varying only in vertical plane.

The data on concentration profiles of different sizes is too extensive, and it is difficult to present all the quantitative trends in detail in this paper. One of the parameters which quantitatively represents the particle size distribution of the solids is the weighted mean diameter as defined in Eq. (23). Thus, distributions of different sizes across rectangular duct, pipe and open channel can be represented by weighted mean diameter profile across the cross-section. The predictions by the Karabelas and modified model for the weighted mean diameter near the bottom and top, i.e., at $y/D = 0.1$ and 0.9 in pipe, $y/H = 0.1$ and 0.9 for rectangular duct and open channel have been presented in Fig. 11. The percentage standard deviation σ between measured and predicted concentration profiles and weighted mean diameter profiles were also calculated. An evaluation of standard deviation for all the experimental data showed that approximately 80% of the experimental data had a standard deviation less than 5% from the predicted overall concentration and weighted mean diameter profiles.



(a) Cvf = 4.0 %, Vm = 2 m/s

(b) Cvf = 4.0 %, Vm = 2.89 m/s

(c) Cvf = 13.5 %, Vm = 1.75 m/s

(d) Cvf = 18.6 %, Vm = 2.75 m/s

(e) Cvf = 25.0 %, Vm = 1.41 m/s

(f) Cvf = 18 %, Vm = 3.08 m/s

Fig.8 Some measured and predicted (by modified and Karabelas (9) model) composite concentration profiles in pipe

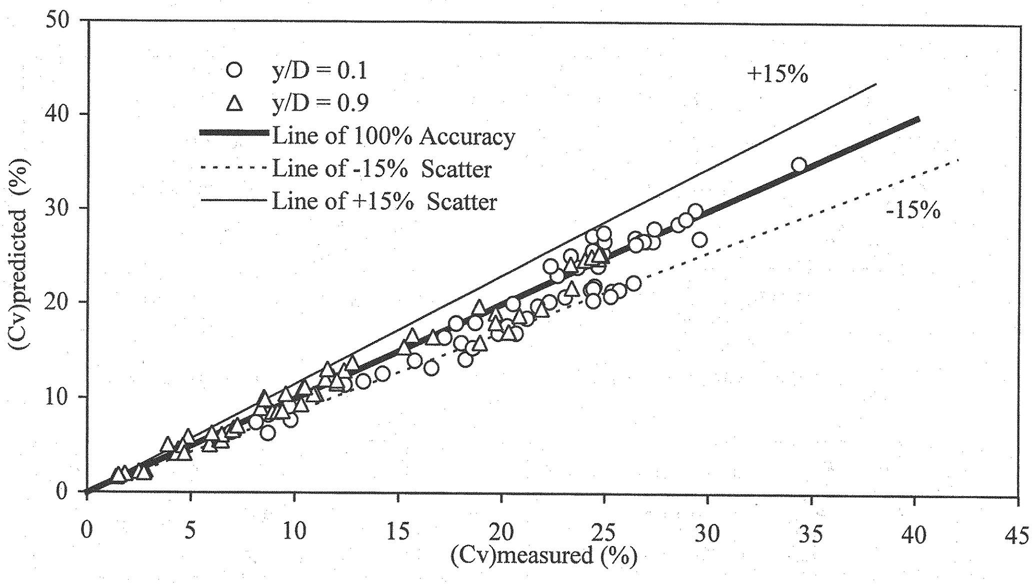


Fig. 9 Comparison between measured and predicted (by modified model) overall concentration profiles for Kaushal (10), Mukhtar (19) and Seshadri et al. (26) data in pipe

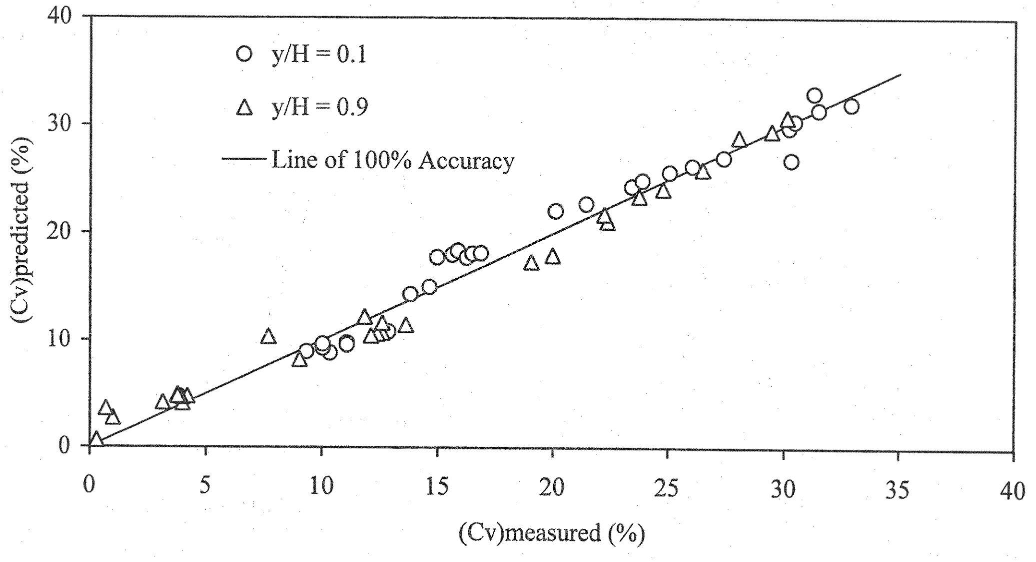


Fig. 10 Comparison between measured and predicted (by modified model) overall concentration profiles for Vanoni (28), Samaga et al. (25), Morales (18) and Winterwerp et al. (31) data in open channel

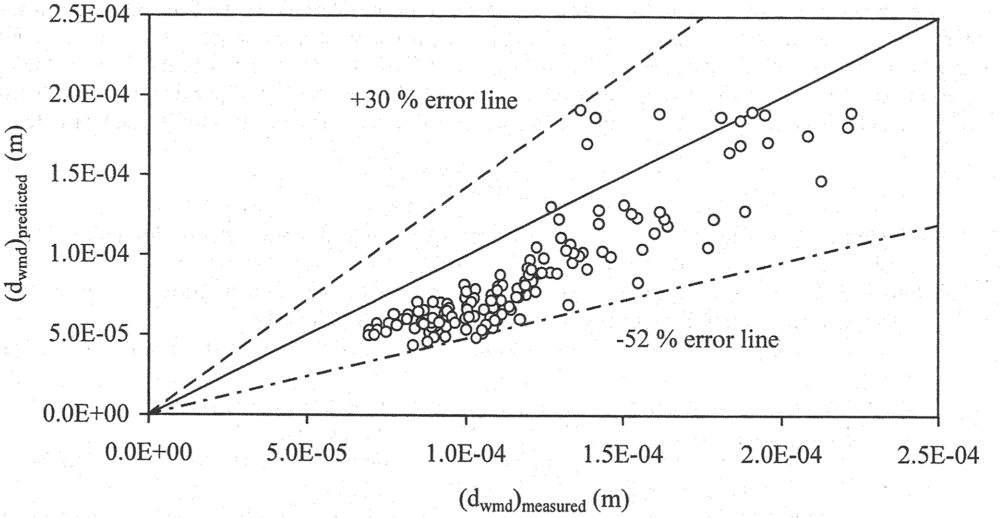


Fig. 11(a) Overall weighted mean diameter predicted by Karabelas (9) model at y/D = 0.1 and 0.9 in pipe, at y/H = 0.1 and 0.9 in rectangular duct and open channel

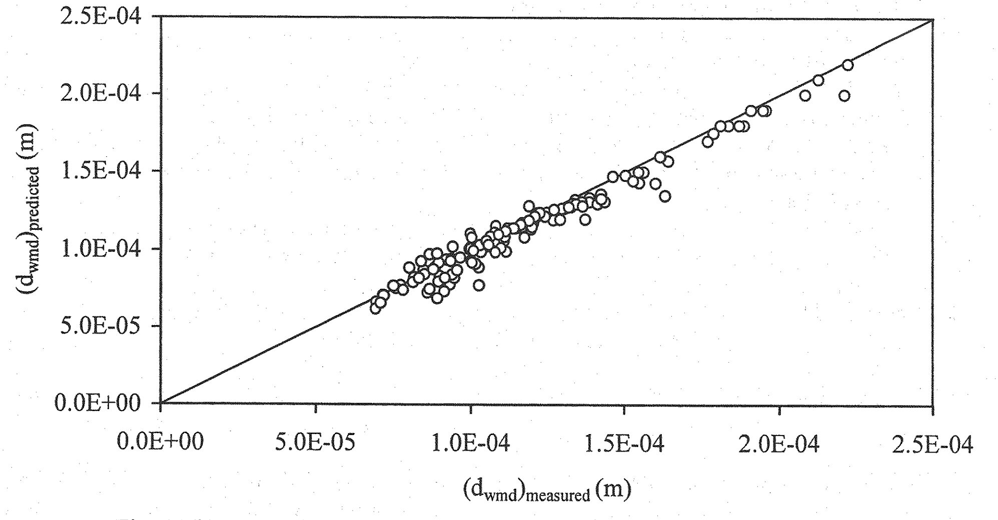


Fig. 11(b) Overall weighted mean diameter predicted by proposed model at y/D = 0.1 and 0.9 in pipe, at y/H = 0.1 and 0.9 in rectangular duct and open channel

CONCLUSIONS

The modified Karabelas model proposed in the present paper produces a good fit not only between measured and predicted overall concentration profiles but also between measured and predicted distributions of individual particle sizes in the flow of multisized particulate slurry through pipe, rectangular duct and open channel. At high solid concentrations, it is essential to consider the effects of local concentration on the particle settling and particle diffusivity as variable with local concentration and particle size. Further study is recommended to determine the dependence of particle diffusion coefficient β on particle shape and other fluid - particle relationships besides efflux concentration and particle size.

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APPENDIX - NOTATION

The following symbols are used in this paper:

C_D = drag coefficient;

C_j = concentration by volume of j th particle size;

C_{vf} = efflux concentration by volume;

C_{vif} = concentration by volume of i th size fraction in efflux;

C_{vss} = static settled concentration by volume;

D = diameter of pipe;

d_i = mean particle diameter of i th size fraction;

d_{wmd} = weighted mean diameter;

d_{wmdf} = weighted mean diameter of efflux sample;

H = depth of flow in open channel or height of rectangular duct;

i_m = pressure drop due to slurry;

k_j = parameter $w_j / \xi u_*$;

R = radius of pipe;

R_{ed} = particle Reynolds number;

u_* = friction velocity;

V_m = average flow velocity;

w = settling velocity;

w_{jo} = unhindered settling velocity of j th size fraction;
w_j = hindered settling velocity of j th size fraction;
W = width of rectangular duct or open channel;
y = height from bottom of the duct;
y' = y/R for pipe, y/H for rectangular duct and open channel;
Z = parameter $1.8\, w/\beta u_*$;
ε_s = mass transfer coefficient or particle diffusivity;
ε_l = momentum diffusivity of liquid or liquid diffusivity;
β = ratio of mass transfer coefficient to liquid momentum diffusivity;
κ = von Karman constant;
ξ = dimensionless eddy diffusivity of liquid.

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