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INVESTIGATION ON THE SCALE OF RAINFALL SPATIAL VARIABILITY TO BE CONSIDERED IN RUNOFF SIMULATION

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SYNOPSIS

A scale of rainfall spatial variability to be considered in a runoff simulation is investigated using mesh-type rainfall data and a distributed runoff model. First, a runoff simulation is conducted by means of a mesh-type rainfall data as it is, and the result is regarded as a standard result. In the next step, the rainfall data is averaged in some area and a runoff simulation is conducted by using this averaged rainfall data. If little difference is found between the standard result and the result obtained by using the averaged data, we do not have to explicitly consider the rainfall spatial variability in the area where the data is averaged, and so we can use the average rainfall data as an input of the runoff simulation. Conversely, if the difference is large, we have to consider the rainfall variability in the averaging area when conducting a runoff simulation. These numerical experiments indicate that it is important to find out a scale of rainfall spatial variability which affects the results of runoff simulations and the scale changes with the size of drainage area.

INTRODUCTION

To what extent is the scale of rainfall spatial variation to be considered in a runoff simulation? For example, to what extent is the spatial resolution of radar rainfall data which might be necessary to reproduce a discharge hydrograph for a basin of 1000 km^2? How many meters should we choose as the interval of the rain gauge in order to reproduce a discharge hydrograph in a small experimental basin? We do not yet have any clear answers to these questions. Also, we have developed a method to derive a lumped relationship between storage volume and outflow discharge of basin slope systems (Ichikawa et al. (1), (2), (4)). This method is based on the assumption that rainfall intensity is spatially constant, and should be applied to the extent that rainfall intensity can be regarded as a constant. Therefore we have to know how the extent is. In this study, a rainfall spatial variation scale to be considered in a runoff simulation is investigated using radar rainfall data and a distributed runoff model.

The methodology of the investigation is as follows. First, a runoff simulation is conducted by using a mesh-type rainfall data as it is, and the result is assumed to be a standard result. In the next step, the rainfall data is averaged in some area and a runoff simulation is conducted by using this averaged rainfall data. If little difference is found between the standard result and the result obtained by using the averaged data, we do not have to explicitly consider the rainfall spatial variation in the area where the data is averaged, and so we can use the average rainfall data as an input of the runoff model. Conversely, if the difference is large, we have to consider the rainfall variation in the averaging area when conducting a runoff simulation.

As a preliminary analysis, numerical experiments are conducted using mesh-type rainfall data generated by means of a random field generation computer program. Since this program can control statistical

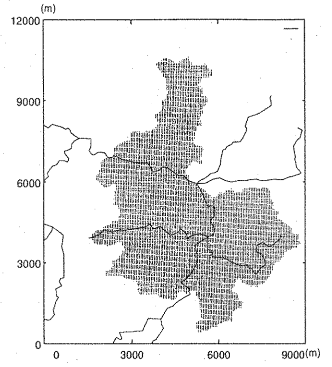

Fig. 1 Study area

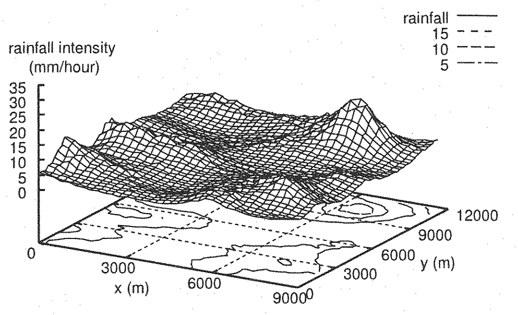

Fig. 2 Example of generated rainfall field ($\mu = 5.0$, $\delta = 0.5$, $\alpha = 1500.0$)

characteristics of generated rainfall data, we can investigate the relationship between the statistical characteristics and results of runoff simulations. When this analysis is completed, a similar analysis is conducted using observed radar rainfall data.

The distributed runoff model used in this study represents a hill slope system as a set of small rectangular planes (Shiiba et al. (5)), which are called 'slope units', and routes water flow from upstream slope units to downstream slope units by applying a kinematic wave model considering field capacity of soil layer to each slope unit (Ichikawa et al. (3)). These slope units are generated from a digital elevation data whose grid spacing is 50 m. The input data of this runoff model is a mesh-type rainfall data. When the model calculates the water flow of a slope unit, it uses a rainfall intensity of the mesh which overlaps with the slope unit.

ANALYSIS USING GENERATED RAINFALL DATA

In this chapter, numerical experiments are conducted in order to investigate the relationship among rainfall averaging scale, characteristics of rainfall field (average rainfall intensity, coefficient of variation, correlation length) and runoff simulation error. In this analysis, runoff simulations are conducted for slope systems of the area to be studied, but river flow routing is not conducted. The analysis that considers the effects of river flow is discussed in the next chapter.

Generating mesh-type rainfall data

In Fig. 1, the hatched area shows the area to be investigated (33 km^2) and the solid lines show a river network. This area is a part of Daido River basin (190 km^2), which is located to the south of Biwa Lake. The area is a mountainous region and is almost covered by forest trees. Mesh-type rainfall data are generated over the rectangular area (9000 m $\times$ 12000 m) which covers the study area. The rainfall data are generated using a random field generation program developed by Tachikawa and Shiiba (6). The size of each mesh is 250 m $\times$ 250 m. It means 36 $\times$ 48 mesh data is generated over the rectangular area. Each generated data has a log normal distribution and also has a spatial correlation expressed by the Gauss function.

There are three parameters to generate random mesh-type rainfall data, which are spatial average rainfall intensity, μ [mm/hour], coefficient of variation of rainfall intensity, δ, and correlation length, α [m]. α is the length where the spatial correlation coefficient is $1/e$ and can be regarded as a representative scale of spatial variation of rainfall field. Fig. 2 shows the generated rainfall data with $\mu = 5.0$, $\delta = 0.5$, and $\alpha = 1500.0$. Fig. 3 is a histogram of rainfall intensity of the generated data shown in Fig. 2. Fig. 3 indicates that the generated data has log normal distribution. Fig. 4 shows the spatial correlation coefficient of the generated data. There is a considerable difference between theoretical values (solid line) and calculated values (dots) in the range that the length is more than 2500m. This finding means that it is more difficult to generate data which has strong spatial correlation than to generate data which has

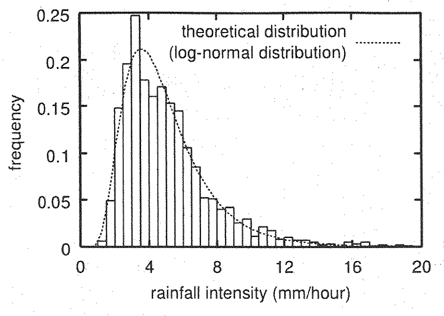

Fig. 3 Histogram of rainfall intensity

Table 1 Parameter values for rainfall field generation

case	μ (mm/hour)	δ	α(m)
1	5.0	1.0	500.0
2	5.0	1.0	1500.0
3	5.0	0.5	500.0
4	5.0	0.5	1500.0
5	10.0	1.0	500.0
6	10.0	1.0	1500.0
7	10.0	0.5	500.0
8	10.0	0.5	1500.0

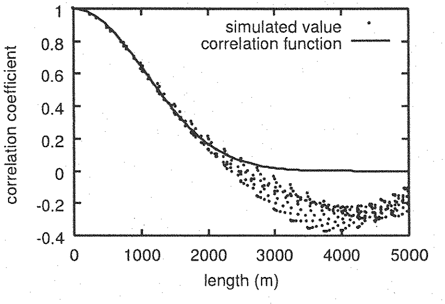

Fig. 4 Spatial correlation of generated rainfall field

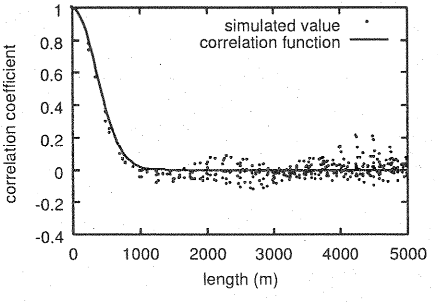

Fig. 5 Spatial correlation for $\alpha = 500$

weak one with the same variance. Fig. 5 shows the spatial correlation coefficient of a generated data with $\mu = 5.0$, $\delta = 0.5$, and $\alpha = 500.0$. Reducing α from 1500.0 to 500.0 makes the difference between theoretical and calculated values smaller. In this chapter the data which do not completely follow the theoretical structure of spatial correlation are used as they are.

The rainfall data are generated using the parameter values shown in Table 1. For each case, five data are generated in order to eliminate a chance factor. The data generated in this process are referred to as the data of level 1.

Averaging the rainfall field

The rainfall data of level 1 are spatially averaged over: 750m $\times$ 750m (referred to as level 2), 1500m $\times$ 1500m (level 3), 3000m $\times$ 3000m (level 4) and 9000m $\times$ 12000m (level 5). Each averaged data is multiplied by a constant so that the areal mean rainfall intensity in the area (Fig. 1) becomes same as the one calculated from the rainfall data of level 1.

Runoff simulation

Runoff simulations are conducted by using the rainfall field data generated by the procedure mentioned above. First, runoff simulations are conducted using the level 1 rainfall field data for each case shown in Table 1. For each case, five rainfall field data are generated and the runoff simulations are conducted five times using the generated data. The results of these simulations are considered as standard results for evaluating the simulation error due to rainfall averaging. Next, runoff simulations are conducted using the level 2 rainfall field data in the same way as the simulations using the level 1 data. Runoff simulations using the rest of the rainfall data (level 3, 4 and 5) are also conducted. The rainfall inputs are given for 80,000 seconds at a constant intensity and the simulation length is 120,000 seconds.

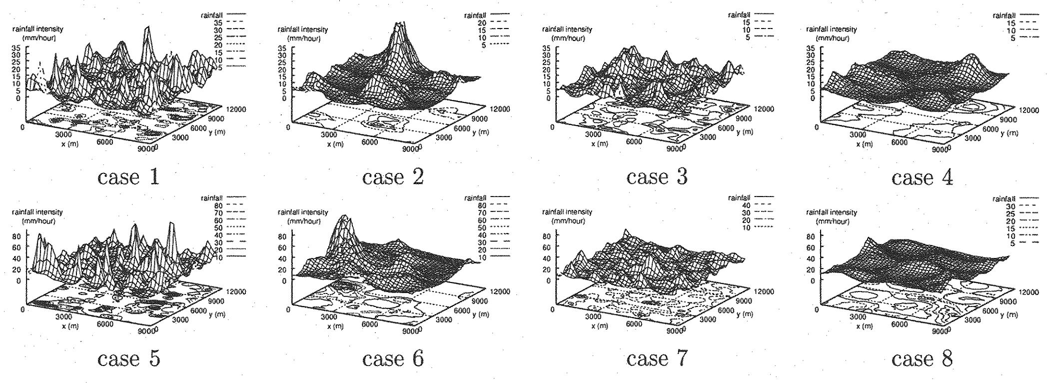

Fig. 6 Generated rainfall fields

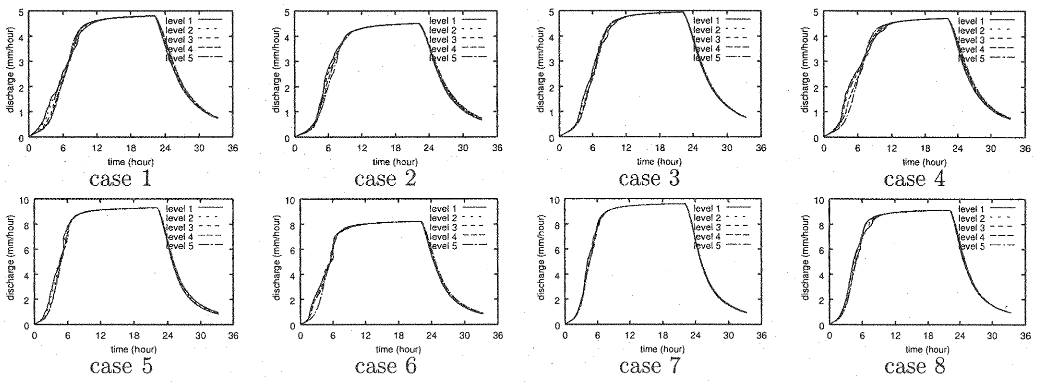

Fig. 7 Runoff simulation results

Evaluating runoff simulation error

Runoff simulation error ϵ_r is evaluated by comparing the simulation results of level 2 - 5 with the results of level 1, which is considered to be the standard result. ϵ_r is written as:

$$\epsilon_r = \frac{1}{5}\sum_{i=1}^{5}\left\{\sqrt{\frac{\sum_{j=1}^{N^i}(Q^i_{1,j} - Q^i_{k,j})^2}{N^i}}/R^i_m\right\} \quad (1)$$

where i shows a simulation number for each case, j shows calculation time, R^i_m is the average rainfall intensity of ith rainfall data, N^i is the number of calculation steps of the ith simulation result, $Q^i_{k,j}$ is runoff height at time j of the ith simulation result obtained by using level k data.

Simulation results and discussion

Fig. 6 shows the rainfall field of level 1 of each case, and Fig. 7 shows the runoff simulation results of each case.

Fig. 8 shows comparisons of ϵ_r between the case of μ (spatial average rainfall intensity) = 5.0 and 10.0. The vertical axis of the graphs represents the calculation error (ϵ_r) and the horizontal axis is the ratio of rainfall averaging scale (level 2: 750 m, level 3: 1500 m, level 4: 3000 m, and level 5: 9000 m) to the mesh size of the original rainfall data (250 m), plotted on a log scale. For all of the cases, the calculation errors become greater as the rainfall averaging area is extended. Moreover, there are no substantial differences of ϵ_r between the cases being compared, except for the result plotted on the upper right graph. Because ϵ_r is an index standardized by dividing by the rainfall intensity, the differences between the calculation results (differences of runoff height) are in proportion to the rainfall intensity.

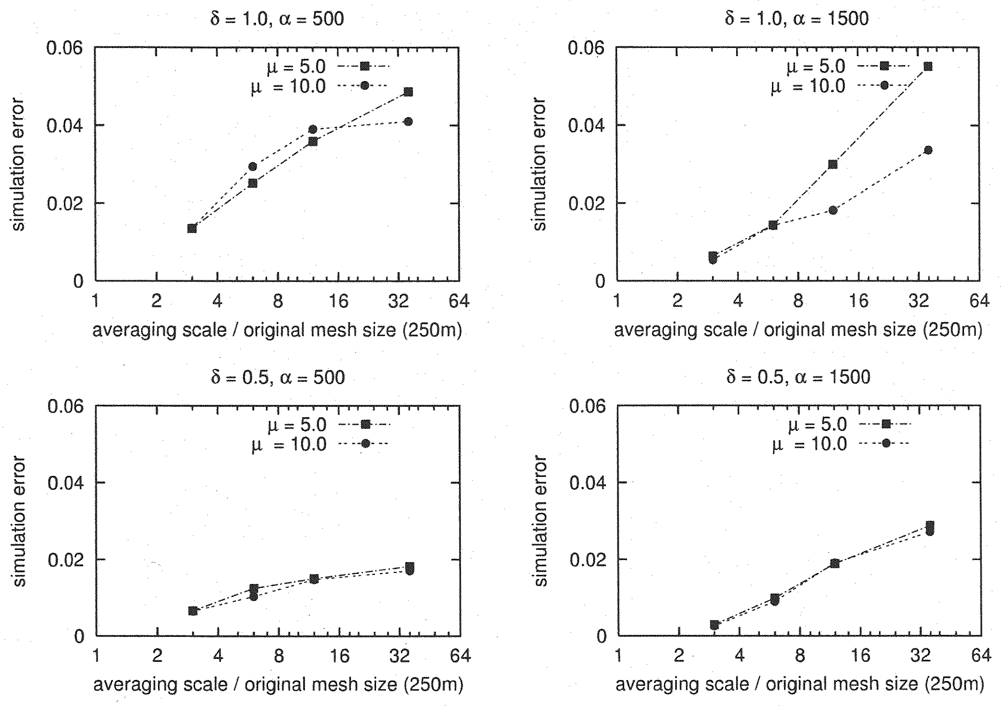

Fig. 8 Comparisons of ϵ_r between $\mu = 5.0$ and 10.0

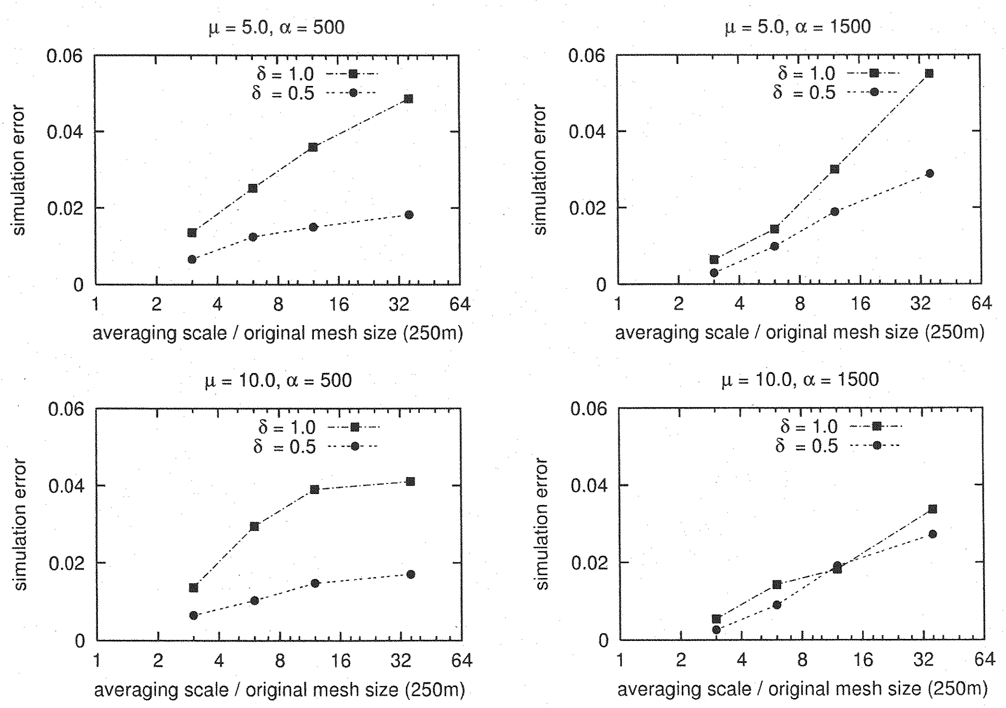

Fig. 9 Comparisons of ϵ_r between $\delta = 1.0$ and 0.5

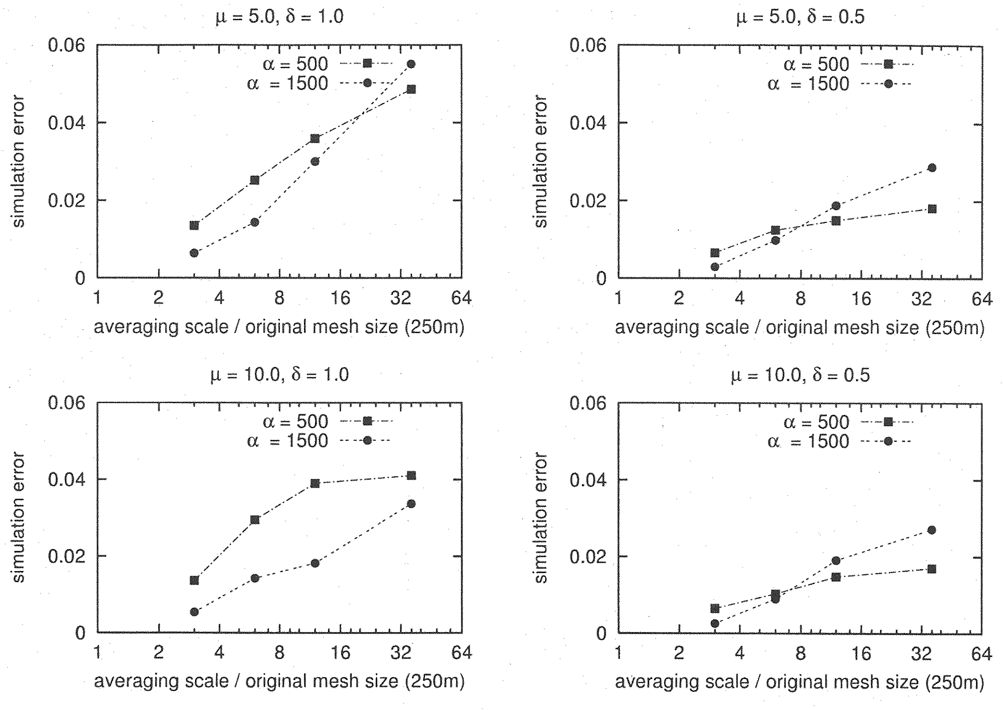

Fig. 10 Comparisons of ϵ_r between $\alpha = 500$ and 1500

Fig. 9 shows comparisons of ϵ_r between the case of δ (coefficient of variation of rainfall intensity)= 0.5 and 1.0. From this figure, it can be deduced that the calculation errors for the case of $\delta = 1.0$ are greater than the case of $\delta = 0.5$.

Fig. 10 shows comparisons of ϵ_r between the case of α (correlation length) = 500 and 1000. From this figure, it can be seen that the simulation errors for the case of $\alpha = 1500$ are smaller than those for the case of $\alpha = 500$ when the ratio of averaging scale to the original mesh size is small (level 2 and 3), while the errors for the case of $\alpha = 1500$ often become greater than those for the case of $\alpha = 500$ when the ratio is large (level 5 in the upper left graph; level 4 and 5 in the upper and lower right graphs). This finding suggests (1) that the simulation error due to spatial averaging of rainfall is small if the averaging scale does not exceed the spatial correlation length, which is a representative length of spatial variation of a rainfall field, while the error due to averaging increases considerably if the averaging scale exceeds the spatial correlation length, and (2) that it is necessary to find out a scale of rainfall spatial variability which affects the results of runoff simulations.

ANALYSIS USING OBSERVED RADAR RAINFALL DATA

In this chapter, numerical experiments using observed radar rainfall data are conducted in the same way as the previous chapter. A runoff simulation is conducted by using an observed rainfall radar data as it is, and the result is assumed to be a standard result. Then the observed radar data is spatially averaged (no temporal data processing is added), and runoff simulations are conducted by using the averaged radar data. By comparing these simulation results, the relationship among spatial averaging scale, size of drainage area and calculation error is examined. The model used in this analysis is basically the same as the one used in the analysis described in the previous chapter, but a river flow routing is also conducted in this analysis.

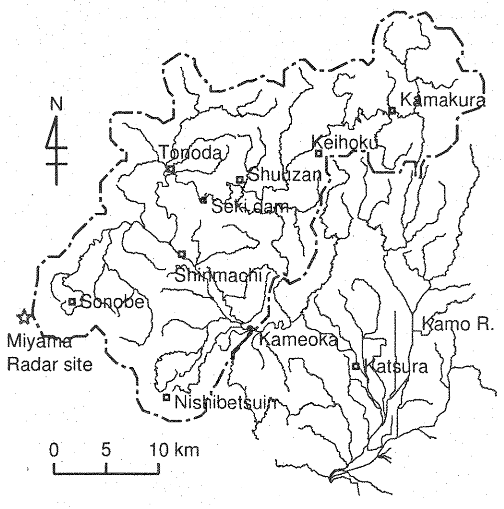

Fig. 11 Katsura River basin

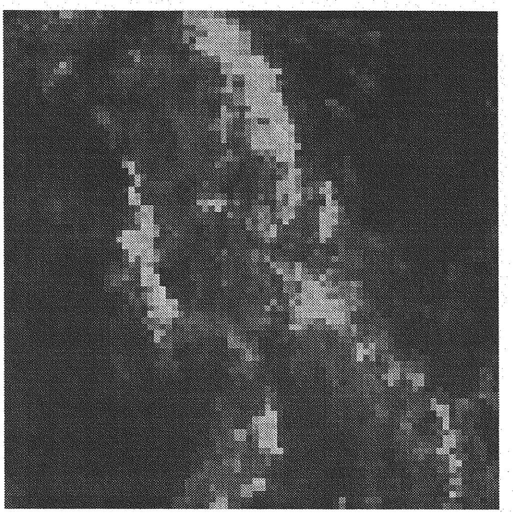

Fig. 12 Rainfall field observed by Miyama Radar

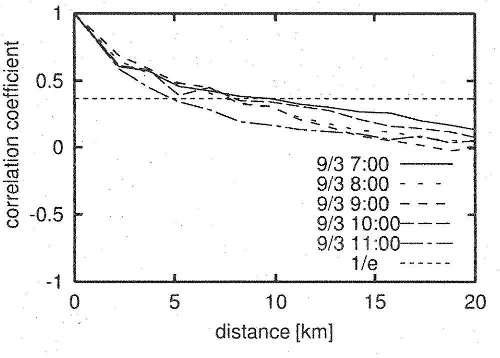

Fig. 13 Spatial correlation coefficients calculated from radar data

Study basin and radar data

The basin to be studied is Katsura River basin (710.2 km^2), which is shown in Fig. 11. The basin is located in a mountainous area, and the slope of the upper stream is about 0.01 and that of the lower stream is about 0.005. The analysis uses the data obtained by Miyama Radar, which was installed in the west of the basin. The period of analysis is from Sept. 2 to 5, 1989. Although the Hiyoshi dam reservoir is located in Katsura River basin, we did not take this dam reservoir into consideration in out analysis, because it was built in 1998, after the period of analysis. The radar data used in the analysis contains some missing observations from 18:45, Sept. 2 to 01:05, Sept. 3. However, we used this data for the following reasons: 1) this period had the most rainfall in 1989; 2) we could not obtain other appropriate data; and 3) the main purpose of this study was not to reproduce past floods exactly but to investigate the effects of rainfall spatial variation on runoff phenomena. Runoff simulations were conducted with the data whose missing part was filled up by linear interpolation. The radar reflectivity (Z) was converted to rainfall intensity (R) by Z-R relationship ($Z = BR^{\beta}$; $B = 80$, $\beta = 1.7$). Furthermore, the rainfall intensity was calibrated by means of data obtained by several rain gauges. The spatial resolution and time interval of the radar data was 3 km and 5 minutes, respectively. Fig. 12 shows the rainfall field observed by Miyama Radar at 09:05, Sept. 3 (black: no rainfall, white: high rainfall intensity). Fig. 13 is a plot of spatial correlation coefficients of rainfall fields calculated from the radar data observed at 07:00 to 11:00, Sept. 3, and shows that correlation length of the rainfall field is 5 $\sim$ 10 km.

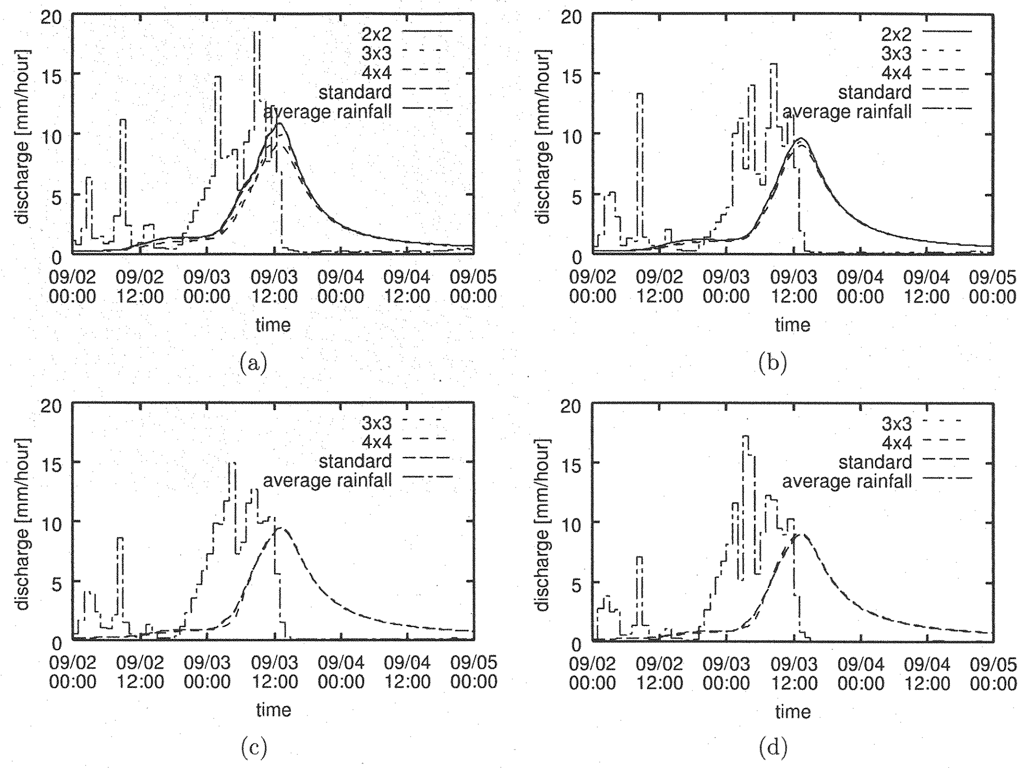

Fig. 14 Simulated hydrographs obtained at (a) a drainage area of 74.5 km^2, (b) a drainage area of 278.4 km^2, (c) a drainage area of 582.0 km^2, and (d) a drainage area of 710.2 km^2

Results and discussions

The simulation results are shown in Fig. 14. The lines labeled as "standard" in the figures show the standard results, that is, the simulation results obtained by using the observed radar data as it is. The lines labeled as "2×2", "3×3" and "4×4" show the simulation results using the radar data averaged over (A) 2 × 2 radar meshes (6 km × 6 km), (B) 3 × 3 radar meshes (9 km × 9 km) and (C) 4 × 4 radar meshes (12 km × 12 km), respectively.

From these results, the runoff calculation error due to spatial averaging of the rainfall data, ϵ, is evaluated as:

$$\epsilon = \{\int_{t_s}^{t_e} |Q(t) - Q_s(t)|dt\}/\{\int_{t_s}^{t_e} Q_s(t)dt\} \qquad (2)$$

where t_s is integration starting time, t_e is integration ending time, $Q_s(t)$ is discharge calculated by using the observed radar data, and $Q(t)$ is discharge calculated by using the averaged radar data.

Fig. 15 shows the relation between ϵ and the drainage area. An increase of the drainage area reduces the value of ϵ and extending averaging area increases the value of ϵ. The curved lines in this figure are the envelope curves of the calculation errors ((A): broken line, (B): dotted line, (C): solid line). The envelope curves are given by

$$\epsilon = \exp(ax^c + b) \qquad (3)$$

where x is the drainage area (km^2), a, b and c are constants. In Fig. 15, the straight line is also drawn to denote $\epsilon = 0.05$, which we considered to be a tolerable level. The drainage areas where this line intersects with the envelope curves are (A): 52 km^2, (B): 202 km^2 and (C): 465 km^2. For case (A), in which the radar data is averaged over 6 km × 6 km, the minimum size of drainage area in which the error of runoff

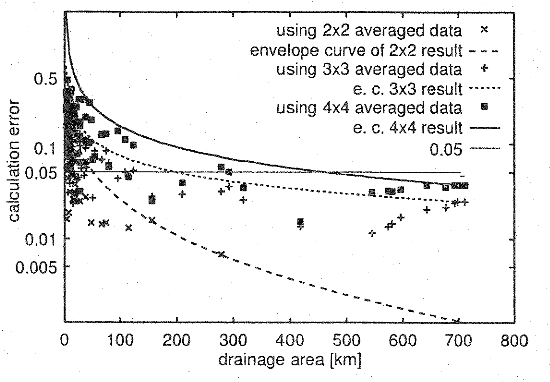

Fig. 15 Calculation error and drainage area

simulation can be regarded as relatively small is about 50 km^2. This means that the radar resolution of 6 km is required in order to precisely simulate outflow discharge from about 50 km^2 drainage area. Similarly, from case (B) and (C), it can be deduced that the radar resolutions of 9 km and 12 km are required in order to obtain precise discharge at the outlets of drainage areas of 200 km^2 and 450 km^2, respectively. These experiments indicate the importance to find out a scale of rainfall spatial variability which affects the accuracy of runoff simulations. Furthermore, they imply that the scale varies with the size of drainage area.

The analyses conducted in this study, however, do not have universal applications because they are based on one rainfall event of one river basin, and implicitly assume that the spatial variability of rainfall field is sufficiently captured by the radar system used in this study (spatial resolution: 3 km). Further investigations based on various rainfall events and river basins are necessary.

CONCLUSION

The focus of this study was to investigate a rainfall spatial variation scale to be considered in a runoff simulation by using mesh-type rainfall data and a distributed runoff model. As a preliminary analysis, numerical experiments were conducted using generated rainfall data in order to investigate a relationship among rainfall averaging scale, characteristics of rainfall field and runoff simulation error. The results of our analyses showed (1) that the runoff simulation error is in proportion to the rainfall intensity, (2) that the error becomes greater as the coefficient of variation of rainfall increases, and (3) that it is important to find out a scale of rainfall spatial variability which affects the results of runoff simulations. Next, numerical experiments using observed radar rainfall data were conducted in the same way as the preliminary analysis. These experiments indicated the importance to find out a scale of rainfall spatial variability which affects the accuracy of runoff simulations, which was already pointed out in the previous analysis. Furthermore, they implied that the scale changes with the size of drainage area.

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