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NUMERICAL PREDICTION METHOD FOR THE PROGRESS OF DETACHMENT OF ALGAE DUE TO GRAVEL PARTICLE CONTACT

By

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SYNOPSIS

A numerical method for predicting the progress of algal detachment due to gravel contact in a flow is developed. This numerical method is constructed on the basis of the Discrete Element Method, which can compute three-dimensional motion of each gravel particle independently. Moreover, an experiment is also conducted in a laboratory stream. In this experiment, an algae-covered block is placed in a testing flow channel and a certain amount of spherical particles or gravel continued to be added into the flow. The apparent status of algae is recorded by means of a digital video camera and a simple threshold technique is applied to the captured still image in order to measure the area of attached algae. The developed numerical method is applied to simulate the experiment for its validation. The velocity profiles of particles and time series of the attachment rate of algae measured in the experiment are compared with the numerical results.

INTRODUCTION

Hyperplastic periphyton caused by eutrophication often causes environmental problems because it is unpleasant to view and often emits a putrid smell. In order to preserve a riverine environment, it is preferable to detach hyperplastic periphyton from the river bed at fixed intervals. It is known generally that periphyton, such as sessile algae and bacteria, is swept away by the increase of flows due to discharge from dams or floods. Therefore, if the relationship between current velocity and removed quantity or area of periphyton is known, it becomes possible to predict changes in distribution of periphyton on river beds by numerical simulation. Several experimental studies have been made on the effects of current velocity on algal attachment and detachment (e.g. (9),(10)). Horner *et al.* (6) performed research in laboratory streams to evaluate periphytic loss rate with variable velocity. Their experiments demonstrated that sudden increase in current velocity raises instantaneous periphytic loss rates, and they concluded that the relative strength of the attachment of algae is a function of current velocity prevailing during colonization and growth, though algal species certainly differ in their attachment ability. Besides current velocity, it is presumed that transported gravel also have considerable effects for algal detachment. Kitamura *et al.* (7) conducted an experiment in a flume in order to evaluate the effects of gravel addition on algal detachment. They suggested that the detachment rates of algae is related to the impact of the friction contacting particles. However, except for the mentioned study, few studies have focused on the effects of transported gravel on algal detachment, and little is known about its detail at present.

To contribute to the understanding of the effect of bed load on detachment of algae on riverbed, we developed a numerical method that can predict the progress of algal detachment due to particle contact on a flat bed. The numerical method developed in this study was constructed on the basis of the Discrete Element Method (DEM). The DEM is a numerical method which can compute each gravel particle motion individually, and can treat contact forces acting between particle and contacting object easily. It is therefore widely used now in many fields (e.g. (4),(5)). We proposed a model for evaluating the area of algae detached by contacting gravel, and combined it into the DEM. An experiment was also conducted in a laboratory stream to compare and to validate the developed numerical method. The velocity profiles of particles and time series of the attachment rate of algae were compared between experimental results and numerical results.

NUMERICAL PROCEDURE

Particle Motion

The DEM proposed by Cundall and Strack (3) is based on the Lagrangian approach to simulate the motion of granular materials. By applying the Newtonian equation of motion, the translational motion of each particle is calculated as

$$m\frac{d\boldsymbol{u}_p}{dt} = \sum \boldsymbol{F}_E + \sum \boldsymbol{F}_C \tag{1}$$

where m = mass of particle; $\Sigma\, \boldsymbol{F}_E$ = sum of the external forces; $\Sigma\, \boldsymbol{F}_C$ = sum of the contact forces (particle-particle and particle-wall) acting on the particle; $\boldsymbol{u}_p$ = velocity of the particle;

Similarly, the rotational motion of each particle is calculated as

$$I\frac{d\boldsymbol{\omega}_p}{dt} = \boldsymbol{T} \tag{2}$$

where I = moment of inertia; ω_p = angular velocity; $\boldsymbol{T}$ = torque; These equations are integrated to obtain the velocities in the x, y and z direction and the angular velocity, and the new coordinates of particle can be found adding the original coordinates with the incremental displacement obtained by integrating the obtained velocities.

The external forces considered in this study are gravitational, Magnus and fluid drag forces. The Basset history term and the virtual mass are also taken in account, though the Saffman lift force and the force due to the pressure gradient are neglected as their contribution is minimal.

The Magnus force acting on a rotating spherical object is shown as

$$\boldsymbol{F}_L = \frac{\pi}{8} C_l d^2 \rho_f \left|\boldsymbol{u}_r\right| \frac{\boldsymbol{u}_r \times \boldsymbol{\omega}_r}{\left|\boldsymbol{\omega}_r\right|} \tag{3}$$

where C_l = Magnus force coefficient; d = diameter of particle; ρ_f = density of fluid; $\boldsymbol{u}_r$ = relative velocity vector; ω_r = relative angular velocity vector;

The fluid drag force on a single spherical particle is expressed as

$$\boldsymbol{F}_D = \frac{\pi}{8} C_d\, d^2 \rho_f\, \boldsymbol{u}_r \left|\boldsymbol{u}_r\right| \tag{4}$$

where C_d = fluid drag coefficient; Since the fluid drag coefficient taking the effect of surrounding particles into account is unknown, we used the fluid drag coefficient for a single particle proposed by Schiller *et al.*(11).

The virtual mass of the particle relative to the ambient fluid is given by

$$\boldsymbol{F}_{VM} = \frac{\pi}{12} \rho_f\, d^3 \frac{d\boldsymbol{u}_r}{dt} \tag{5}$$

and the Basset history term is given by

$$\boldsymbol{F}_{BT} = \frac{3d^2}{2} \sqrt{\pi \rho_f \mu} \int_0^t \frac{d\boldsymbol{u}_r}{dt} \frac{dt'}{\sqrt{t-t'}} \tag{6}$$

where μ = viscosity of fluid;

The contact forces act normally and tangentially to the particle surface. Cundall and Strack (3) modeled these forces as follows:

$$\boldsymbol{F}_{Cn} = -\kappa_n \boldsymbol{\delta}_n - \eta_n \boldsymbol{v}_n \tag{7}$$

$$F_{Ct} = -\kappa_t \delta_t - \eta_t v_t \tag{8}$$

where δ_n , δ_t = particle displacements in the normal and tangential directions; v_n, v_t = particle velocities in the normal and tangential directions; k_n , k_t = stiffness of the spring in the normal and tangential directions; η_n , η_t = damping coefficient in the normal and tangential directions; The stiffness is determined from material properties and geometric conditions of the two contacting particles. The normal stiffness is derived from Hertz theory (12) and can be written as follows:

$$\kappa_n = \frac{4}{3}\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1}\left(\frac{r_1 + r_2}{r_1 r_2}\right)^{-\frac{1}{2}} \tag{9}$$

where ν = Poisson's ratio; E = modulus of restitution; r = radius of particle; subscripts 1 and 2 mean two types of elements; The stiffness for a particle-wall contact is obtained by setting $r_2 = \infty$. The damping coefficient is derived from a second order damped system by Cundall (2) as

$$\eta_n = 2\sqrt{m\kappa_n}$$

In this paper, the tangential stiffness is assumed to be proportional to the normal stiffness with proportionality constant s = 0.25, as proposed by Kiyama et al. (8).

$$\kappa_t = s\,\kappa_n \tag{10}$$

$$\eta_t = \eta_n \sqrt{s} \tag{11}$$

Model for Detachment of Algae Due to Contacting Particles

Algal adhesion force on a substrate seems to be dependent on algal species, growth and current velocity prevailing during colonization and growth. Therefore, supposing that detachment of algae due to particle contact is mainly caused by friction, the force required to detach algae can be written as follows:

$$f_d = \alpha f_{fric} \tag{12}$$

where α = proportional coefficient related to species and growth of algae; f_{fric} = friction force; Assuming that an area of algae detached by a particle is proportional to the contact surface area between a particle and algal surface, the amount of detached area by a particle is given by

$$A_d = \gamma\, A_{cont} \tag{13}$$

where γ = proportional coefficient; A_{cont} = contact surface area; Supposing that the coefficient γ is proportional to the work done by the force to detach algae, the coefficient γ can be written as

$$\gamma = \beta\, f_d \Delta L \tag{14}$$

where β = proportional coefficient (Nm^{-1}); ΔL = distance the particle moved during the short time interval Δt; The friction force (f_{fric}) equals the coefficient of friction (μ_f) times the vertical force (f_n), and ΔL equals Δt times the root mean square speed of particle, which is given by a combination of velocities in the horizontal directions. Combining these relations and Eq.(12),(14), Eq.(13) then becomes

$$A_d = C\,\mu_f\, f_n \sqrt{v_x^{\,2} + v_y^{\,2}}\, \Delta t \tag{15}$$

where $C = \alpha\,\beta A_{cont}$ (m/N); v_x , v_y = particle velocities in the x and y directions; We define the unknown parameter C as detachment coefficient here. The vertical force f_n is estimated from Eq.(7) and the detachment coefficient is determined from experimental results.

In order to compute the progress of detachment of algae, the following steps are taken. First, place the initial algae-attached area partition on the computing channel floor and divide the partition into n_x-by-n_y cells, which contain information about cell position and the area of attached algae (Fig.1). Next, look for particle contacting on cell(i,j), where i = 1 to n_x, j = 1 to n_y, at the time of t. If a contacting particle exists, the area detached by the particle during the short time interval Δt is estimated from Eq.(4). Then, the estimated amount is deducted from the amount of the area of algae on cell(i,j) at time t. If the area of algae on cell(i,j) becomes less than zero, calculation of detachment on the cell(i,j) is discontinued from that time. This operation is repeated for all cells and the total amount of an area of algae at time $t+\Delta t$ is calculated by adding up each amount of cell information about the area of attached algae. Consequently, the attachment rate of algae, the ratio of total area of existing algae to total area of initial attached algae, at time $t+\Delta t$ can be written as follows:

$$Q^{t+\Delta t} = \frac{\sum_{i,j} A_{i,j}^{t+\Delta t}}{\sum_{i,j} A_{i,j}^{0}} = \frac{\sum_{i,j} \left(A_{i,j}^{t} - \sum_{n} (A_d)_n\right)}{\sum_{i,j} A_{i,j}^{0}} \tag{16}$$

where $Q^{t+\Delta t}$ = attachment rate at time $t+\Delta t$; $A_{i,j}^{t+\Delta t}$ = an area of attached algae in cell(i,j); $A_{i,j}^{0}$ = initial area of attached algae in cell(i,j); n = total number of particles contacting on cell(i,j) at time t;

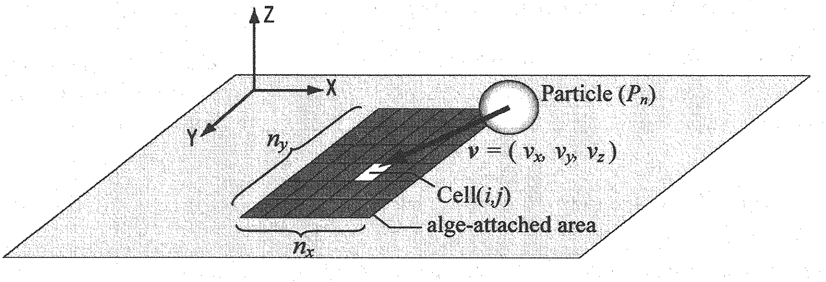


Fig. 1. Model for progress of algal detachment by contact particle

SIMULATION OF THE EXPRIMENT

Experimental Apparatus and Procedure

An experiment was performed for the purpose of validation and comparison of the developed numerical method. The experiment was conducted in a acrylic channel with a length of 7m, a width of 0.2m, and a water depth of 0.15m (Fig.2). Water in the channel was circulated from downstream to upstream by a centrifugal pump, and the water flow rate was controlled by means of a valve at the recirculation pipe. An algae-covered block was placed on the bottom of downstream and a certain amount of aluminum spherical particles or gravel (non-spherical) continued to be added into the flow. The conditions of experiment are given in Table.1. Under the near bottom velocity conditions in this experiment, it was confirmed by a preliminary experiment that little algae was detached by current flow.

The algae used in this experiment were developed in laboratory channel in which river water was contained.

Unglazed blocks with a length of 0.1 m, a width of 0.2m and a height of 0.03m, were placed in the channel for approximately 2 weeks under conditions of current velocity of 0.2 m/s, lighting for 18 hours a day, and sufficient nutrient supply. Consequently, the blocks were covered with algae. It was observed by means of an optical microscope that some green algae, diatom (*Synedra*) and blue-green algae (*Phormidium*) were dominant on the test block.

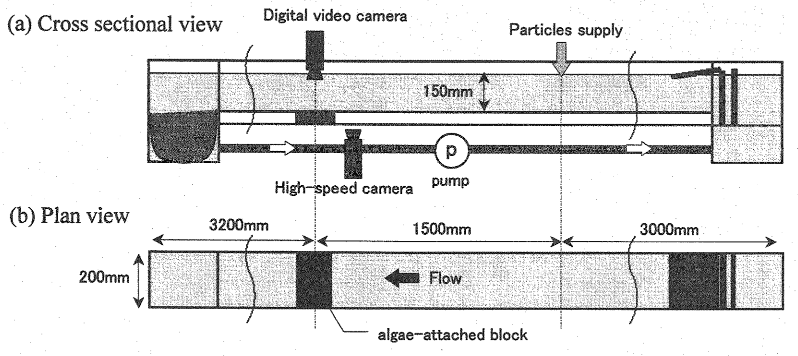


Fig.2. Experimental setup

Table.1 Conditions of experiment

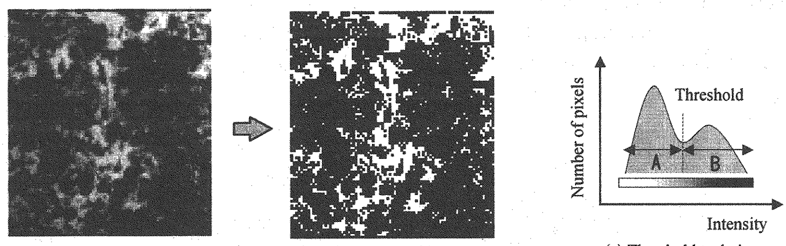
TEST No.	Near bottom velocity(m/s)	Shape of particle	Diameter (mm)	Mixing rate of particles (g/s)	Mixing rate of particles (particles/min, approx.)
G3	0.6	gravel	4 (avg.)	3.8	3000
G4	0.6	gravel	4 (avg.)	1.44	1200
G5	0.4	gravel	4 (avg.)	1.44	1200
L6	0.6	sphere	5	11.84	3000
L7	0.6	sphere	5	4.52	1200
L8	0.4	sphere	5	4.52	1200
S9	0.6	sphere	3	2.69	3000
S10	0.6	sphere	3	1.02	1200
S11	0.4	sphere	3	1.02	1200

It was observed in the experiment that attached algae were gradually detached from the block by contact of particles which were transported by flowing water. The apparent status of algae was recorded by means of a digital video camera, and a simple threshold technique was applied to the captured still image in order to measure an area of algae. The threshold technique can transform an image into a two-toned image, algae region and background, by comparing each pixel against a threshold value (Fig.3). In order to determine a threshold value easily, we used light-colored block for algal substrate because of its sufficient contrast from algae-covered area. An area of attached algae can be determined by counting the number of pixels within segment B in Fig.3. Thus, the attachment rate, the ratio of the total area of existing algae to the initial total area of attached algae, can be derived from following equation:

$$Q = N_B / N_{B0} \qquad (14)$$

where N_B = number of pixels within segment of algae (segment B); N_{B0} = initial value of N_B; The attachment rate variations with time were evaluated by analyzing time series of still images with this threshold technique.

Experiment L8 and S11 resulted in failure. The detachment speed observed in case L8 and S11 were higher than that of L9 and S11 respectively in spite of their lower current velocity condition and same particle size. The reason for this unreasonable results is presumed that the attachment ability of algae on test blocks used in experiment L8 and S11 was not uniform. The experimental results of time series of attachment rate in case of L8 and S11 were therefore not compared with predicted results in the present paper.



(a) Original image (b) Segmented image (c) Threshold technique

Fig 3. Segmented image based on threshold

Conditions of Simulation

The conditions of simulation, such as particle size, mixing rate of particles, location of algae-covered block, initial area of attached algae and water depth, were set to same conditions of the experiment. Table 2 shows the physical properties of the particle, wall and algae-attached block. Although the dynamic friction coefficient between particle and algae-attached block should be directly measured by experiment, it could not be estimated due to a lack of equipment for this study. The value of the coefficient is therefore assumed to be 0.3 in this paper.

In the DEM calculation, the fluid flow domain is usually divided into cells and fluid velocity is averaged in the cell using a weight function. In this study, the fluid velocity in the cell was determined from the experimental data shown in Fig.4. This vertical profiles of flow velocity were measured by acoustic doppler velocimeter. Since the fluid flow in the experiment can be treated as steady flow, the vertical velocity profile in the entire simulated flow field was set to the experimental value shown in Fig.4.

Moreover, n_x and n_y, the number of the division of algae attached area in the x and y directions, were set to 100 and 200 respectively.

Table.2 Physical properties

Young's moudulus (Pa)	Side and buttom wall	7.13×10^{10}
	Aluminum particle	7.03×10^{10}
	Gravel, algae-atached block	3.92×10^{10}
Poisson's ratio	Side and buttom wall	0.220
	Aluminum particle	0.345
	Gravel, algae-atached block	0.300
Density (kg/m^3)	Aluminum particle	3600
	Gravel	2650
Coefficient of dynamic friction	Particle-Particle	0.230
	Particle-Wall	0.176
	Particle-algae-attached block	0.300

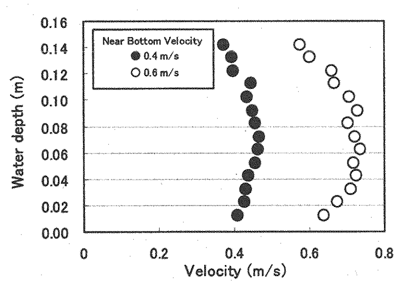
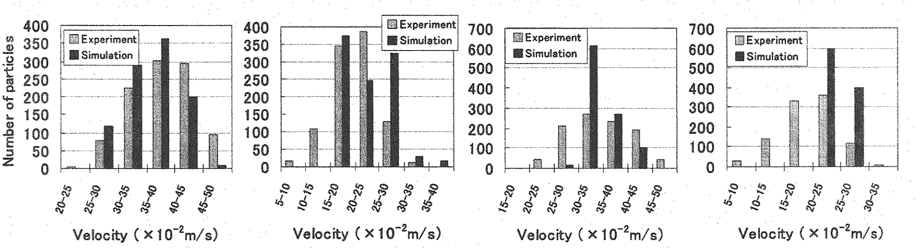


Fig 4. Vertical velocity profile of fluid flow

Particle Velocity

The velocity profiles of particles were compared between experimental results and numerical results for the evaluation of numerical particle motion. In our experiment, Particle Tracking Velocimetry (PTV) technique was applied to measure individual particle velocities in horizontal directions (V_x,V_y) . PTV is one of flow measurement methods which uses image processing and is capable of tracing individual particle paths. In the experiment, images of moving particles were captured by a high-speed camera positioned under the bottom of channel, and the four-flame particle-tracking algorithm (1), which is one of the main PTV algorithms, was used to determine the correct particle velocities.

Fig.5 shows the distribution of the magnitude of particle velocities in horizontal directions sampled randomly in the case of L7, L8, S10 and S11. The each total number of velocity data sampled experimentally and numerically was a thousand per the respective case. Fig.5(a) and (b) show that the predicted particle velocities are agree with the experimental results in the case of L7, L8, though Fig.4(c) and (d) show that numerical results have a distribution in a slightly higher velocity range compared with experiments in the case of S10, S11.



(a) L7 (b) L8 (c) S10 (d) S11

Fig.5. Distribution of the magnitude of particle velocities

Parameter Estimation

Experimental time series of attachment rate data were applied to determine the detachment coefficient *C*, which is defined in Eq.(15). Since numerical particle velocities agreed well with experimental results as shown in the previous section, the data of experiment L6, which was conducted under the conditions of near bottom velocity of 0.6 m/s, particle diameter of 5mm, particle mixing rate of 3000 particles/s, were used for determination of parameter *C*. The simulation was conducted with the coefficient *C* varying from 0.05 to 0.25 in increments of 0.05. Time series of attachment rate of algae obtained experimentally are plotted with the numerical curves in Fig.6. Findings from Fig.6 indicate that the numerical curve agree well with the experimental results adequately when $C = 0.15$. Thus simulations of other experimental cases in which spherical particles were used were carried out with this value $C = 0.15$.

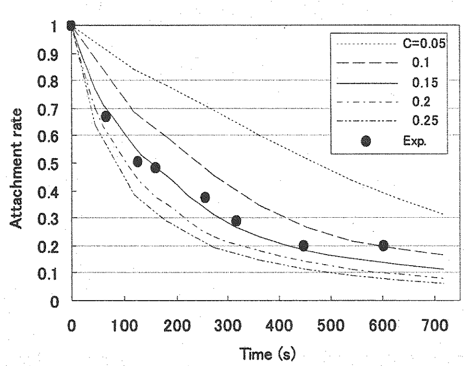


Fig.6. Estimation of the attachment coefficient *C*

Comparison of the Results

The simulations of experiment L6-L7, S9-S10 were conducted with the value of the detachment coefficient C=0.15, as determined previous paragraph. Fig.7 shows the attachment rate variations with time obtained by simulations and experiments. It was found from the figures that detachment of algae was increased under conditions of higher particle concentration, larger diameter of particle and faster current velocity. It was also found that the predicted time series of the algal detachment rate agreed with the experimental results in case of L6, L7, S9 and S10.

The simulations of experiment G3-G5 in which non-spherical gravel was used for algal detachment were also carried out. The model of Discrete Element Method used in this study is intended for spherical particles. Therefore, in the case of simulating G3-G5, the motion of particles is likely to be miss-estimated, though the gravel particle used in experiment G3-G5 has round shape and the influence of particle shape seems not to be enormous. Additionally, it is obvious that contact surface area which is included in the detachment coefficient C is different between a spherical particle and non-spherical gravel. In order to cope with these circumstances, the detachment coefficient C was re-estimated with the result of experiment G4 in the same way as described previously. As a result, it was found that the value of C=0.2 was suitable for simulating the algal detachment by gravel contact. Therefore, simulations for experiment G3-G5 were conducted with the value of C=0.2. Time series of the attachment rate in the case of G3-G5 obtained experimentally was plotted with the corresponding numerical curves in Fig.8. It was found by comparing Fig.7 and Fig.8 that gravel was much effective for algal detachment because of its large contact surface area, though its average size was smaller than that of a spherical particle used in L6-L7. Findings revealed that numerical results of G3 and G4 agreed with the experimental results. However, in the case of G5, experimental detachment was retarded as compared with numerical results. When experiment G5 was conducted, it was observed that small part of gravel stopped tmoving and accumulated on the bottom of channel before reaching the algae-attached block. Therefore, the progress of algal detachment was delayed. In contrast, under the condition of experiment G5, there was no accumulation in the simulation because the numerical method treats a gravel particle as a spherical particle, and the force of friction acting on a spherical particle was much less than the resultant force of external forces, such as the fluid drag force. We concluded that the disagreement between the experiment and the simulation in the case of G5 was due to these circumstances.

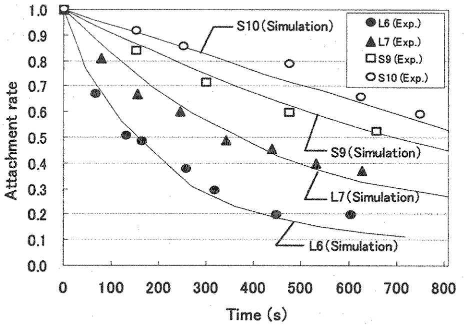


Fig.7 Time series of attachment rate (L6-L7,S9-S10)

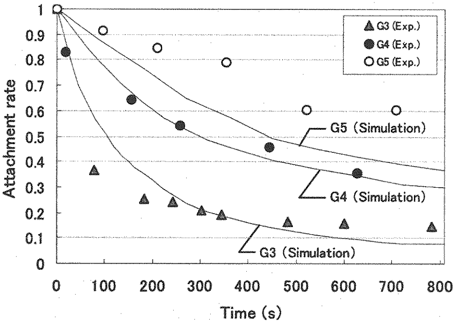


Fig.8 Time series of attachment rate (G3-G5)

CONCLUDING REMARKS

A numerical method for predicting the progress of algal detachment due to contact of gravel in a flow was developed. The developed numerical method was constructed on the basis of the Discrete Element Method, which can compute three-dimensional motion of each gravel particle independently. A model for evaluating the area of algae detached by contacting gravel particle was also proposed and combined into the numerical method. In addition, an experiment was conducted in a laboratory stream. The apparent status of algae was recorded by means of a digital video camera and a simple threshold technique was applied to obtained images. As a result, the variation with time of algal attachment rate, the ratio of total area of existing algae to initial area of attached algae, was measured experimentally. The simulation of the experiment was performed with the developed numerical method for the purpose of comparison and validation of the proposed model. Therefore, the validity of the model was confirmed by attaining an agreement with the measured particle velocity profiles and time series of attachment rate. In this simulation, the unknown detachment coefficient C defined in the proposed model was determined from the experimental result. The value of the coefficient C is presumed to change according to species and growth of attached algae. It is therefore not possible to surmise that the value of C determined in this simulation is applicable to other cases of algal detachment. Further studies are required to examine the applicability of this numerical method to other general cases.

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APPENDIX - NOTAION

The following symbols are used in this paper:

A_{cont} = contact surface area;

$A_{i,j}$ = area of attached algae in cell(i,j)

A_d = area of detached algae;

C = detachment coefficient;

C_d = fluid drag coefficient;

C_l = Magnus force coefficient;

d = diameter of particle;

E = modulus of restitution;

$\boldsymbol{F}_{BT}$ = Basset history term;

$\boldsymbol{F}_C$ = contact force;

$\boldsymbol{F}_D$	=	fluid drag force;
$\boldsymbol{F}_E$	=	external force;
f_{fric}	=	friction force;
$\boldsymbol{F}_L$	=	Magnus force;
f_n	=	vertical force acts on algae surface;
$\boldsymbol{F}_{VM}$	=	virtual mass of the particle relative to the ambient fluid;
I	=	moment of inertia;
k	=	stiffness of the spring;
ΔL	=	distance the particle moved during the short time interval;
$\mathrm{N_{B0}}$	=	initial number of pixels within segment of algae;
$\mathrm{N_B}$	=	number of pixels within segment of algae;
n_x, n_y	=	number of the division of algae attached partition along the x-axis and y-axis;
Q	=	attachment rate;
r	=	radius of particle;
$\boldsymbol{T}$	=	torque;
Δt	=	short time interval;
$\boldsymbol{u}_p$	=	velocity of the particle;
$\boldsymbol{u}_r$	=	relative velocity;
v	=	particle velocity;
α	=	proportional coefficient related to species and growth of algae;
β	=	proportional coefficient;
γ	=	proportional coefficient;
δ_n , δ_t	=	particle displacements in the normal and tangential directions;
η	=	damping coefficient;
μ	=	viscosity of fluid;
μ_f	=	coefficient of friction;
ν	=	Poisson's ratio;
ρ_f	=	density of fluid;
ω_p	=	angular velocity;
ω_r	=	relative angular velocity;

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